Bernoulli Distribution

If it assumes only two values 0 [failure], 1 [success] with the corresponding properties with P and Q respectively $P+Q=1$

x	0	1
$P(X=x)$	Q	P

Mean and variance of Bernoulli Distribution

Mean, $\mu = E(x) = \sum_i x_i . P_i$

$= 0(Q) + 1(P) = P$

$E(x^2) = \sum_i x_i^2 . P_i$

$= (0^2)(Q) + (1)^2(P)$

$= P$

variance, $\sigma = E(x)^2 - \mu^2$

$= P - P^2$

$= P(1-P)$

$= PQ$

Standard Deviation $= \sqrt{PQ}$

Discrete Random Variable X.

$P(X=x) = {}^nC_x . P^x . Q^{n-x}$, $x = 0, 1, 2, \dots n$

Probability Mass Function

X is Binomial varia

$X \sim B(n, p)$

Binomial Distribution

Mean ; $\mu = E(X)$

$$= \sum x \cdot P(X=x)$$

$$= \sum_{x=0}^{n} x \cdot {}^nC_x \cdot p^x \cdot q^{n-x}$$

$$= np(q+p)^{n-1}$$

$$= np$$

variance $\sigma^2 = npq$

10 Coins are thrown simultaneously. Find the probability of of getting atleast (i) 7 heads (ii) 6 heads

$n = 10$

$p = \frac{1}{2}$

$q = \frac{1}{2}$

Let X denotes No.of heads

$X \sim B(n,p)$

Probability Mass function of Binomial distance is

$P(X=x) = {}^nC_x \, p^x . q^{n-x}$;

(i) atleast 7

$$P(X \geq 7) = P(X=7 \text{ or } 8 \text{ or } 9 \text{ or } 10)$$

$$= P(X=7) + P(X=8) + P(X=9) + P(X=10)$$

$$= {}^{10}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{10-7} + {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^{10-8} +$$

$${}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^{10-8} + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^{10-10}$$

$$= \left(\frac{1}{2}\right)^{10} \left[{}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right]$$

$\left[{}^nC_r = {}^nC_{n-r}\right]$ $= \left(\frac{1}{2}\right)^{10} \left[{}^{10}C_3 + {}^{10}C_2 + {}^{10}C_1 + 1\right]$

$$= \left(\frac{1}{2}\right)^{10} \left[1+10+45+120\right] = \left(\frac{1}{2}\right)^{10} (176)$$

(ii) Atleast 6 heads

$P(x \geq 6) = P(x = 6 \text{ (or) } 7 \text{ (or) } 8 \text{ (or) } 9 \text{ (or) } 10)$

$= P(x=6) + P(x=7) + P(x=8) + P(x=9) + P(x=10)$

$= 10C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^{10-6} + 10C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{10-7} +$

$10C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^{10-8} + 10C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^{10-9} +$

$10C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^{10-10}$

$= \left(\frac{1}{2}\right)^{10} [10C_6 + 10C_7 + 10C_8 + 10C_9 + 10C_{10}]$

$= \left(\frac{1}{2}\right)^{10} [10C_4 + 10C_3 + 10C_2 + 10C_1 + 1]$

$= \left(\frac{1}{2}\right)^{10} [1 + 10 + 45 + 120 + 210]$

$= \left(\frac{1}{2}\right)^{10} [386]$

A dice is thrown 6 times if getting an even number is a success find the probabilities of atleast 1 success (ii) Less than or equal to 3 successes (iii) four successes

Probability of getting even number when a die is thrown

$P = \frac{3}{6} = \frac{1}{2}$

$n = 6$

$S = \{1, 2, 3, 4, 5, 6\}$

$q = 1 - P = \frac{1}{2}$

$X \sim B(n, p)$ Let x denotes No. of success

Probability Mass function of Binomial distance is

$P(X = x) = nC_x \, p^x \cdot q^{n-x} \quad ; x = 0, 1, 2, \ldots n.$

$$\frac{\cancel{6}^{3}\times 5\times \cancel{4}\times \cancel{3}}{\cancel{4}\times \cancel{3}\times \cancel{2}\times 1} \quad 15$$

