Analysis of Population Dynamics Between Trees and Humans

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Contents

1 Abstract

One of the most salient environmental trends in the past half-century is the rapid decline of global forest coverage in conjunction with a rapid rise of human population. Here we proposed and analyzed a nonlinear mathematical model to study the relationship between forest coverage and human population. Based on the changes in human population and forest coverage over time, we identified two drivers of deforestation: the natural death rate of trees and human consumption of trees. There was evidence of an inverse relationship between the human population and forest coverage, and our initial model was built accordingly. We then modified the model by adding seasonal dynamics and carrying capacity. Through numerical simulations conducted from the model, we used the results to predict the human population and forest coverage for the next 75 years. An analysis of our model showed that as human population increases, total forest coverage decreases, resulting in an unstable relationship that eventually leads to Earth's forests reaching an indelible state from which they cannot recover. To stabilize this unstable relationship, we proposed two optimal solutions, the first of which was a direct method that put a quota on trees cut per year, and the second of which was an indirect method that implemented a one-child policy.

2 Introduction

Our ecosystems are abundant with a plethora of unique relationships between different species. One such unique relationship is that which that exists between humans and the myriad of plant species that comprise the Earth's forests. Both humans and the Earth's forests experience positive and negative impacts on their own populations brought on by the other. One particular case of this is the act of deforestation, which is the process of clearing our earth of its forests on a massive scale, resulting in habitat loss and damage to the land's quality. Over the past two decades, the issue of deforestation has become one of the most prominent social and economic problems worldwide. Rising human populations tend to put a great deal of pressure on natural resources, particularly in the case of forests. This is primarily due to the growth of infrastructure, agricultural expansion, logging, and other human activities, resulting in the global loss of over 120,000 square kilometers of forest coverage every year [2].

In this paper, we analyze the relationship between the rise in the human population and the decrease in the forest area. A nonlinear mathematical model was proposed and analyzed through the study of depletion in forest areas caused by population growth. A continuous-time model was considered for this behavioral relationship—including various constant parameters. The main purpose at the beginning of this paper is to dissect these parameters and understand how they affect the population count and change overall forest coverage. The latter is affected by natural disasters and human interactions with the forest. Human interaction with forests is seen through two primary actions: (1) Cutting trees for agriculture, infrastructure, materials, etc. (2) Planting trees to counterbalance the effects of deforestation.

More precisely, we suggested a model for deforestation due to the increase in human population and natural factors. This model was described by a system of ordinary differential equations with the following two variables: human population and forest area. Furthermore, we modified the model with the aim of increasing realism by implementing a number of new parameters that allowed us account for seasonal dynamics and forest carrying capacity. The model was then analyzed through equilibrium points, using stability theory of differential equations and from performing numerical simulations. From our analysis, we concluded that the rapid shrinkage of global forest coverage required immediate intervention, and that the current unsustainable rate of deforestation will almost certainly lead to insurmountable future problems.

3 Variables, Parameters and Model Equation

Prior to the creation of our mathematical model, several initial assumptions must first be asserted. In order to ascertain the nature of these assumptions, we first collected data on global forest coverage and human population from The World Bank, a non-profit organization with an open database on a variety of different global statistics [1][2]. After quick analysis of this data, it becomes clear that there is a relatively linear relation between our two populations in that as the total human population increases, forest population decreases.

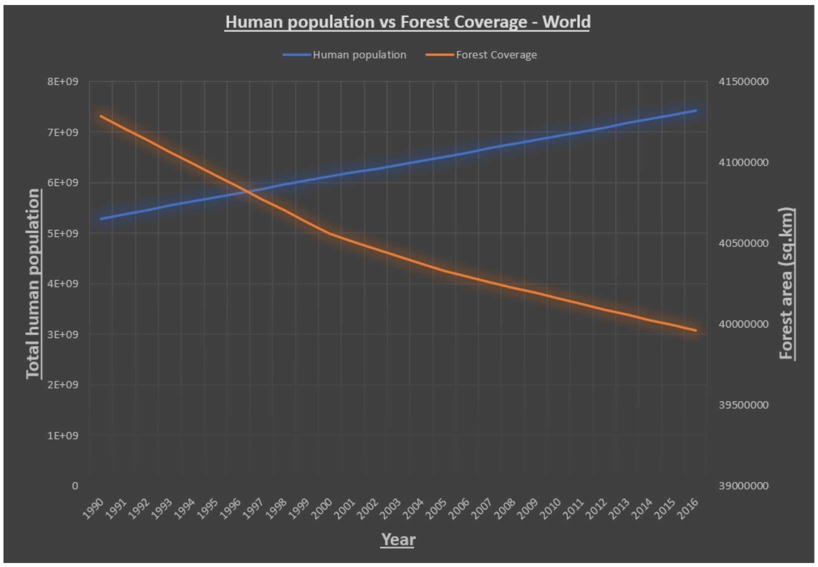


Figure 1: Global Human Population vs Global Forest Coverage
[1][2]

