

## CSCI 532 – Algorithm Design Assignment 8

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**Question 1.** Why don't we allow a minimum degree of  $t = 1$ ?

**Solution:**

Minimum degree  $t$  means every node other than root node should have at least  $t-1$  keys and  $t$  children.

Minimum degree  $t =$  minimum  $t-1$  keys and maximum  $2t-1$  keys, minimum  $t$  children and maximum  $2t$  children

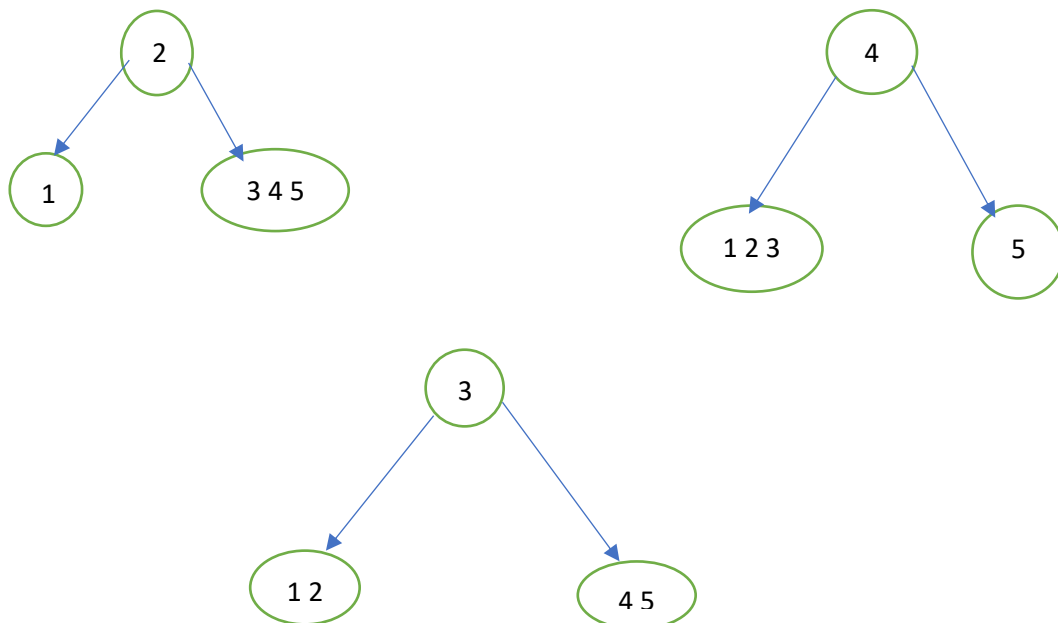
When  $t=1$ , minimum becomes 0 keys and maximum 1 key, minimum 1 child and maximum 2 children

Any degree can not have 0 keys and 1 child, because every node with 1 key will have 2 children

**Question 2.** Show all legal B-trees of minimum degree 2 that represent  $\{1; 2; 3; 4; 5\}$ .

**Solution:**

Every node except root node should have minimum of  $t-1$  keys and maximum of  $2t-1$  keys. All the leaves should stay in the same depth. The possible B-trees are:



**Question 3.** Given an adjacency-list representation of a directed graph, how long does it take to compute the out-degree of every vertex? How long does it take to compute the in-degrees?

**Solution:**

To compute the out degree of each vertex, it would  $O(|E|)$

To compute the in degree of each vertex, we will have to scan through all of the adjacency lists and keep counters for how many times each vertex has appeared. The cost is  $O(|E| + |V|)$ .

**Question 4.** Give an adjacency-list representation for a complete binary tree on 7 vertices. Give an equivalent adjacency-matrix representation. Assume that vertices are numbered from 1 to 7 as in a binary heap.

**Solution:**

Assuming the vertices from 1 to 7, it is considered 7 x 7 matrix, the adjacency-matrix representation is

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

**Question 5.** What is the running time of BFS if we represent its input graph by an adjacency matrix and modify the algorithm to handle this form of input?

**Solution:**

The time taken to iterate all the edges is  $O(V^2)$  and the running time of BFS is  $O(V + V^2)$