(iii) 4 successes

$$P(X=4) = 6C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{6-4}$$

$$= (15)\left(\frac{1}{2}\right)^6$$

$$= 15/64 = \frac{15}{64}$$

(i) Atleast one success

$$P(X\geq 1) = 1 - P(X=0)$$

$$= 1 - 6C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^6$$

$$= 1 - \frac{1}{2^6}$$

$$= 1 - \frac{1}{64}$$

$$= \frac{63}{64}$$

(ii) ≤ 3 successes $P(X\leq 3)$

$$= P(X=0 \text{ or } 1 \text{ or } 2 \text{ or } 3)$$

$$= P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= 6C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^6 + 6C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^5 + 6C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^4$$

$$6C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3$$

$$= \left(\frac{1}{2}\right)^6\left[6C_0 + 6C_1 + 6C_2 + 6C_3\right]$$

$$= \left(\frac{1}{64}\right)\left[1+6+15+20\right]$$

$$= \frac{42}{64} = \frac{21}{32}$$

Probability of a defective bolt is $\frac{1}{8}$. find mean and variance of distribution of defective bolts of 640

Given that $n = 640$

$P = \frac{1}{8}$ [Probability of a defective bolt]

we know that the mean and variance of binomial distribution is

Mean $\mu = np$

$= 640 \times \frac{1}{8}$

$= 80$

variance $\sigma^2 = npq$

$= 640 \times \frac{1}{8} \times \frac{7}{8}$

$= 70$.

Determine the binomial distribution for which mean is 4 and variance 3

Given that mean and variance of binomial distribution are 4, 3

$np = 4$, $npq = 3$

$\Rightarrow \frac{\cancel{np}q}{\cancel{np}} = \frac{3}{4} = \boxed{q = \frac{3}{4}}$

$\Rightarrow P = 1 - q = 1 - \frac{3}{4} = \frac{4-3}{4} = \frac{1}{4} \Rightarrow \boxed{P = \frac{1}{4}}$

$np = 4 \Rightarrow n\left(\frac{1}{4}\right) = 4 \Rightarrow \boxed{n = 16}$

Mean and Variance of a binomial distribution are 4 and $\frac{4}{3}$ respectively. Find $P(x \geq 1)$

Given that mean and variance of binomial distribution are $4, \frac{4}{3}$

$np = 4$; $npq = \frac{4}{3}$; $P = 1 - \frac{1}{3}$

$n \cdot \frac{2}{3} = 4$; $nq = \frac{4}{3}$; $= \frac{2}{3}$

$n = 6$; $q = \frac{1}{3}$

$$P(x \geq 1) = 1 - P(x=0)$$

$$= 1 - 6C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^6$$

$$= 1 - \frac{1}{36} = \frac{728}{729}$$

The Mean and variance of a binomial variate x are 16 and 8 find $P(x \geq 1)$ and $P(x > 2)$

$np = 16$ $\quad$ $npq = 8$ $\quad$ $p = 1 - \frac{1}{2}$

$n\left(\frac{1}{2}\right) = 16$ $\quad$ $16\,q = 8$ $\quad$ $p = \frac{1}{2}$

$q = \frac{1}{2}$

$n = 32$

(i) $P(x \geq 1) = 1 - P(x=0)$

$$= 1 - 32C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{32}$$

$$= 1 - \frac{1}{2^{32}}$$

(ii) $P(x>2) = 1 - \{P(x=0) + P(x=1) + P(x=2)\}$

$$= 1 - 32C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{32} + 32C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{31} + 32C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{30}$$

$$= 1 - \left(\frac{1}{2}\right)^{32}\left[32C_0 + 32C_1 + 32C_2\right]$$

Assume that 50 % of the all Engineering students are good in mathematics. Determine the probability that among 18 engineering students (i) Exactly 10 (ii) Atleast 10 (iii) At most 8

(iv) Atleast 2 and Atmost 9

Given that $n = 18$

[Probability of a student who is good in Mathematics] $P = 50\% = \frac{1}{2}$

$q = \frac{1}{2}$

Let x denotes no of students who are good in Mathematics

$X \sim B(n, p)$

The P.M.f of binomial distribution is.

$$P(X = x) = n_{C_x}\, p^x q^{n-x} \; ; \; x = 0; 1, 2, \ldots n$$

(i) $P(X = 10) = 18 C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^{18-10} = 18 C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^{8}$

(ii) $P(X \geq 10) = P(X = 10) + \ldots + P(X = 18)$

(iii) $P(X \leq 8) = P(X = 0) + \ldots + P(X = 8)$

(iv) $P(2 \leq X \leq 9)$.