This negative effect that an increasing human population has on forest coverage leaves us to make some assumptions. First, notice the negative slope of the graph for forest coverage decreases in magnitude in the year 2000 (a period of time widely considered to be the dawn of the environmentalism movement) yet the slope of the graph for human population remains constant. This leaves us to assume that humans must have some positive effect on forest coverage along with the clear negative impact they impose. So we can let b be the number of trees consumed per human and c be the number of trees produced per human with $b > c$.

Along with this it is safe to assume that trees will die off naturally without influence from human interaction. Likewise trees will naturally reproduce. So we can let n be the natural death rate of trees and l be the natural birthrate of trees. Humans of course, must also have a natural growth in their population, and thus we will denote r to be the natural growth rate of the human population.

Since our data shows that human populations rapidly grow as they consume trees for agricultural or industrial purposes, our model assumes that humans must benefit from the consumption of trees. Since there is a limited amount of resources it must also follow that as more trees are consumed, the less humans can benefit which would lead to a decrease in human growth. Thus we can let α be the number of humans gained from tree consumed and γ be the number of humans lost per tree lost with $\alpha > \gamma$.

Note that because humans do not have infinite resources to work with, it must also follow that humans must have a carrying capacity for their population, which we will denote as K_H.

To summarise we present the following table,

Parameter	
Variable Name	Meaning
b	number of trees used per human(km^2/$person$)
c	number of trees planted per human(km^2/$person$)
n	natural death rate of trees
l	natural birth rate of trees
α	number of human gained per tree(km^2) consumed
γ	number of human loss per tree(km^2)$loss$
r	natural human growth rate
K_H	human capacity

Table 1: Initial parameters

Using this we can begin to construct our mathematical model. First we will initialize our variables.

Let F be the global forest coverage in (km^2).

Let H be the number of people alive.

Then using our assumptions and parameters we can formulate the following continuous time model,

$$\begin{aligned} \frac{dF}{dt} &= (c-b)H - (n-l)F \\ \frac{dH}{dt} &= r\frac{K_H - H}{K_H}H + \alpha bH + \gamma[(l-n)F + (c-b)H]H \end{aligned} \tag{1}$$

Parameter		
Variable Name	Base value	Reference
H	5281340078	#1(Data set)
F	41282694.9	#1(Data set)
b	0.00000651	#1(Data set)
c	0.00000044	#1(Data set), #4(deforestation usage)
n	0.0006	Assumed value
l	0.0002	Assumed value
α	1250.0	Assumed value
γ	30.0	Assumed value
r	0.0126	#1(Data set)
K_H	10000000000	#4(human capacity)

Table 2: Initial parameters and base values

Using python as a tool to simulate our initial model in conjunction with the actual data, we were able to produce the following:

