Unit-3

1:

x	1	2	3	4	5	6	7	8
$P(x)$	k	$2k$	$3k$	$4k$	$5k$	$6k$	$7k$	$8k$

i) Find k => $\sum_i P_i(X) = 1$

$$k + 2k + 3k + 4k + 5k + 6k + 7k + 8k = 1$$

$$36k = 1$$

$$\boxed{k = 0.0278}$$

ii) $P(X \le 2)$ => $P(X = 1 \text{ or } 2)$

$$= P(1) + P(2)$$

$$= k + 2k = 3k = 3(0.0278)$$

$$= 3 \times \frac{1}{36} = \frac{1}{12} = 0.0833$$

iii) $P(2 \le x \le 5)$ => $P(X = 2 \text{ or } 3 \text{ or } 4 \text{ or } 5)$

$$= P(2) + P(3) + P(4) + P(5)$$

$$= 2k + 3k + 4k + 5k$$

$$= 14k = \frac{14}{36} = 0.3889$$

2.

X	0	1	2	3	4	5	6	7	8
$P(X)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

i) Find $a \Rightarrow$ $a+3a+5a+7a+9a+11a+13a+15a+17a=1$

$$a = \frac{1}{81} = 0.0123$$

ii) $P(X<3)$ & $P(X\geq 3) \Rightarrow$ $P(X<3)$ is $P(X=0$ or 1 or $2)$

i.e $\Rightarrow$ $P(0)+P(1)+P(2)$

$= a+3a+5a = 9a$

$= \frac{9}{81} = \frac{1}{9} = 0.111$

$P(X\geq 3) = P(X=3$ or 4 or 5 or 6 or 7 or $8)$

$= 7a+9a+11a+13a+15a+17a$

$= 72a = \frac{\cancel{72}8}{\cancel{81}9} = \frac{8}{9} = 0.889$

iii) $P(0<X<5) = P(X=1$ or 2 or 3 or $4)$

$= P(1)+P(2)+P(3)+P(4)$

$= 3a+5a+7a+9a$

$= 24a$

$= \frac{24}{81} = 0.2963$

3.

X	0	1	2	3	4	5	6
$P(x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

i) Find $k \Rightarrow k + 3k + 5k + 7k + 9k + 11k + 13k = 1$

$$k = \frac{1}{49}$$

$$\boxed{k = 0.0204}$$

ii) $P(X<4)$ & $P(X \geq 5) \Rightarrow$ $P(X<4) \Rightarrow P(X = 0 \text{ or } 1 \text{ or } 2 \text{ or } 3)$

$$\Rightarrow P(0) + P(1) + P(2) + P(3)$$

$$= k + 3k + 5k + 7k$$

$$= 16k = \frac{16}{49} = 0.3265$$

$P(X \geq 5) \Rightarrow P(X = 5 \text{ or } 6)$

$$= P(5) + P(6) = 11k + 13k$$

$$= \frac{24}{49} = 0.4898$$

iii) $P(3 < X \leq 6) \Rightarrow P(X = 4 \text{ or } 5 \text{ or } 6)$

$$= P(4) + P(5) + P(6)$$

$$= 9k + 11k + 13k$$

$$= 33k = \frac{33}{49} = 0.6734$$

1) A discrete probability distribution of r.v. 'X', is as follows,

$$\begin{array}{lcccccccc} x: & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ P(X=x): & 0 & k & 2k & 2k & 3k & k^2 & 2k^2 & 7k^2+k \end{array}$$

Find k

$P(X<6)$, $P(X\geq 6)$

$P(0<x<5)$

A. 'X' is a discrete r.v, then

$$\sum_i P_i = 1$$

i.e, $0+k+2k+2k+3k+k^2+2k^2+7k^2+k = 1$

$$10k^2+9k=1$$

$$10k^2+10k-k-1=0$$

$$10k(k+1)-1(k+1)=0$$

$$k=\frac{1}{10}, -1$$

$P_i \geq 0, \forall i$, $\boxed{k=\frac{1}{10}}$

ii)

<u>Mean:</u> $\mu = E(X) = \sum_i x_i \, P(X = x_i)$

$\mu = 0(0) + 1(k) + 2(2k) + 3(2k) + 4(3k) + 5(k^2) + 6(2k^2) + 7(7k^2 + k)$

$= 66k^2 + 30k$

$= (66)\left(\frac{1}{10}\right)^2 + 30\left(\frac{1}{10}\right)$

$= 0.66 + 3 = 3.66$

$E(X^2) = \sum x_i^2 \cdot P(X = x_i)$

$= k + 8k + 18k + 48k + 25k^2 + 72k^2 + 343k^2 + 49k$

$= 124k + 440k^2$

$= 12.4 + 4.4 = 16.8$

iii)