**Fit a binomial distribution to the following data Hence find theoritical frequencies [expected freq]

x :	0	1	2	3	4	5	6
f :	13	25	52	58	32	16	4

x	f	fx	freq Expected
0	13	0	6
1	25	25	28
2	52	104	56
3	58	174	60
4	32	128	36
5	16	80	12
6	4	24	2
	200	535	

$\Sigma f_i = 200$

$\Sigma f_i x_i = 535$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{535}{200} = 2.675$$

We know that mean of binomial distribution np

$np = 2.675$

Here $n = 6$

$$\Rightarrow p = \frac{2.675}{6} = 0.445$$

$q = 1 - 0.446 = 0.554$

The binomial distribution to be fitted using the given data is

$$N(q+p)^n$$

$$= 200(0.554 + 0.446)^6$$

$$= 200\left[6C_0(0.554)^6 + 6C_1(0.554)^5(0.446) + 6C_2(0.554)^4(0.446)^2 + 6C_3(0.554)^3(0.446)^3 + 6C_4(0.554)^2(0.446)^4 + 6C_5(0.554)(0.446)^5 + 6C_6(0.446)^6\right]$$

$$= (200)\left[0.0289 + 0.1396 + 0.2811 + 0.3017 + 0.1822 + 0.0586 + 0.0079\right]$$

$$= 5.78 + 27.92 + 56.22 + 60.3 + 36.44 + 11.72 + 1.58$$

Fit a binomial distribution to the following data

x	:	0	1	2	3	4	5
f	:	2	14	20	34	22	8

$n=5$

A probability of a man hitting a target is $\frac{1}{3}$ $p=\frac{1}{3}$, $Q=\frac{2}{3}$

(i) If he fires 5 times, what is the probability of his hitting the target atleast twice $P(X\geq 2)=1-\{P(X=0)+P(X=1)\}$

(ii) How many times must he fire so that the probability of his hitting the target atleast once is more than 90% $P(X\geq 1)>90\%$

$1-P(X=0)>90\%$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Poisson Distribution

It is a limiting case of Binomial Distribution.

(i) n being very large i.e $n \to \infty$

(ii) p is very small $\Rightarrow np = \lambda$ (finite)

Definition :- A discrete random variable X assumes only non negative values and having p.m.f

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \dots$$

then X is said to follow poisson distribution with parameter λ, it is denoted by $X \sim P(\lambda)$

Example :- (i) No. of telephone calls receive by the operate at telephone exchange

(ii) No. of accidents occur in national highway

Mean $= E(X)$

$= \sum x \cdot P(X=x)$

$= \lambda$

Variance $= E(X)^2 - \mu^2$

$= \lambda$

Mean $= \lambda$

A Hospital which were receives average of 4 emergency calls in 10 minute interval. What is the probability that

(i) there are atmost two emergency calls in 10 minute interval $x \le 2$

(ii) there are exactly three emergency calls in 10 minute interval $x = 3$

Let X denotes no. of emergency calls received

$X \sim P(\lambda)$

P.m.f of poisson distribution is, $P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}; x = 0, 1, 2, \dots$

$\lambda = 4$ calls/10 min.

$$= \frac{e^{-4} \cdot 4^x}{x!}; \quad x = 0, 1, 2, \dots$$

(i) At most 2 calls

$$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{e^{-4} 4^0}{0!} + \frac{e^{-4} 4^1}{1!} + \frac{e^{-4} 4^2}{2!}$$

$$= e^{-4}[1+4+8]$$

$$= 13e^{-4} = (13)(0.0183) = 0.2381$$

(ii) Exactly 3 calls

$$P(X=3) = P(X=3)$$

$$= \frac{e^{-4} 4^3}{3!}$$

$$= \frac{32e^{-4}}{3} = \frac{32}{3}(0.0183) = 0.1954$$

It has been found that 2% of the items produced by certain machine were defective. What is the probability that in the shipmate 400 of such items

$n = 400$

$P = 2\% = 0.02$

$\frac{3}{100} \times 400$

(i) 3% or more 12 $X \ge 12$

$\frac{2}{100} \times 400$

$X \le 8$

(ii) 2% or less 8 will be defective

Let x denotes no. of defective items

Given that probability for getting a defective item $P = 2\% = 0.02$

$n = 400$

$\lambda = np = 8$

Clearly $X \sim P(\lambda)$

P.m.f of poisson distribution is $P(X=x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}, x = 0, 1, 2, \ldots$

$$= \frac{e^{-8} \cdot 8^x}{x!}, x = 0, 1, 2, \ldots$$

(i) 3% or more

3% of $400 = \frac{3}{100} \times 400 = 12$

$P(X \geq 12) = 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10) + P(X=11)]$

$$= \frac{e^{-8}.8^0}{0!} + \frac{e^{-8}.8^1}{1!} + \frac{e^{-8}.8^2}{2!} + \frac{e^{-8}8^3}{3!} + \frac{e^{-8}8^4}{4!} + \frac{e^{-8}8^5}{5!} + \frac{e^{-8}8^6}{6!} + \frac{e^{-8}8^7}{7!} + \frac{e^{-8}8^8}{8!} + \frac{e^{-8}8^9}{9!} + \frac{e^{-8}8^{10}}{10!} + \frac{e^{-8}8^{11}}{11!}$$