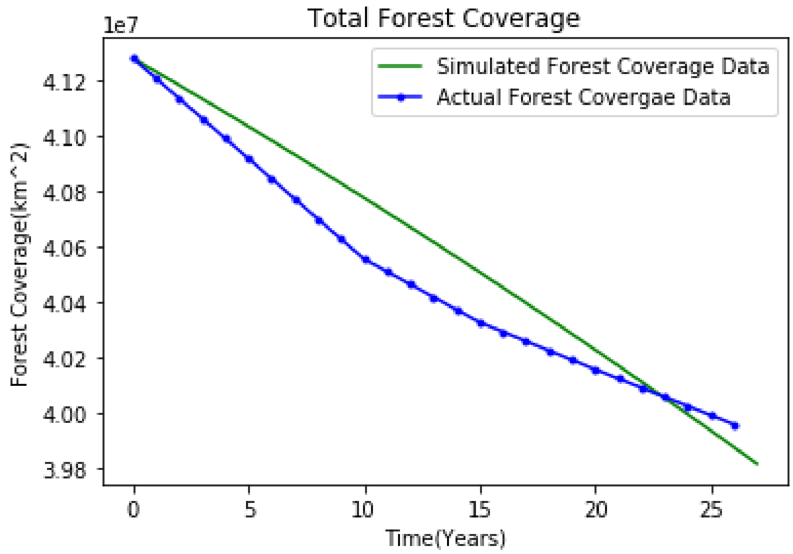


Figure 2: Simulated Model Vs. Real Data For Forest Coverage

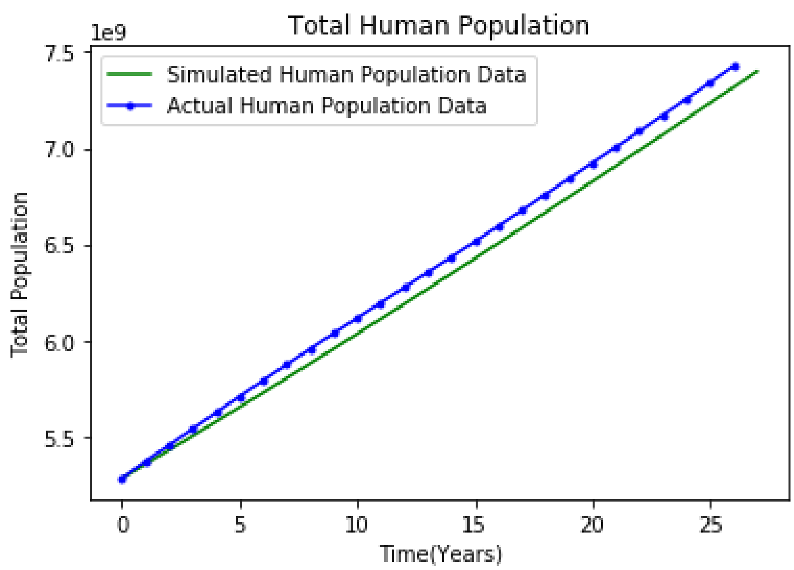


Figure 3: Simulated Model Vs. Real Data For Human Population

4 Model Modifications

In order to make the model more realistic, the annual growth rate of trees should be adjusted. By using the sine function, the growth rate of trees can show seasonal dynamics with an annual periodicity. We can do this by asserting the following., $I = \frac{|sin(h)|}{1200}$,where h represents a quarter of one year and I is the annual growth rate of trees. We divide our sin function by 1200 to keep our growth rate consistent with our data.

A carrying capacity for forest coverage should also be implemented. It should follow that the carrying capacity of trees should be affected by the environment including the deforestation rate. So let K_F be the new carrying capacity for forest coverage. Including this into our model, our equation then becomes,

$$\begin{aligned} \frac{dF}{dt} &= (c-b)H + I(\frac{K_F - F}{K_F})F \\ \frac{dH}{dt} &= r(\frac{K_H - H}{K_H})H + \alpha bH + \gamma[(l-n)F + (c-b)H] \end{aligned} \tag{2}$$

Where K_F represents the carrying capacity of forest coverage and K_H is the carrying capacity of people.

Parameter		
Variable Name	Base value	Reference
H	5281340078	#1(Data set)
F	41282694.9	#2(Data set)
b	0.00000651	#1(Data set), #4(deforestation usage)
c	0.00000044	#1(Data set),
n	0.0006	Assumed value
l	0.0002	Assumed value
α	1250.0	Assumed value
γ	30.0	Assumed value
r	0.0126	#1(Data set)
I	0.000590	#2(Imperical Form)
K_H	10000000000	#4(human capacity)
K_F	44000000	#5(forest capacity)

Table 3: Initial parameter value for new model

By taking our modified model and running a new simulation using python we obtained the following graphs:

