UNIT – I : HandOut

Algorithm: A step by step process to perform a particular task.

Algorithm Features: Input/Output, Finiteness, definiteness, Effectiveness, Generality

Performance Analysis: Determining an estimate of the time and memory requirement of the algorithm.

- Time estimation is called time complexity analysis
- Memory size estimation is called space complexity analysis.

Notations used to represent Time Complexity:

- **Big-O Notation** : f(n)=**O(g(n))** iff there exist a positive constant C and non-negative integer n_0 such that f(n) □ Cg(n) for all n□n0.
- **Omega Notation:** f(n) = □ **(g(n))** iff there exist a positive constant C and non-negative integer n_0 such that f(n) ≥ Cg(n) for all n> n_0.
- **Theta Notation:** □*(g) = O(g)* □□□□*(g) :* function *f* is bounded both from above and below by the same function *g*
  - *F(n) =* □*(g) iff* $c_1 g(n) < f(n) < c_2 g(n)$ for all *n>N*

Recursion: Process of calling a function by itself.

Algorithm fact(n)	Recursive Alg.: gcd(a,b)	Recursive Alg.: fib(n)
{	begin	begin
if((n==0) || (n==1)	if(b==0)	if (n <= 1)
return 1;	return a;	return n;
else	else	else
return(n*fact(n-1));	return gcd(b, a mod b)	return (fib(n-1) + fib(n-2));
}	end	end

Recursion Types: Direct recursion, Indirect recursion.

Linear Recursion, Binary Recursion, Tail Recursion

UNIT - I

Algorithm: Algorithm is a step by step process used to perform a particular task. Algorithm can be writen in General English i.e not by using any specific computer programming language.

Eg: Algorithm: To perform the addition of two numbers

Step 1: Start

Step 2: Read the values of a,b

Step 3: c = a + b

Step 4: print c

Step 5: Stop

Algorithm Features: The various characteristics of an algorithm are,

Input/Output, Finiteness, definiteness, Effectiveness, Generality

Input/Output : Algorithm should consist zero or more but only finite number of inputs and atleast one output.

Finiteness: An algorithm must terminate after a finite number of steps and further each step must be executable in finite amount of time that it terminates (in finite number of steps) on all allowed inputs.

Definiteness: Every step should be precisely defined means each and every step of the algorithm should be simple and clear i.e. easy to understand without any ambiguity.

Effectiveness: Each step of the Algorithm must be feasible i.e., it should be practically possible to perform the action. In other words, algorithm can be converted into code.

Performance Analysis: Determining an estimate of the time and memory requirement of the algorithm.

- Time estimation is called time complexity analysis
- Memory size estimation is called space complexity analysis.

Space Complexity:

Space Complexity of an algorithm is the amount of memory it needs to run to completion.
The space needed by any algorithm is the sum of the following components.

1. A **fixed part**, that is independent of the characteristics of inputs and outputs. This part includes space for instructions(code), space for simple variables, & fixed size component variables, space for constants etc.
2. A **variable part**, which consists of space needed by component variables whose size, is dependent on the particular problem instance being solved, space for reference variables and recursion stack space etc.

The Space requirement *S(P)* of an algorithm *P* may be written as
$S(P) = c + S_P(\text{instance characteristics})$ where *'c'* is a constant.

When analyzing the space complexity of an algorithm first we estimate $S_P(\text{instance characteristics})$. For any given problem, we need to determine which instance characteristics to use to measure the space requirements.

Example 1:

```
1    Algorithm abc(a, b, c)
2    {
3       return a+b+b*c + (a+b –c) / ( a+b)
4    + 4.0;
     }
```

It is characterized by values of *a, b, c*. If we assure that one word is needed to store the values of each *a, b, c , result* and also we see $S_P(instance\ characteristics)=0$ as space needed by *abc* is independent of instance characteristics; So 4 words of space is needed by this algorithm.

Example 2:

1
2
3
4
5
6
7
	Algorithm sum(*a, n*)
	{
	s := 0.0;
	
	for *i* := 1 **to** *n* **do**
	s := s+a[i];
	return *s*;
	}
		This algorithm is characterized by *'n'* (number of elements to be summed). The space required for *'n'* is 1 word. The array *a[]* of float values require atleast *'n'* words. So, we obtain
		$\boldsymbol{S_{sum}(n) \geq n+3}$
		(*n* words for *a*, 1 word for each of *n, i, s*)

Time Complexity:

The time complexity of an algorithm is the amount of computer time it needs to run to completion.