$= e^{-8}[1 + 8 +$

If x is a poisson variate such that $P(1) = P(2)$. find mean of the distribution, $P(4)$, $P(X \geq 1)$, $P(1 < X < 4)$.

since x is a poisson variate then p.m.f is

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots$$

given that $P(1) = P(2) \Rightarrow P(X=1) = P(X=2)$

$$\frac{e^{-\lambda}.\lambda^1}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$$

$$2\lambda = \lambda^2$$

$$\lambda = 2$$

$$P(4) = P(X=4) = \frac{e^{-2}.2^4}{4!} = \frac{2}{3e^2} = 0.0902$$

$$P(X \geq 1) = 1 - P(X=0)$$

$$= 1 - \frac{e^{-2}.2^0}{0!}$$

$$= 1 - e^{-2} = 1 - \frac{1}{e^2} = 0.8647$$

$$P(1 < X < 4) = P(X=2) + P(X=3)$$

$$= \frac{e^{-2}.2^2}{2!} + \frac{e^{-2}.2^3}{3!} = \frac{10}{3e^2} = 0.4511$$

If x is a poison variant such that $P(X=1)\frac{3}{2} = P(X=3)$

find $P(X>1)$ $P(X\leq 3)$ $P(2\leq X\leq 5)$

since x is a poison variant them p.m.f is

$$P(X=x) = \frac{e^{-\lambda}.\lambda^{x}}{x!} \quad ; x=0,1,2,\ldots$$

Given that $\quad P(X=1)\frac{3}{2} = P(X=3)$

$$\frac{3}{2}\frac{e^{-\lambda}.\lambda^{1}}{1!} = \frac{e^{-\lambda}.\lambda^{3}}{3!}$$

$$9\lambda = \lambda^{3}$$

$$\lambda(\lambda^{2}-9) = 0$$

$$\boxed{\lambda=0,\pm 3}$$

$\boxed{\lambda=3}$ $\because \lambda$ is nonzero & non negative fit a poisson distribution to the following data and find expected frequencies

x :	0	1	2	3	4	5	6	7
f :	305	365	210	80	28	9	2	1

x	f	fx
0	305	0
1	365	365
2	210	420
3	80	240
4	28	112
5	9	45
6	2	12
7	1	7
	1000	1201

$N = \Sigma f_i = 1000$

$\Sigma f_i x_i = 1201$

$$\text{Mean} = \frac{\Sigma f_i x_i}{N} = \frac{1201}{1000}$$

$$= 1.201$$

$$\simeq 1.2$$

we know that mean of the poisson distribution is λ

$$\boxed{\lambda = 1.2}$$

Now, the expected frequencies are obtained by $N.P(X=x)$

$$N.P(X=x) = N.\frac{e^{-\lambda}\lambda^x}{x!} \;;\; x = 0,1,2,.$$

$$= (1000)\frac{e^{-1.2}(1.2)^x}{x!} \;;\; x = 0,1,2,.$$

$$= (1000)\frac{e^{-1.2}(1.2)^0}{0!} + (1000)\frac{e^{-1.2}(1.2)^1}{1!} + (1000)\frac{e^{-1.2}(1.2)^2}{2!} +$$

$$(1000)\frac{e^{-1.2}(1.2)^3}{3!} + (1000)\frac{e^{-1.2}(1.2)^4}{4!} + (1000)\frac{e^{-1.2}(1.2)^5}{5!} +$$

$$(1000)\frac{e^{-1.2}(1.2)^6}{6!} + (1000)\frac{e^{-1.2}(1.2)^7}{7!}$$

x :	0	1	2	3	4	5
f :	142	156	69	27	5	1

Normal Distribution

A continuous random variable x is said to follow Normal distribution if its probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty$$

$\mu \rightarrow$ mean ; $\sigma \rightarrow$ standard deviation of Normal distribution

This is also called as Gaussian distribution

Normal Distribution is a limiting case of binomial distribution

(i) $n \rightarrow \infty$

(ii) neither P nor Q very small

$x = \mu$

Symmetric about $x = \mu$

Mean = Median = Mode

unimodal

x-axis is an asymptote

$$P(a \le x \le b) = \int_a^b \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$P(x_1 \le x \le x_2) = \int_{x_1}^{x_2} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