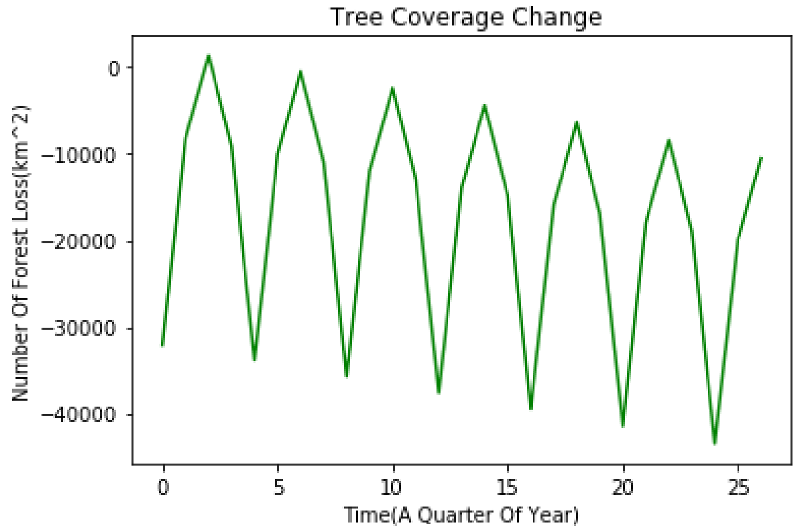


Figure 4: Seasonal Dynamic Of Growth Rate Of Forest

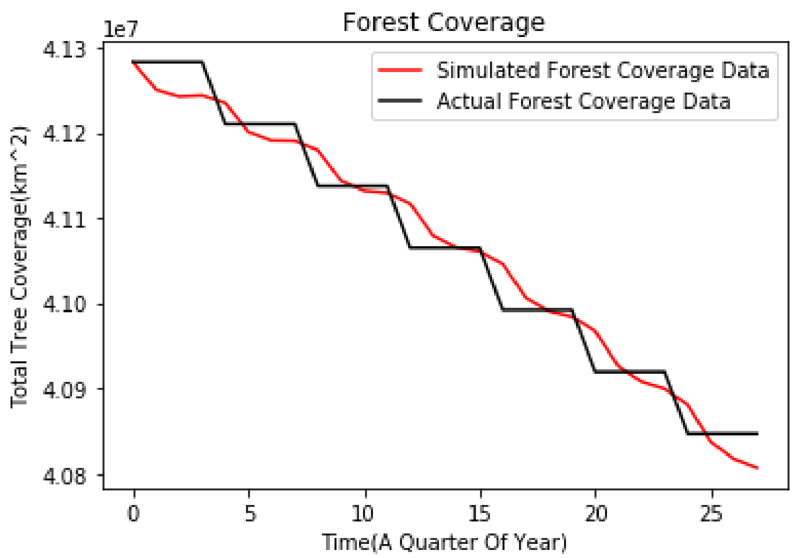


Figure 5: Simulated Model Vs. Real Data For Forest Coverage

5 Analysis

5.1 Equilibrium point

To further observe properties of this system we will start by implementing a stability analysis.

To begin, let $\frac{dF}{dt} = 0$ and $\frac{dH}{dt} = 0$

Putting our model equations in terms of our state variables we get the following:

$$\begin{aligned} F &= \frac{1}{\gamma(n-l)}[r - \frac{rH}{K_H} + \alpha b + \gamma(c-b)]H \\ H &= \frac{I}{c-b}(\frac{F-K_F}{K_F})F \end{aligned} \tag{3}$$

The above equations need to be simplified, so to do this we define the following...

$$\begin{aligned} m &\equiv \gamma(n-l) \\ P &\equiv r + \alpha b + \gamma(c-b) \\ Q &\equiv \frac{I}{c-b} \end{aligned} \tag{4}$$

Then the equations are simplified to be:

$$\begin{aligned} F &= \frac{H}{m}(P - \frac{rH}{K_H}) = -\frac{r}{mK_H}H^2 + \frac{P}{m}H \\ H &= QF(\frac{F-K_F}{K_F}) = \frac{Q}{K_1}F^2 – QF \end{aligned} \tag{5}$$

The equilibrium points are the interceptions of the graphs of F and H. However, both the population of humans and forest coverage should be greater than zero. Then

$$\begin{aligned} F &= H(\frac{P}{m} - \frac{r}{mK_H}H) = 0 \\ H &= F(\frac{QF}{K_F} - Q) = 0 \end{aligned} \tag{6}$$

Since both F and H are not zero, then:

$$\begin{aligned} \frac{P}{m} - \frac{r}{mK_H}F &= 0 \\ \frac{QF}{K_F} - Q &= 0 \end{aligned} \tag{7}$$

Then the equilibrium points are:

$$\begin{aligned} F_* &= K_F \\ H_* &= \frac{PK_H}{r} \end{aligned} \tag{8}$$

5.2 Stability analysis

In order to find the stability at the equilibrium point, the continuous time model should be written in it's matrix form. So taking,

$$\begin{aligned}\frac{dF}{dt} &= -mF + PH - \frac{r}{K_H}H^2 \\ \frac{dH}{dt} &= IF - \frac{I}{K_F}F^2 + (c-b)H\end{aligned} \tag{9}$$

