Unit-2

1.

x	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	y	$y_i - \bar{y}$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
100	1	1	98	3	9	3
101	2	4	99	4	16	8
102	3	9	99	4	16	12
102	3	9	97	2	4	6
100	1	1	95	0	0	0
99	0	0	92	-3	9	0
97	-2	4	95	0	0	0
98	-1	1	94	-1	1	1
96	-3	9	90	-5	25	15
95	-4	16	91	-4	16	16
990	0	54	950	0	96	61

$\bar{x} = \frac{990}{10} = 99$ $\qquad$ $\bar{y} = \frac{950}{10} = 95$

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \cdot \sum_{i=1}^{n} (y_i - \bar{y})^2}}$$

$$= \frac{61}{\sqrt{54 \times 96}} = \frac{61}{72} = 0.8472$$

2.

x	y	$(x_i-\bar{x})$	$(x_i-\bar{x})^2$	$(y_i-\bar{y})$	$(y_i-\bar{y})^2$	$(x_i-\bar{x})(y_i-\bar{y})$
65	67	-3.25	10.5625	-1.5	2.25	4.875
66	68	-2.25	5.0625	-0.5	0.25	1.125
67	64	-1.25	1.5625	-4.5	20.25	5.625
67	68	-1.25	1.5625	-0.5	0.25	0.625
68	72	-0.25	0.0625	3.5	12.25	-0.875
69	70	0.75	0.5625	1.5	2.25	1.125
71	69	2.75	7.5625	0.5	0.25	1.375
73	70	4.75	22.5625	1.5	2.25	7.125
546	548		49.5		40	21

$\bar{x} = \frac{546}{8} = 68.25$

$\bar{y} = 68.5$

$r = \frac{21}{\sqrt{49.5 \times 40}} = 0.4719 \simeq 0.472$

3.

x	y	Ranks of X	Ranks of Y	$d_i = x_i - y_i$	d_i^2
40	45	11	7.5	3.5	12.25
46	45	10	7.5	2.5	6.25
54	50	9	6	3	9
60	43	8	9	-1	1
70	40	7	11	-4	16
80	75	6	1	5	25
82	55	5	5	0	0
85	72	3.5	2	1.5	2.25
85	65	3.5	4	-0.5	0.25
90	42	2	10	-8	64
95	70	1	3	-2	4
					140

10.4 20.8

$$\rho = 1 - \frac{6\{\Sigma d_i^2 + CF\}}{n(n^2-1)}$$

$$C.F = \frac{2(2^2-1)}{12} + \frac{2(2^2-1)}{12}$$

$$= \frac{2\times3}{12} + \frac{2\times3}{12} = 1$$

$$= \frac{1-6\{140+1\}}{11(120)}$$

$$= \frac{1-(6\times141)}{120\times11}$$

$$1 - \frac{141}{220} = 0.3591$$

$$= 0.36$$

4.

x	y
65	67
66	68
67	64
67	68
68	72
69	70
71	69
73	76

Same data as 2nd question
(Malli evadu chesthadu Bokka)

$\bar{x} = 68.25$

$\bar{y} = 68.5$

Eqn to regression line, Y on X is

$$Y - \bar{Y} = b_{YX}(X - \bar{X})$$

$$b_{YX} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$$

$$Y - 68.5 = 0.424(X - 68.25)$$

$$= \frac{21}{49.5} = 0.424$$

$$Y = 0.424X - 28.95165 + 68.5$$

$$\boxed{Y = 0.424X + 39.54835}$$

Eqn X on Y is,

$$X - \bar{X} = b_{XY}(Y - \bar{Y})$$

$$X - 68.25 = 0.525(Y - 68.5)$$

$$\boxed{X = 0.525Y + 32.2875}$$

$$b_{XY} = \frac{21}{40} = 0.525$$

5.

x	y	xy	x^2
1	14	14	1
2	27	54	4
3	40	120	9
4	55	220	16
5	68	340	25
15	204	748	55

$n = 5$,

Normal eqns are,

$$\sum_i y_i = na + b \sum_i x_i$$

$$\sum_i x_i y_i = a \sum_i x_i + b \sum_i x_i^2$$

$$\Downarrow$$

$$204 = 5a + 15b - (1)$$

$$748 = 15a + 55b - (2)$$

$(1) \times 3 \Rightarrow 612 = 15a + 45b - (3)$

$(2) - (3) \Rightarrow 748 = 15a + 55b$

$612 = 15a + 45b$

$136 = 10b$

$\boxed{b = 13.6}$

$\boxed{a = 0}$

Reqd straight line is, $y = a + bx$

$\boxed{y = 13.6x}$

6.

x	y	x_i^2	x_i^3	x_i^4	$x_i y_i$	$x_i^2 y_i$
10	14	100	1000	10000	140	1400
12	17	144	1728	20736	204	2448
15	23	225	3375	50625	345	5175
23	25	529	12167	279841	48	13225
20	21	400	8000	160000	420	8400