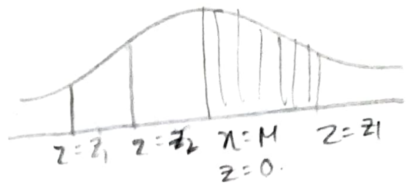
$z = \frac{x-\mu}{\sigma}$ $\quad z \sim N(0,1)$

as $x \to \mu$, $z = 0$

as $x \to x_1$, $z = \frac{x_1-\mu}{\sigma} = z_1$

$$P(\mu \le x \le x_1) = P(0 \le z \le z_1)$$

$$= \int_0^{z_1} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}z^2} \sigma\, dz$$



$$= \frac{1}{\sqrt{2\pi}} \int_0^{z_1} e^{-\frac{z^2}{2}} dz$$

Area under the standard normal curve between $z=0$ and $z=z_1$
$= A(z_1)$

How to find $P(x_1 \le x \le x_2)$

when $x = x_1$, $z = \frac{x_1-\mu}{\sigma} = z_1$, say

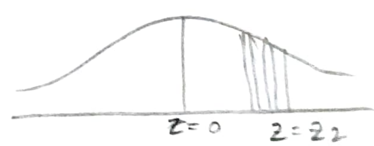
when $x = x_2$, $z = \frac{x_2-\mu}{\sigma} = z_2$, say

$\therefore P(x_1 \le x \le x_2) = P(z_1 \le z \le z_2)$

$P(z_1 \le z \le z_2)$

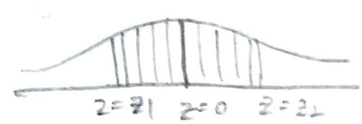
case(i): $z_1 > 0$, $z_2 > 0$

$P(z_1 \le z \le z_2) = A(z_2) - A(z_1)$



case(ii): $z_1 < 0$ and $z_2 > 0$

$P(z_1 \le z \le z_2) = A(z_1) + A(z_2)$



We have $z = \frac{x-\mu}{\sigma} = \frac{x-30}{5}$

We have $x = 28 = \frac{28-30}{5} = -0.4$

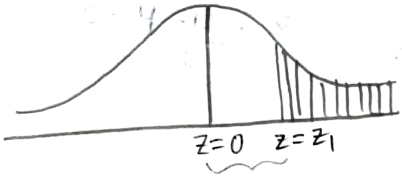
We have $x = 40 = \frac{40-30}{5} = 2$

$P(28 \le x \le 40) = P(-0.4 \le z \le 2) = A(0.4) + A(2)$

$= 0.1554 + 0.4772 = 0.6326$

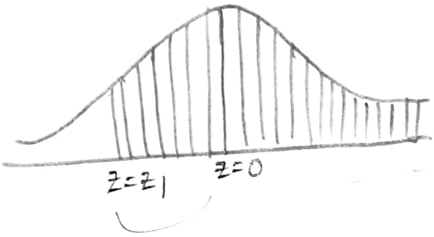
To find $P(Z \geq Z_1)$

case (i) $Z_1 > 0$



$\therefore P(Z \geq Z_1) = 0.5 - A(Z_1)$

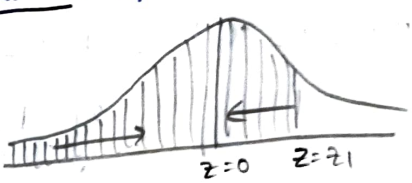
case (ii) $Z_1 < 0$



$\therefore P(Z \geq Z_1) = 0.5 + A(Z_1)$

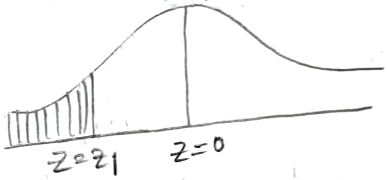
To find $P(Z \leq Z_1)$

case (i) $Z_1 > 0$



$P(Z \leq Z_1) = 0.5 + A(Z_1)$

case (ii) : $Z_1 < 0$



$\therefore P(Z \leq Z_1) = 0.5 - A(Z_1)$

If x is a normal variate, find $P(X \geq 45)$ with mean $\mu = 30$, $\sigma = 5$

$X \sim N(\mu, \sigma^2)$

$\mu = 30$

$\sigma = 5$

$$Z = \frac{x-\mu}{\sigma} = \frac{x-30}{5}$$

when $x = 45$, $Z = \frac{45-30}{5} = 3$

$$P(X \geq 45) = P(Z \geq 3)$$
$$= 0.5 - A(3)$$
$$= 0.5 - 0.4987$$
$$= 0.0013$$

Suppose the weights of 800 male students are normally distributed with mean $\mu = 140$ pounds and standard deviation $\sigma = 10$ pounds. find the no. of students whose weights are (i) between 138 and 148 pounds
(ii) More than 152 pounds.