Where

$$\begin{aligned}P &= \gamma + \alpha b + \gamma(c-b) \\ I &= \frac{|sin(h)|}{1200} \\ m &= \gamma(n-l)\end{aligned} \tag{10}$$

We convert this into matrix form:

$$\begin{bmatrix} F' \\ H' \end{bmatrix} = \begin{bmatrix} -m & P - \frac{rH}{K_H} \\ I - \frac{IF}{K_F} & c-b \end{bmatrix} \cdot \begin{bmatrix} F \\ H \end{bmatrix} \tag{11}$$

So define:

$$M = \begin{bmatrix} -m & P - \frac{rH}{K_H} \\ I - \frac{IF}{K_F} & c-b \end{bmatrix} \tag{12}$$

In order to determine the general structure of the solution sets of F and H, the eigenvalues of the row reduced echelon form of the matrix M should be calculated and we expect that the sign of the eigenvalues should be determined by the parameters. The row reduced echelon form of the matrix M is the following:

$$M = \begin{bmatrix} I - \frac{IF}{K_H} & c-b \\ 0 & \frac{PI}{K_F} - \frac{rHI}{K_F} + \frac{m(c-b)}{K_F - F} \end{bmatrix} \tag{13}$$

Then the eigenvalues are:

$$\begin{aligned}\lambda_1 &= I - \frac{IF}{K_F} \\ \lambda_2 &= \frac{PI}{K_F} - \frac{rHI}{K_F} + \frac{m(c-b)}{K_F - F}\end{aligned} \tag{14}$$

The sign of λ_1 is determined by the ratio between the population of trees and its carrying capacity.

At the equilibrium points, matrix M becomes:

$$M(F_*, H_*) = \begin{bmatrix} -m & 0 \\ 0 & c-b \end{bmatrix} \tag{15}$$

This is a diagonal matrix and the eigenvalues are:

$$\begin{aligned}\lambda_1 &= -m \\ \lambda_2 &= c-b\end{aligned} \tag{16}$$

Notice that, $\lambda_1 = -m = \gamma(l-n)$. Then the sign of $-m$ is determined by the birth rate and death rate of trees at the equilibrium point since γ is strictly positive. Likewise for $\lambda_2 = c-b$ the sign of this eigenvalue is dependent on the number of trees planted and the number of trees consumed per human. However, the values of the parameters in the table 3 can not be used since the forest coverage cannot reach the carrying capacity. So now, the differential equations can be linearized at their equilibrium point as:

$$\begin{bmatrix} F' - F_* \\ H' - H_* \end{bmatrix} = \begin{bmatrix} -m & 0 \\ 0 & c-b \end{bmatrix} \cdot \begin{bmatrix} F - F_* \\ H - H_* \end{bmatrix} \tag{17}$$

Since both the derivative of F and H are negative, then at the equilibrium point, both the population of human and trees should decrease.

5.3 Limits and constraints

In this section, we will show there is no upper limit for the total population of Human and Forest even though both of them have the carrying capacity. The proof is as follows...

We will be analyzing the limits of our state variables H(t) and F(t) as t tends to infinity. To do this we will first define a function $G(F, H, t)$ such that:

$$G(F, H, t) = \frac{dF}{dt} + \frac{dH}{dt} = (c - b + P)H + (I - m)F - (\frac{rH^2}{K_H} + \frac{IF^2}{K_F}) \tag{18}$$

Rewrite the left side of the equation as a function of $(H + F)$. Then: $G = \phi_2(H + F) - \phi_1(H^2 + F^2)$.
Where:

$$\begin{aligned} \phi_1 &= max\{\frac{r}{K_H}, \frac{I}{K_F}\} \\ \phi_2 &= min\{c - b + P, I - m\} \end{aligned} \tag{19}$$

Then take the limit of time t on both side:

$$\lim_{t\to+\infty} G = \lim_{t\to+\infty} [\phi_2(H + F) - \phi_1(H^2 + F^2)] \tag{20}$$

As time t approaches infinity, the limit of I can be determined:

$$\lim_{t\to+\infty} I = \lim_{t\to+\infty} \frac{|sin(h)|}{1200} \tag{21}$$

So, the constraint for I is:

$$I \in [0, \frac{1}{1200}]$$

Then the set of ϕ_1 and ϕ_2 becomes:

$$\begin{aligned} \phi_1 &= max\{\frac{r}{K_H}, \frac{1}{1200K_F}\} \\ \phi_2 &= min\{c - b + P, 1 - m\} \end{aligned} \tag{22}$$

But, $\lim_{t\to+\infty}[\phi_2 H - \phi_1 H^2 + \phi_2 F - \phi_1 F^2]$ does not have a minimum value since both of the function of F and H concave down. Then, there is no upper limit for the total number of trees and human. However, the lower limit for F and H should both be greater than zero.