- The time *T(P)* taken by a program *P* is the sum of the *compile time* and *Run time*. Compile time does not depend on instance characteristics and a compiled program will be run several times without recompilation, so we concern with just run time of the program. Run time is denoted by $\boldsymbol{T_P(instance\ characteristics)}$.
- The time complexity of an algorithm is given by the number of steps taken by the algorithm to compute the function. The number of steps is computed as a function of some subset of number of inputs and outputs and magnitudes of inputs and outputs.
- A program step is loosely defined as a syntactically or semantically meaningful segment of a program that has an execution time that is independent of the instance characteristics.
- The number of steps any problem statement is assigned depends on the kind of statement.
  - For example, comments 0 steps.
  - Assignment statements 1 step
  - Iterative statements such as for, while & repeat-until 1 step for control part of the statement.
- Operation Counts
  - Based on compiler used: number of adds, multiply, compares etc.
  - Success depends on our ability to identify the operations that contribute most to the time complexity
- Step Counts
  - Account for all time spent in all parts of the program/function
  - Function of instance characteristics
  - Definition: A **program step** is syntactically or semantically meaningful segment of a program for which the execution time is independent of the instance characteristics
    - Ex: 10 additions can be one step, 100 multiplications – one step
- Instance characteristics
  - *Number of inputs, outputs etc.*

- *More complex example:*
  - Number of swaps performed by a Bubble sort
  - Depends on the instance characteristic array size (n), but also on the values: the numbers of swaps varies from *0* to *n-1*
- Operation count is not uniquely determinable by the chosen instance characteristics
- Need best, worst, and average counts

- Number of steps needed by a program to solve a particular problem is determined in two ways.

Method 1 : (Step count calculation using a variable)

- In this method, a new global variable count with initial value '0' is introduced into the program.
- Statements to increment count are also added into the program.
- Each time a statement in the original program is executed, count is incremented by the step count of the statement.

Example:

```
1     Algorithm Sum (a, n)
2     {
3        s:=0.0;
4        count :=count + 1;  //count is global;
5     initially 0
6        for i := 1 to n do
7        {
8           count:=count+1;  //for for
9           s:=s+a[i]; count:=count+1; //for
10    assignment
11       }
12       count :=count + 1;  //for last time of for
13       count :=count + 1;  //for the return
14       return s;
      }
```

- The change in the value of *count* by the time this program terminates is the number of steps executed by the algorithm.
- The value of count is increment by ***2n*** in the for loop.
- At the time of termination the value of count is ***2n+3***.
- So invocation of Sum executes a total of **2n+3** steps.

Example 2:

```
Algorithm RSum(a, n)
{
   count:=count+1; //for the if
conditional
   if ( n ≤ 0) then
   {
        count:=count+1;  //for the return
        return 0.0;
   }
   else
   {
        count:=count+1; //for addition,
function call, return
        return RSum(a,n-1)+a[n];
   }
}
```

- Let $t_{RSum}(n)$ be the increase in the value of *count* when the algorithm terminates.
- When n=0, $t_{RSum}(0) = 2$
- When n>0,*count* increases by 2 plus $t_{RSum}(n-1)$
- When analyzing a recursive program for its step count, we often obtain a recursive formula.
- These Recursive formulas are referred to as *recurrence relations*

To solve it, use repeated substitutions

$$
\begin{aligned}
t_{RSum}(n) &= 2 + t_{RSum}(n-1) \\
&= 2 + 2 + t_{RSum}(n-2) \\
&= 2(2) + t_{RSum}(n-2) \\
&: \\
&= n(2) + t_{RSum}(0) \\
&= 2n + 2, \qquad n \geq 0
\end{aligned}
$$

So the step count of *RSum* is $2n+2$.
This step count is telling *the run time for a program with the change in instance characteristics*.

Method 2 : (Step count calculation by building a table)

- In this method, the step count is determined by building a table. We list total number of steps contributed by each statement in the table. The table is build in this order.
- Determine the number of steps per execution (s/e) of the statement and the total number of times (frequency) each statement is executed.
- (*The s/e of a statement is the amount by which the count changes as a result of the execution of that statement.)*
- The total contribution of the statement is obtained by combining these two quantities.
- Step count of the algorithm is obtained by adding the contribution of all statements.

Asymoptotic Notations used for Time Complexity:

The various notations used to measure time complexity are:

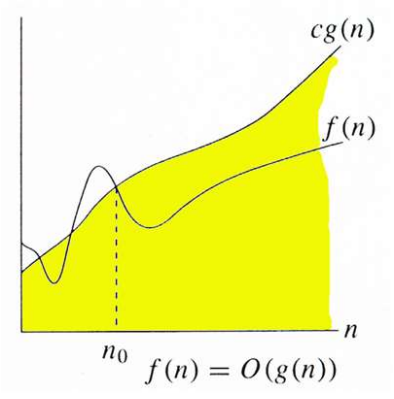
- Big-Oh notation(O)
- Omega Notation( )
- Theta Notation( )

Big-O Notation :

- Let n be a non-negative integer representing the size of the input to an algorithm
- The Big O notation defines an upper bound of an algorithm; it bounds a function only from above.
- Big-Oh notation is used widely to characterize running time and space bounds in terms of some parameter *n*, which varies from problem to problem.
- Constant factors and lower order terms are not included in the big-Oh notation.
- For example, consider the case of Insertion Sort. It takes linear time in best case and quadratic time in worst case. We can safely say that the time complexity of Insertion sort is $O(n^2)$. Note that $O(n^2)$ also covers linear time.