Let X denotes weights of the students
given that, $X \sim N(\mu, \sigma^2)$

$\mu = 140$

$\sigma = 10$

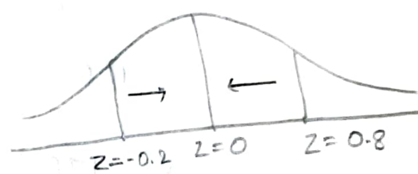
$n = 800$

(i) $P(138 \le X \le 148)$

$$z = \frac{x-\mu}{\sigma} = \frac{x-140}{10}$$

when $x = 138$, $z = \frac{138-140}{10} = -0.2$

when $x = 148$, $z = \frac{148-140}{10} = 0.8$

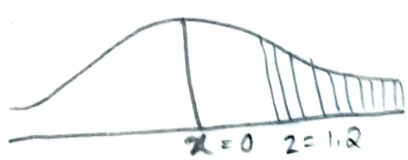


$$P(138 \le X \le 148) = P(-0.2 \le Z \le 0.8)$$
$$= A(0.2) + A(0.8)$$
$$= 0.0793 + 0.2881$$
$$= 0.3674$$

The no. of students where weights between 138 Pounds and 148 Pounds is $n \cdot P(138 \le X \le 148) = 800 \times 0.3674$
$$= 293.92 \simeq 294$$

(ii) $P(X \ge 152)$

when $X = 152$, $z = \frac{152-140}{10} = 1.2$



$$P(X \ge 152) = P(Z \ge 1.2)$$
$$= 0.5 - A(1.2)$$
$$= 0.5 - 0.3849$$
$$= 0.1151$$

The no. of students whose weights more than 152 pounds is

$= n \cdot P(X \geq 152)$

$= 800 \times 0.1151$

$= 92.08$

$\simeq 92$

Given that the mean height of students in a class is 158cm with standard deviation of 20cm. find how many students height lies between 150cm and 170cm, If there are 100 students in a class

Let x denotes heights of the students in a class

Given that $X \sim N(\mu, \sigma^2)$

$\mu = 158cm$ Mean height

$\sigma = 20\,cm$ standard deviation

$n = 100$ No.of students

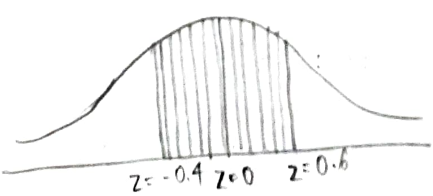
$P(150 \leq X \leq 170)$

we have $Z = \frac{x-\mu}{\sigma} = \frac{x-158}{20}$

when $x = 150$ $Z = \frac{150-158}{20} = \frac{-8}{20} = -0.4$

when $x = 170$ $Z = \frac{170-158}{20} = \frac{12}{20} = 0.6$

$P(150 \leq x \leq 170) = P(-0.4 \leq z \leq 0.6)$



$= A(0.4) + A(0.6)$

$= 0.1554 + 0.2258$

$= 0.3812$

No.of students whose heights lies between 150cm & 170cm is

$n \cdot P(150 \leq x \leq 170)$

$= 100 \times 0.3812$

$= 38.12 \simeq 38$

A sales tax office reported that the average sales of 500 business that he has to deal with during the year is 36000 with a standard deviation of 10000 ruppes. Assuming that the sales in his business are normally distributed find

(i) No. of business has the sales of which are above 40000 rupees

(ii) The percentage of bussiness the sales of which are likely to range b/w 30000 and 40000 rupees.

Let x denotes sale of business

Given that $x \sim N(\mu, \sigma^2)$

Average sales $\mu = 36,000$

Standard deviation $\sigma = 10,000$

No. of business $n = 500$

(i) $P(x > 40000)$

we have $z = \dfrac{x-\mu}{\sigma} = \dfrac{x - 36000}{10000}$

when $x = 40000$ $z = \dfrac{40000-36000}{10000} = 0.4$

$P(x > 40000) = P(Z > 0.4)$

$= 0.5 - A(0.4)$

$= 0.5 - 0.1554$

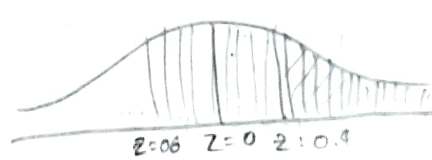
$= 0.3446$

No. of Companies making more than 40000 rupees business

$= n\,P(x > 40000)$

$= 500 \times 0.3446$

$= 172.3 \simeq 172$



(ii) $P(30000 \le X \le 40000)$

when $x = 30000$ $z = \dfrac{30000-36000}{10000} = -0.6$

when $x = 40000$ $z = \dfrac{40000-36000}{10000} = 0.4$

$P(30000 \le x \le 40000) = P(-0.6 \le z \le 0.4)$

$= A(-0.6) + A(0.4)$

$= 0.2258 + 0.1554$

$= 0.3812$

$= 38.12\%$

Find the mean and standard deviation of a normal distribution in which 7% of items are under 35 and 89% are under 63.