For any planer systems, there exits a Hamiltonian for that system [7]. By this fact, we can determine some of the relations between parameters. First, we define a function $\bar{H} = \bar{H}(F, H)$.By letting

$$\begin{aligned} F' &= \frac{\partial \bar{H}(F,H)}{\partial H} \\ H' &= \frac{\partial \bar{H}(F,H)}{\partial F} \end{aligned} \tag{23}$$

Integrate $\partial \bar{H}(F, H)$ with respect to H, then we have the following:

$$\int \partial \bar{H}(F,H) = \int (-mF + PH - \frac{rH^2}{K_H})dH$$

This is an indefinite integral, and there should be a function of F after the integration.So, we have

$$\bar{H}(F,H) = -mHF + \frac{PH^2}{2} - \frac{rH^3}{3K_H} + \psi_1(F)$$

Take the derivative of the equation above with respect to F, we have:

$$\frac{\partial \bar{H}(F,H)}{\partial F} = -mH + \psi_1'(F)$$

But,

$$\frac{\partial \bar{H}(F,H)}{\partial F} = (b-c)H + \frac{IF^2}{K_F} - IF$$

Then,

$$-mH + \psi_1'(F) = (b-c)H + \frac{IF^2}{K_F} - IF$$

So,

$$\psi_1'(F) = \frac{IF^2}{K_F} - IF$$

Then

$$\psi_1(F) = \frac{IF^3}{3K_F} - \frac{IF^2}{2} + CNST$$

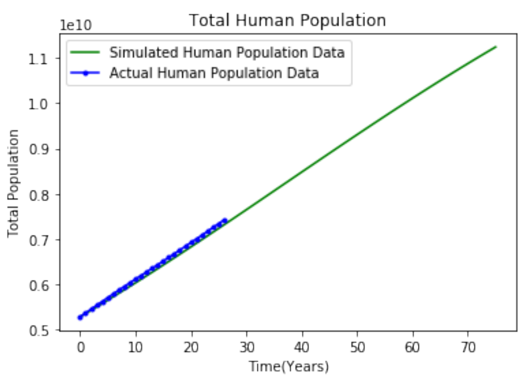
Also, if there exists a Hamiltonian for the system, then $m = c - b$ n the growth rate is fixed and the system becomes autonomous

For example, if we take any specific season in a year, then the growth rate is fixed and the system becomes autonomous. According to the result from above, m should be equal to c-b. Also, $m = \gamma(n-l) = (c-b)$ which means $\gamma = \frac{c-b}{n-l}$. This equality means that the humans loss due to tree population depletion is determined by the growth rate of trees both naturally and influenced by humans.

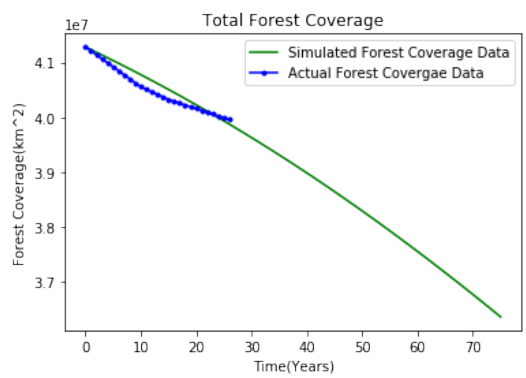
6 Conclusion

In this paper, we create and solve a mathematical model based off information collected from the World Bank from the years 1990-2016. The primary goal for doing this was to be able to utilize the aforementioned model to analyze the relationship between forest coverage and human population, as well as generating a prediction about the future trends of both variables. After extracting data on total human population and forest coverage in sq. km for every single country from the World Bank database, we noticed some clear trends, namely that there was a nearly-linear inverse relationship between forest coverage and human population. Based on these trends, we assumed that change in total forest area depended on the average net natural growth rate of forests as well as the average net human effect on forests. We also assumed that the change in total human population was dependant on the net natural growth rate of humans and the average net forest effect on humans. Implementing these assumptions resulted in a continuous-time model with a linear equation for the change in total forest area and a non-linear equation for the change in total human population. The non-linearity of the second equation was a result of its carrying capacity component.