<u>Variance:</u>

$V(X) = E(X^2) - \mu^2 = 16.8 - (3.66)^2$

$= 3.4044$

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5)

x :	-3	-2	-1	0	1	2	3
$P(X=x)$:	k	0.1	k	0.2	$2k$	0.4	$2k$

Find k, mean, variance

A. <u>i)</u> $k + 0.1 + k + 0.2 + 2k + 0.4 + 2k = 1$

$6k + 0.7 = 1$

$k = \frac{\cancel{0.3}\,0.1}{\cancel{6}\,2}$

$k = 0.1 \times 0.5$

$\boxed{k = 0.05}$

ii) Mean $\mu = E(X) = \sum_i x_i P(x = x_i)$

$\mu = -3k - 0.2 - k + 2k + 0.8 + 6k$

$= -0.15 - 0.2 - 0.05 + 0.1 + \overset{0.8}{\cancel{0.4}} + 0.3$

$= 0.8$

$E(X^2) = \sum x_i^2 P(x = x_i)$

$= 9k + 0.4 + k + 2k + 1.6 + 18k$

$= 0.45 + 0.4 + 0.05 + \overset{0.1}{\cancel{0.2}} + 1.6 + 0.$

$= \cancel{3.4}\ 3.5$

iii) Variance:

$V(X) = E(X^2) - \mu^2 = \overset{3.5}{\cancel{3.4}} - 0.64$

$= \cancel{2.76}\ 2.86$

6) The p.d.f of a continuous random variable 'X' defined by $f(x) = k(1-x^2)$

for $0 < x < 1$

$= 0$, otherwise

Find i) k ii) Mean iii) Variance. iv) $P(\frac{1}{2} < x < \frac{3}{4})$

A. $\because f(x)$ is p.d.f of 'X',

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

i) i.e,

$$\int_{-\infty}^{0} 0 \cdot dx + \int_{0}^{1} k(1-x^2)dx + \int_{1}^{\infty} 0 \cdot dx = 1$$

$$k\left(x - \frac{x^3}{3}\right)_0^1 = 1$$

$$k\left(\frac{2}{3}\right) = 1$$

$$\boxed{k = \frac{3}{2}}$$

ii) Mean $\mu = E(X) = \int_{-\infty}^{0} x(0)dx + \int_{0}^{1} k(1-x^2)x\,dx + \int_{1}^{\infty} 0.(x)dx$

$$= k\left(\frac{x^2}{2} - \frac{x^4}{4}\right)_0^1$$

$$= k\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{k}{2} = \frac{3}{8}$$

$$E(x^2) = K\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1$$

$$= K\left(\frac{1}{3} - \frac{1}{5}\right) = K\left(\frac{2}{15}\right) = \frac{\cancel{3}}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{15}5} = \frac{1}{5}$$

iii) <u>Variance:</u> $Var(X) = \frac{1}{5} - \frac{9}{64}$

$$\cancel{= \frac{16-45}{80} = \frac{29}{80}} = \frac{\overset{64}{\cancel{320}} - 45}{320} = \frac{19}{320}$$

7. $f(x) = \begin{cases} kxe^{-\lambda x}, x \geq 0, \lambda > 0 \\ 0, \text{otherwise} \end{cases}$

i) Value of k =

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_{-\infty}^{0} 0.\,dx + \int_{0}^{1} kxe^{-\lambda x}\,dx + \int_{1}^{\infty} kxe^{-\lambda x}\,dx = 1$$

$$1 = k\left[xe^{-\lambda x}(-\lambda) + e^{-\lambda x}\right]_0^1 + k\left[xe^{-\lambda x}(-\lambda) + e^{-\lambda x}\right]_1^{\infty}$$

$$k\left[e^{-\lambda}(-\lambda) + e^{-\lambda} - 1\right] + k\left[0 - \left(e^{-\lambda}(-\lambda) + e^{-\lambda}\right)\right] = 1$$

$$k\left[e^{-\lambda}(-\lambda) + e^{-\lambda} - 1\right] + k\left[-e^{-\lambda}(-\lambda) - e^{-\lambda}\right] = 1$$

$$k\left[\cancel{e^{-\lambda}(-\lambda)}\right] + \cancel{k(e^{-\lambda})} - k + k\left[-\cancel{e^{-\lambda}(-\lambda)}\right] - \cancel{k(e^{-\lambda})} = $$

$$-k = 1$$

$$\boxed{k = -1}$$

ii) Mean : $\mu = E(x) = \int_{-\infty}^{0} 0.\,dx + \int_{0}^{1} kx^2e^{-\lambda x}\,dx + \int_{1}^{\infty} kx^2e^{-\lambda x}\,dx$

$$= k\left[x^2e^{-\lambda x}(-\lambda) + e^{-\lambda x}(2x)\right]_0^1$$

$$+$$

$$k\left[x^2e^{-\lambda x}(-\lambda) + e^{-\lambda x}(2x)\right]_1^{\infty}$$

$$= k\left[e^{-\lambda}(-\lambda) + 2e^{-\lambda}\right] + k\left[-\left(e^{-\lambda}(-\lambda) + 2e^{-\lambda}\right)\right]$$

$$= 0$$