

**CSCI 532 – Algorithm Design**  
**Assignment 2**

Name: Badarika Namineni

CWID: 50233279

**Question 1:** What are the minimum and maximum number of elements in a heap of height  $h$ ?

**Solution:**

The no. of edges in the longest possible path from the parent node to the last leaf node is the height of the node.

For given height  $h$ , the maximum number of elements in the heap is  $2^{h+1}-1$ .

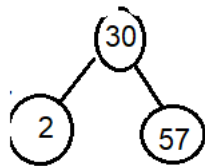
The minimum number of elements in the heap is  $2^h$ .

**Question 2:** Show that an  $n$ -element heap has height  $(\lg n)$ .

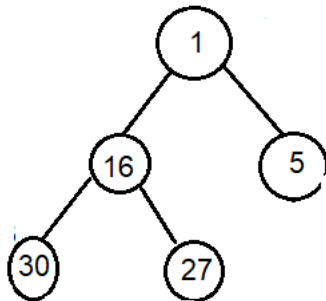
**Solution:**

Let's consider a binary tree

- 1) In the case of 3 nodes, the height of the node is 1. [The no. of edges from the parent node to the leaf node]



- 2) In the case of 5 nodes, the height of the node is 2



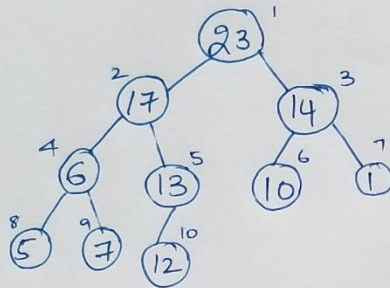
Likewise, in the case of 7 nodes the height is 2, for 15 nodes the height is 3 and for 31 nodes the height is 4.

Hence, for the  $n$  nodes the height is  **$(\log n)$**

**Question 3:** Is the array with values  $\{23, 17, 14, 6, 13, 10, 1, 5, 7, 12\}$  a max-heap?

**Solution:**

3) Given array is  $\{23, 17, 14, 6, 13, 10, 1, 5, 7, 12\}$   
It can be written in tree as



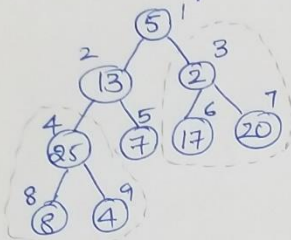
To prove it is a max-heap, all the parent nodes has to be greater than their child nodes.  
But,  $A[6]$  the parent node is smaller than its child node  $A[9]$ .

Hence, it is not a max-heap.

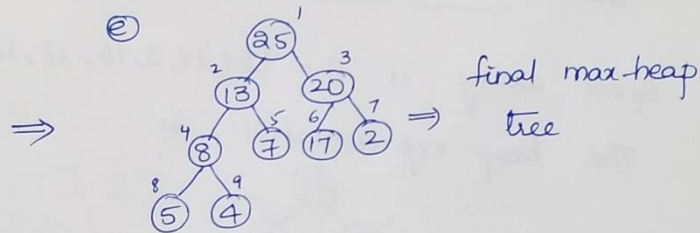
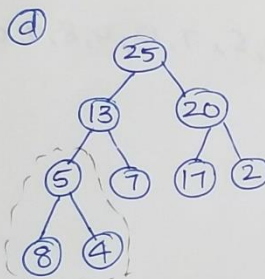
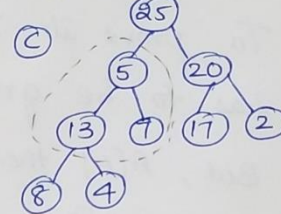
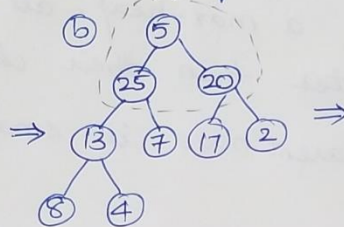
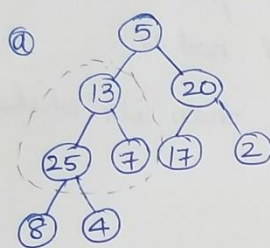
**Question 4:** Illustrate the operation of heapsort on the array  $A = \{5, 13, 2, 25, 7, 17, 20, 8, 4\}$ .

**Solution:**

The given array is  $A = \{5, 13, 2, 25, 7, 17, 20, 8, 4\}$   
The binary tree representation of the array is



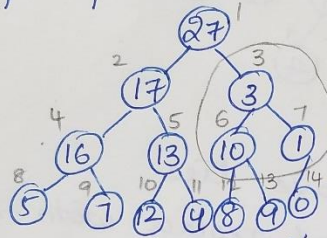
Now, let's sort the tree to get max-heap sort.