Our initial model for human population growth compared quite favorably to the actual data that we collected from the world bank. The initial model for total forest coverage also compared quite well to the actual data for forest coverage, with the exception of a decrease in magnitude of the slope for actual data relative to simulated data circa 1999. In order to make our model more accurate, we adjusted it so that seasonal dynamics were included. This was done by multiplying the net natural forest growth rate by a factor I, which represented the general seasonal dynamics of forests. We also added a carrying capacity for trees since the idea of an infinite amount of trees growing in a fixed area isn't very realistic. Using our new model to predict the future global human population and total forest coverage respectively resulted in the following two graphs:



(a) Fig 6. Human population projection for the next 75 years



(b) Fig 7. Forest coverage projection for the next 75 years

After conducting an analysis of our adjusted model, we concluded that there was nearly linear negative correlation between forest coverage and human population. This model could potentially be used to simulate the future population/forest coverage of any given country in the world, thus allowing the relevant organizations to utilize the correct optimal controls in order to achieve the desired annual forest coverage change. In order to stabilize the unstable relationship between forest coverage and human population, we proposed two optimal solutions, the first of which was a direct method that put a quota on trees cut per year, and the second of which was an indirect method that implemented a one-child policy. Both the former and the latter would decrease c, the human consumption of the forest as well as increase the natural forest growth rate, resulting in more total forest coverage. As future work, we intend to adjust the model further by simulating the effects of human intervention for deforestation, as well as the effects of natural disasters and socioeconomic factors on forest coverage.

7 References

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8 Appendix

8.1 Code for old model in python

```
from pylab import *
from math import*
import numpy as np
import matplotlib.pyplot as plt

##########################Old equation#########################
#parameter
#Ht = #number of human
#Ft = #number of trees
#b = #number of trees used per human(km^2/person)
#c = #number of trees planted per human(km^2/person)
#n = #natural death rate of trees
#l = #natural birth rate of trees
#alpha = #number of human gained per tree(km^2) consumed
#gamma = #number of human loss per tree(km^2) loss
#r = Malthusian parameter
#K_H = human capacity
b = 0.00000651 #3
c = 0.00000044 #3
n = 0.0006
l = 0.0002
alpha = 1250
gamma = 30
run_time = 27
K_H = 10000000000 #2
r = 0.0126
t=[1990,1991,1992,1993,1994]
ActDataF = [41282694.9, 41210027.5, 41137360.5, 41064693.4, 40992025.9, 40919358.7,  40846691.3, 407740
ActDataH = [5281340078, 5369210095, 5453393960, 5538448726, 5622575421,5707533023, 5790454220, 58730717


def initialization():
    global Ht, Ft, HtResult, FtResult
    Ht = 5281340078 #1
    Ft = 41282694.9 #1
    HtResult = [Ht]
    FtResult = [Ft]

def observe():
    global Ht, Ft, HtResult, FtResult
    HtResult.append(Ht)
    FtResult.append(Ft)

def update():
    global Ht, Ft, HtResult, FtResult
    nextFt = c*Ht - b*Ht - n*Ft +l*Ft  + Ft
    nextHt = r*(K_H-Ht)/K_H*Ht+alpha*b*Ht + gamma*((l-n)*Ft + (c-b)*Ht) + Ht
    Ht = nextHt
    Ft = nextFt
```

```
initialization()
for t in range(run_time):
    update()
    observe()

plt.plot(HtResult, color='green', label = "Simulate Data for human")
plt.plot(ActDataH, c = 'blue', marker = ".", label = "Actual Data for human")
plt.title('Total human population')
plt.xlabel('time')
plt.ylabel('number of people in line');
plt.legend()
show()

plt.plot(FtResult, color='green',label= "Simulate Data for tree")
plt.plot(ActDataF, c = 'blue', marker = ".",label = "Actual Data for tree")
plt.title('Total tree coverage')
plt.xlabel('time')
plt.ylabel('number of trees(km^2)');
plt.legend()
show()

plot(HtResult, FtResult)
plt.title('Comparsion beteen tree coverage and total population')
plt.xlabel('Total population')
plt.ylabel('number of trees(km^2)');
show()
```