- **Def:** The function $f(n)=O(g(n))$ iff there exists positive constants c and n_0 such that
  - $f(n) \leq c*g(n)$ for all n, $n \geq n_0$.

- Figure shows that, for all values n at and to the right of n_0, the value of function $f(n)$ is on or below $c.g(n)$.



- When computing the complexity,
  - *f(n) is the actual time formula*
  - *g(n) is the simplified version of f*
  - Since f(n) stands often for time, we use T(n) instead of f(n)
  - In practice, the simplification of T(n) occurs while it is being computed by the designer
- If T(n) is the sum of a constant number of terms, drop all the terms except for the most dominant (biggest) term;
- Simplification Methods:
  - Drop any multiplicative factor of that term
  - What remains is the simplified g(n).
  - $a_m n^m + a_{m-1} n^{m-1} + \ldots + a_1 n + a_0 = O(n^m)$.
  - $n^2 - n + \log n = O(n^2)$

Example : *7n – 2* is *O(n)*

Proof: By the big-Oh definition, we need to find a real constant $c>0$ and an integer constant $n_0 \geq 1$ such that $7n - 2 \leq cn$ for every integer $n \geq n_0$.

Possible choice is $c=7$ and $n_0=1$.

Example: $20n^3+10n \log n+5$ *is* $O(n^3)$

Proof: $20n^3+10n \log n+5 \leq 35n^3$, for $n \geq 1$

In fact, any polynomial $a_k n^k + a_{k-1} n^{k-1} + \ldots + a_0$ will always be $O(n^k)$.

Here is a list of functions that are commonly encountered when analyzing algorithms. The slower growing functions are listed first. k is some arbitrary constant.

Notation	• Name
O(1)	Constant
O(log n)	Logarithmic
O(n)	Linear
O(n log n)	Polylogarithmic
O(n^2)	Quadratic
O(n^3)	Cubic
O(n^k) (k≥1)	Polynomial
O(k^n) (k>1)	exponential

- Instead of always applying the big-Oh definition directly to obtain a big-Oh characterization, we can use the following rules to simplify notation.

- Example:

  $f(x) = x^2 + 2x + 1$.

  For $x > 1$ we have:

$x^2 + 2x + 1 <= x^2 + 2x^2 + x^2$

$x^2 + 2x + 1 <= 4x^2$

Therefore, for C = 4 and k = 1:

$f(x) <= Cx^2$ whenever $x > k$.

So, $f(x)$ is $O(x^2)$.

- If $f(x)$ is $O(x^2)$, is it also $O(x^3)$? **Yes.** x^3 grows faster than x^2, so x^3 grows also faster than $f(x)$. Therefore, we always have to find the **smallest** simple function $g(x)$ for which $f(x)$ is $O(g(x))$.
- "Popular" functions g(n) are n log n, 1, 2^n, n^2, n!, n, n^3, log n
- Listed from slowest to fastest growth: 1 log n n n log n n^2 n^3 2^n n!
- **Useful Rules for Big-O**
  - For any **polynomial** $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0$, where $a_0, a_1, \ldots, a_n$ are real numbers, $f(x)$ is $O(x^n)$.
  - If $f_1(x)$ is $O(g_1(x))$ and $f_2(x)$ is $O(g_2(x))$, then
    $(f_1 + f_2)(x)$ is $O(\max(g_1(x), g_2(x)))$
  - If $f_1(x)$ is $O(g(x))$ and $f_2(x)$ is $O(g(x))$, then
    $(f_1 + f_2)(x)$ is $O(g(x))$.
  - If $f_1(x)$ is $O(g_1(x))$ and $f_2(x)$ is $O(g_2(x))$, then
    $(f_1 f_2)(x)$ is $O(g_1(x)\, g_2(x))$.

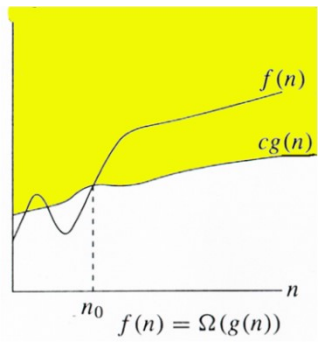
- **[Omega] Ω-notation:**

  Ω-notation provides an asymptotic upper bound.