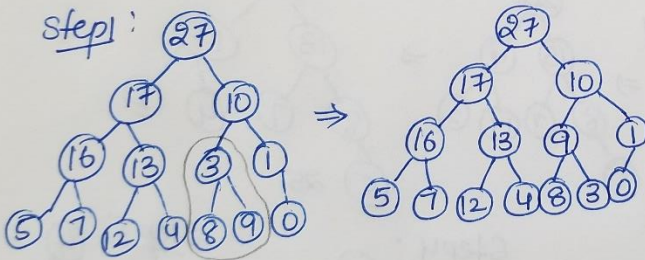
**Question 5:** Illustrate the operation of Max-Heapify(A,3) on the array A = {27,17,3,16,13,10,1,5,7,12,4,8,9,0}

**Solution:**

⑤ Given array is  $A = \{27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0\}$   
The heap representation is



It is not max-heap. let's heapify. let's do

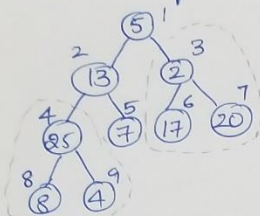


**Question 6:** Using Figure 6.4 as a model, illustrate the operation of HEAPSORT on the array  $A = [5, 13, 2, 25, 7, 17, 20, 8, 4]$ .

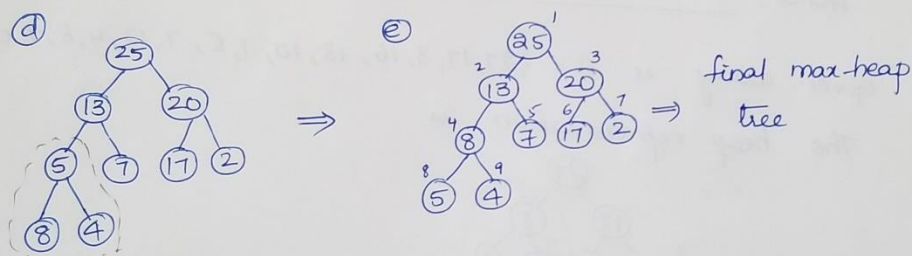
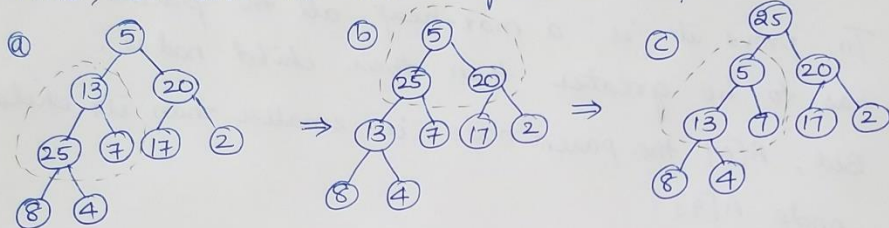
**Solution:**

The given array is  $A = \{5, 13, 2, 25, 7, 17, 20, 8, 4\}$

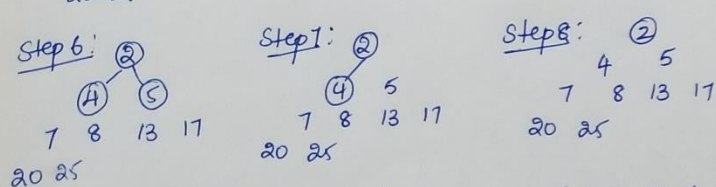
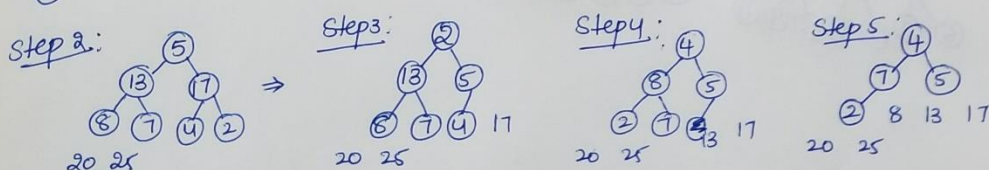
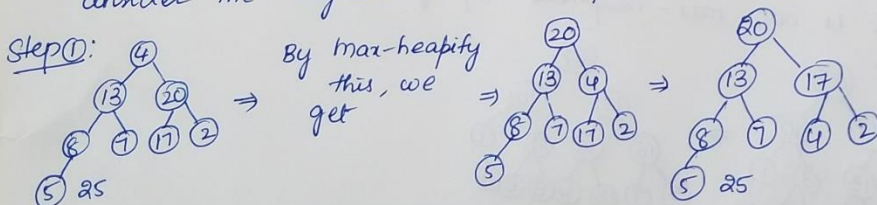
The binary tree representation of the array is



Now, let's sort the tree to get max-heap sort.



Using heapsort for the above tree, consider the highest node and replace with last node (9).



The final sorted array is  $\{2, 4, 5, 7, 8, 13, 17, 20, 25\}$ .