8.2 Code for new model in python

```
from pylab import *
from math import*
import matplotlib.pyplot as plt

########################new equation######################
#parameter
#Ht = #number of human
#Ft = #number of trees
#b = #number of trees used per human(km^2/person)
#c = #number of trees planted per human(km^2/person)
#n = #natural death rate of trees
#l = #natural birth rate of trees
#alpha = #number of human gained per tree consumed
#gamma = #number of human loss per tree loss
#r = Malthusian parameter
#K_H = human capacity
#
b = 0.00000651
c = 0.00000044
l = 0.0000001013
n = 0.0000003036
alpha = 1250
```

```
gamma = 30
run_time = 27
K_H = 10000000000
K_F = 44000000.0
r = 0.0126
ActDataF = [41282694.9, 41210027.5, 41137360.5, 41064693.4, 40992025.9, 40919358.7,  40846691.3, 407740
ActDataF2 = [41282694.9, 41282694.9,41282694.9,41282694.9,41210027.5, 41210027.5,41210027.5,41210027.5,
ActDataH = [5281340078, 5369210095, 5453393960, 5538448726, 5622575421,5707533023, 5790454220, 587307170



def initialization():
    global Ht, Ft, HrResult,FrResult,Hr, Fr, t, HtResult, FtResult,Rresult, ROF, h
    Ht = 5281340078.0
    Ft = 41282694.9
    Hr = 0
    Fr = 0
    HrResult = []
    FrResult = []
    HtResult = [Ht]
    FtResult = [Ft]
    Rresult= []
    ROF = 0
    h = 0


def observe():
    global Ht, Ft, HrResult, FrResult, Hr, Fr, t, HtResult, FtResult,Rresult, ROF, h
    HrResult.append(Hr)
    FrResult.append(Fr)
    HtResult.append(Ht)
    FtResult.append(Ft)
    Rresult.append(ROF)

def update():
    global Ht, Ft, HrResult, FrResult, Hr, Fr, t, HtResult, FtResult, ROF, h
   # ROF = (1.0 - (1.0-(ActData[t-1]-Ft)/ActData[t-1])**(1/h))
    nextFt = c*Ht - b*Ht + 1/1200*abs(math.sin(h))*(K_F-Ft)/K_F*Ft + Ft
    nextHt = r*(K_H-Ht)/K_H*Ht+alpha*b*Ht + gamma*((l-n)*Ft + (c-b)*Ht) +Ht
    Fr = c*Ht - b*Ht + (K_F-Ft)/K_F*(1/1200*abs(math.sin(h)))*Ft
    Hr = r*(K_H-Ht)/K_H*Ht+alpha*b*Ht + gamma*((l-n)*Ft + (c-b)*Ht)
    h += 1/4*math.pi
    Ht = nextHt
    Ft = nextFt

initialization()
for i in range(run_time):
    update()
    observe()

xValue, yValue  = meshgrid(arange(-100, 100, 0.1), arange(-100, 100, 0.1))
xdot = c*yValue - b*yValue + (1/1200*abs(math.sin(h)))*xValue
ydot = r*yValue + alpha*b*yValue + gamma*(1/1200*abs(math.sin(h))*xValue + (c-b)*xValue)
streamplot(xValue, yValue, xdot, ydot)
```

```
show()

plt.plot(HrResult, color='blue')
plt.title('Numbers of human change in population')
plt.xlabel('time(a quarter of year)')
plt.ylabel('number of people increase');
show()
plt.plot(FrResult, color='green')
plt.title(' Tree coverage change')
plt.xlabel('time(a quarter of year)')
plt.ylabel('number of trees loss(km^2)');
show()

plt.plot(HtResult, color ='black', label = "Simulate Data for human")
plt.plot(ActDataH, c = 'blue', marker = ".", label = "Actual Data for human")
plt.legend()
plt.title('Total pupulation')
plt.xlabel('Time(year)')
plt.ylabel('Total population');
show()

plot(FtResult, color ='Red', label = 'Simulate Data for tree')
plot(ActDataF2, color = 'black', label = "Actual Data for tree")
plt.title('tree coverage')
plt.legend()
plt.xlabel('Time(a quarter of year)')
plt.ylabel('Total tree coverage(km^2)');
show()



plot(HtResult,FtResult )
plt.title('Tree coverage vs. Human population')
plt.xlabel('total population')
plt.ylabel('Total tree coverage');
show()
```