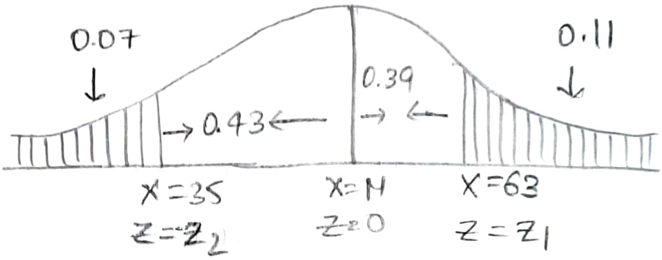
Given that $X \sim N(\mu, \sigma^2)$

$P(X<35) = 7\% = 0.07$

$P(x<63) = 89\% = 0.89$

$P(x>63) = 1-0.89$

$= 0.11$



We have, $z = \frac{x-\mu}{\sigma}$.

when $x=35$ $\quad z = \frac{35-\mu}{\sigma} = -z_1$ say ——(i)

when $x=63$ $\quad z = \frac{63-\mu}{\sigma} = z_2$ say ——(ii)

from the Normal curves, $P(0<z<z_1) = 0.5-0.07$

$= 0.43$

$\Rightarrow z_1 = -1.48$ from Normal tables

$P(0<z<z_2) = 0.5-0.11 = 0.39$

$\Rightarrow z_2 = 1.23$ from Normal tables

from (i), $\frac{35-\mu}{\sigma} = -1.48 \Rightarrow \frac{35}{\sigma} - \frac{\mu}{\sigma} = -1.48 \rightarrow$ (iii)

from (ii), $\frac{63-\mu}{\sigma} = 1.23 \Rightarrow \frac{63}{\sigma} - \frac{\mu}{\sigma} = 1.23 \rightarrow$ (iv)

(iv) − (iii) $\Rightarrow \frac{63}{\sigma} - \frac{35}{\sigma} = 2.71$

$$\frac{28}{\sigma} = 2.71 \Rightarrow \sigma = 10.3$$

from (iv), $\dfrac{63 - \mu}{10.3} = 1.23$

$$\Rightarrow \mu = 63 - (10.3)(1.23)$$

$$= 50.3$$

Sampling Distribution

Population
Sample
Population size → N
Sample size → n
large sample → $n \geq 30$
small sample → $n < 30$

- simple random sampling
- stratified sampling
- systematic sampling

Parameter : pop. mean μ, pop. variance, σ^2.
Statistics : sample mean $\bar{x}$, sample variance s^2

$$\bar{x} = \frac{1}{n} \sum_i x_i \qquad s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

Property Distribution of any statistics is called sampling Distribution
If the statistic is the sample mean than it is called as sample Distribution of Means
If the statistic is the sample variance then it is called as sample Distribution of sample sample

Central limit Theorem

$n, \bar{x} \to \mu, \sigma^2 \qquad z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$ as $n \to \infty$

A sample of size n ~~[illegible]~~ with mean $\bar{x}$, can be taken from a normal population with mean μ and variance σ^2

→ standard deviation of the statistics.
Standard Error of $\bar{x}$ is $\frac{\sigma}{\sqrt{n}}$

Sampling Distribution of Means.
$[N^n]$ Infinite population :- sampling is done with replacement

samples can be taken from a infinite population i.e sampling in done with replacement

Mean of the sampling distribution of Means, $\mu_{\bar{x}} = \mu$

variance of the SD of means, $\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$

[Non]finite population :- samples can be taken from a finite population i.e sampling is done without replacement

Mean of the sampling distribution of means, $\mu_{\bar{x}} = \mu$

variance of the SD of means, $\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right)$

Population correction factor $= \frac{N-n}{N-1}$

** A population consist of five numbers 2, 3, 6, 8, 11 consider all possible samples of size two can be drawn with replacement from the population. find (i) Mean of the population