* ~~Reg~~ Fit 2nd degree polynomial for following data

7

x	y	x_i^2	x_i^3	x_i^4	$x_i y_i$	$x_i^2 y_i$
1	2.3	1	1	1	2.3	2.3
2	5.2	4	8	16	10.4	20.8
3	9.7	9	27	81	29.1	~~58~~ 87.3
4	16.5	16	64	256	66	~~264~~
5	29.4	25	125	625	147	~~735~~
6	35.5	36	216	1296	213	~~1278~~
7	54.4	49	343	2401	380.8	~~2665.6~~
28	153	140	784	4676	848.6	~~5053~~

Normal eqn's,

$$153 = 7a + 28b + 140c \quad -(1)$$

$$848.6 = 28a + 140b + 784c \quad -(2)$$

$$5053 = 140a + 784b + 4676c \quad -(3)$$

Solving (1) & (2)

$(1) \times 4 \Rightarrow 612 = 28a + 112b + 560c$

$(2) \Rightarrow 848.6 = 28a + 140b + 784c$

$$236.6 = 28b + 224c$$

$$b + 8c = 8.45 \quad -(4)$$

$$5053 = 140a + 784b + 4676c$$

$(2) \times 5 \Rightarrow 4243 = 140a + 700b + 3920c$

$$810 = 84b + 756c \quad -(5)$$

$(4) \times 84 \Rightarrow 84b + 672c = 709.8$

$(5) \Rightarrow 84b + 756c = 810$

$$84c = 100.2$$

$$\boxed{c = 1.192}$$

$$b = 8.45 - 9.536$$

$$\boxed{b = -1.086}$$

$$30.408 + 153 = 7a + 166.88$$

$$7a = 16.528$$

$$\boxed{a = 2.361}$$

$$y = 2.361 - 1.086x + 1.192x^2$$

8.

x	y	x_i^2	$\log_e y\ (Y_i)$	$x_i Y_i$
0	1.05	0	0.0488	0
1	2.10	1	0.7420	0.7420
2	3.85	4	1.3481	2.6962
3	8.30	9	2.1162	6.3486
6		14	4.2551	9.7868

Normal eqn's are,

$$\sum_i Y_i = nA + b\sum_i x_i$$

$$\sum_i x_i Y_i = A\sum_i x_i + b\sum_i x_i^2$$

$\Downarrow$

$n = 4$

$$4.2551 = 4A + 6b \quad -①$$

$$9.7868 = 6A + 14b \quad -②$$

$② \times 2 \Rightarrow 19.5736 = 12A + 28b$

$① \times 3 \Rightarrow 12.7653 = 12A + 18b$

$(-) \qquad (-) \quad (-)$

$$6.8083 = 10b$$

$$\boxed{b = 0.6808}$$

$$4.2551 = 4A + 4.0848$$

$$\boxed{A = 0.042575}$$

$$A = \log_e a$$

$$a = e^A$$

$$\boxed{a = 1.0435}$$

Reqd eqn is $\Rightarrow$ $\boxed{y = 1.0435 \times e^{0.6808x}}$

---

9)

x	y	xy	x^2
1	14	14	1
2	33	66	4
3	40	120	9
4	63	252	16
5	76	380	25
6	85	510	36
21	311	1342	91

$\boxed{n = 6}$,

Normal eqn's are,

$$\sum_i y_i = na + b\sum_i x_i$$

$$\sum_i x_i y_i = a\sum_i x_i + b\sum_i x_i^2$$

$$\Downarrow$$

$$311 = 6a + 21b - (1)$$

$$1342 = 21a + 91b - (2)$$

(2) $\times 2 \Rightarrow 2684 = 42a + 182b$

(1) $\times 7 \Rightarrow 2177 = 42a + 147b$

$(-) \quad (-) \quad (-)$

$507 = 35b$

$\boxed{b = 14.4857}$

$\boxed{a = 1.1334}$

Reqd eqn is $\boxed{y = 1.1334 + 14.4857x}$

10)

x	y	$(x_i-\bar{x})$	$(x_i-\bar{x})^2$	$(y_i-\bar{y})$	$(y_i-\bar{y})^2$	$(x_i-\bar{x})(y_i-\bar{y})$
1	9	-4	16	-3	9	12
2	8	-3	9	-4	16	12
3	10	-2	4	-2	4	4
4	12	-1	1	0	0	0
5	11	0	0	-1	1	0
6	13	1	1	1	1	1
7	14	2	4	2	4	4
8	16	3	9	4	16	12
9	15	4	16	3	9	12
45	108		60		60	57

$\bar{x} = 5$

$\bar{y} = 12$

$$r = \frac{57}{\sqrt{60 \times 60}} = \frac{57}{60} = 0.95$$

Lines of regression

$$b_{yx} = \frac{57}{60} = 0.95 \qquad b_{xy} = \frac{57}{60} = 0.95$$

Line for Y on X,

$$Y - \bar{Y} = b_{yx}(X - \bar{x})$$

$$Y - 12 = 0.95(X - 5)$$

$$\boxed{Y = 0.95X + 7.25}$$

Line for X on Y,

$$x - \bar{x} = b_{xy}(Y - \bar{Y})$$

$$x - 5 = 0.95(Y - 12)$$

$$\boxed{X = 0.95Y - 6.4}$$