(ii) SD of the population

(iii) Mean of the sample Distribution of the Means

(iv) SD of the sample distribution of " standard deviation of means

Given population is 2, 3, 6, 8, 11

$N = 5 \qquad n = 2$

Population Mean, $\mu = \frac{2+3+6+8+11}{5} = \frac{30}{5} = 6$

Population variance, $\sigma^2 = \frac{1}{5}\left[(2-6)^2 + (3-6)^2 + (6-6)^2 + (8-6)^2 + (11-6)^2\right]$

$= \frac{1}{5}[16+9+0+4+25] = \frac{54}{5} = 10.8$

Population S.D, $\sigma = \sqrt{10.8} = 3.286$

since sampling is done with replacement

No. of samples obtained $= N^n = 5^2 = 25$

25 visible samples are (2,2) (2,3) (2,6) (2,8) (2,11)

(3,2) (3,3) (3,6) (3,8) (3,11)

(6,2) (6,3) (6,6) (6,8) (6,11)

(8,2) (8,3) (8,6) (8,8) (6,11)

(11,2) (11,3) (11,6) (11,8) (11,11)

The samples Mean of the above 25 visible samples are

sample Distribution of Mean
$$\begin{cases} 2 & 2.5 & 4 & 5 & 6.5 \\ 2.5 & 3 & 4.5 & 5.5 & 7 \\ 4 & 4.5 & 6 & 7 & 8.5 \\ 5 & 5.5 & 7 & 8 & 9.5 \\ 6.5 & 7 & 8.5 & 9.5 & 11 \end{cases}$$

$\mu_{\bar{x}} = 6$

$$\text{Mean} = \frac{120}{25} = 6$$

$$\sigma^2 = \frac{1}{25}\Big[(2-6)^2 + (2.5-6)^2 + (4-6)^2 + (5-6)^2 + (6.5-6)^2 + (2.5-6)^2 + (3-6)^2 + (4.5-6)^2 + (5.5-6)^2 + (7-6)^2 + (8.5-6)^2 + (5-6)^2 + (5.5-6)^2 + (7-6)^2 + (8-6)^2 + (9.5-6)^2 + (6.5-6)^2 + (7-6)^2 + (8.5-6)^2 + (9.5-6)^2 + (11-6)^2\Big]$$

$$= \frac{1}{25}[135] = 5.4$$

$$\sigma_{\bar{x}} = \sqrt{5.4} = (2.32)$$

(2) Without replacement [2, 3, 6, 8, 11]

(i) Mean of the sample Distribution of Mean

(ii) Variance of the sample Distribution of Mean

$N = 2$, $n = 2$

No. of sample obtained $= {}^{N}C_{n} = {}^{5}C_{2} = 10 = K$, say

(2,3) (2,6) (2,8) (2,11)

(3,6) (3,8) (3,11)

(6,8) (6,11)

(8,11)

2.5	4	5	6.5
	4.5	5.5	7
		7	8.5
			9.5

$$\mu_{\bar{x}} = \frac{1}{10}\left[2.5 + 4 + 5 + 6.5 + 4.5 + 5.5 + 7 + 7 + 8.5 + 9.5\right]$$

$$= \frac{1}{10}(60) = 6.$$

$$\sigma^2_{\bar{x}} = \frac{1}{k}\sum_i (\bar{x}_i - \mu_{\bar{x}})^2$$

$$= \frac{1}{10}\left[(2.5-6)^2 + (4-6)^2 + (5-6)^2 + (6.5-6)^2 + (4.5-6)^2 + (5.5-6)^2 + (7-6)^2 + (7-6)^2 + (8.5-6)^2 + (9.5-6)^2\right]$$

$$= \frac{1}{10}(40.5) = 4.05$$

$$\frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right) = \frac{10.8}{2} \times \frac{3}{4} = 4.05$$

Samples of size two are taken from population 4, 8, 12, 16, 20, 24 without replacement. Find (i) Population Mean (ii) Population SD

The population is 3, 6, 9, 15, 27
(i) List all possible samples of size three that can be taken without replacement from the finite population.
(ii) Find the mean of sample distribution of means
(iii) Find the SD of sample distribution of means

$N = 5$, $n = 3$

No. of samples obtained $= {}^{N}C_{n} = {}^{5}C_{3} = 10 = K$, say.

(3, 6, 9) (3, 6, 15) (3, 6, 27)
(3, 9, 15) (3, 9, 27) (3, 15, 27)
(6, 9, 15) (6, 9, 27) (9, 15, 27)
(9, 15, 27)

6, 8, 12, 9, 13, 15, 10, 14, 16, 17

$\text{Mean} = \frac{120}{10} = 12$

Find the Mean and standard deviation of sample distribution of variances for the population 2, 3, 4, 5 by drawing samples of size two with replacement $N=4$ $n=2$