  Def: The function $f(n)=\Omega(g(n))$ iff there exists positive constants c and n_0 such that

  $f(n) \geq c*g(n)$ for all n, $n \geq n_0$

  Figure shows the intuition behind Ω- notation for all values n at or to the right of n_0, the value of $f(n)$ is on or above $c.g(n)$.



- If the running time of an algorithm is $\Omega(g(n))$, then the meaning is, "the running time on that input is atleast a constant times $g(n)$, for sufficiently large n".

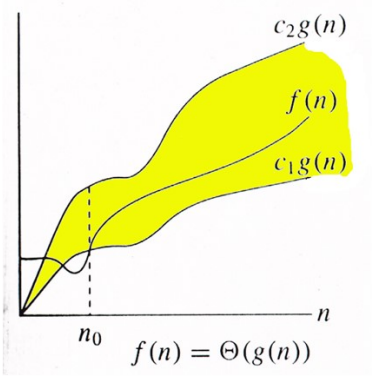
- It says that, $\Omega(n)$ gives a lower-bound on the best-case running time of an algorithm.

[Theta] - notation :

Def: The function f(n) = $\Theta(g(n))$ iff there exists positive constants c_1, c_2 and n_0 such that

$c1*g(n) \leq f(n) \leq c2*g(n)$ for all n, $n \geq n_0$

Following figure gives an intuitive picture of functions $f(n)$ and $g(n)$, where .



- For all values of n at and to the right of n_0, the value of $f(n)$ lies at or above $c1.g(n)$ and at or below $c2.g(n)$.
- For all $n \geq n_0$, the function $f(n)$ is equal to $g(n)$ to within constant factors.
- We can say that $g(n)$ is an asymptotically tight bound for $f(n)$.

SEARCHING

Searching is the (concept) mechanism of searching an element in the list of da ta stored. Basically it is used to find the location or to see whether element is available or not in the list. The application of search occurs when a particular element is required from a set of stored elements. For example, consider a database of telephone directory. It is required to find the address when the telephone number is known.

The efficiency of a search tends to depend upon three things.

- The size and organization of the collection of data we are searching
- the search algorithm being used
- The efficiency of the test used to determine if the search is successful.

The popular searching methods are:

- Linear search
- Binary Search
- Fibonacci search

Any search method requires three components. They are,

- **Search List:** The search list is simply a general term for the collection of elements that we are searching.
- **Key element/ Target element:** This is the term we use to refer to the element that we are searching for. When ever a search is conducted, there are two possible outcomes:
  - The element is found in the list (successful search)
  - The element is not found (unsuccessful search)
- **Test function (or) operator :** These are used to determine how an element in the search list matched the target element. For numbers ==, !=, >= etc and for strings strcmp(), strncmp(), strcmpi() etc are used.

Hence, searching is a terchnique of finding accurate location of an element in the given data list. If the key element is present in the search list then search process is said to be successful and the search is said to be unsuccessful if the given element does not exist in the list.

Linear Search

Linear search, also referred as sequential search is the simplest searching technique. In this, the required element (key element) is searched linear through out the search list.

The search begins atone end of the list and searches for the required element one by one until the element is found or till the end of the list is reached. i.e. from the given list, start the search process by comparing the first element with the key element. If the match is found, then stop the search. Otherwise compare the second element and so on. Continue this process till the key element is

found or there are no more elements in the list (in this case the element is not there in the list). The search is said to be successful if the search element is found and unsuccessful if the search element is not found.

For example, consider the list of 5 elements as, 23 45 67 32 86

- if the key element is 67 then, Start searching by comparing the first element (23) with the key element(67) . As two elements are not matched, move to second(45) and so on. The element is found at position 3.
- If the key element is 58 then search linearly through out the list. In this case, the element is not present in the list.

Algorithm: Linear_Search(A, key,n)

```
// A is the linear array with n elements and key is the key element
{
        i <-- 0;  // initialize linear search
        while ( a[i] ≠key) do
                i <--  i+1;
        if ( i=n) then
          print ' unsuccessful search'
        else
           print ' element found at ', i
}
```

Example:

```
/* program for linear serach using non-resursive function*/
#include<stdio.h>
#include<conio.h>
int l_search(int[],int,int);
void main()
{
 int a[10],i,n,key,x;
 clrscr();
 printf("Enter Integer Array size :");
 scanf("%d",&n);
 printf("\n Enter elements \n");
 for(i=0;i<n;i++)
   scanf("%d",&a[i]);
 printf("\n Enter Key element to search :");
   scanf("%d",&key);
 x=l_search(a,key,n);
 if(x==-1)
   printf("unsuccessful search");
 else
  printf("key element found at:%d",x);
 getch();
}
int l_search(int a[],int key,int n)
{
 int i;
 for(i=0;i<n;i++)
 {
  if(key==a[i])
    return i;
 }
 return -1;
}
```

```
/* program for linear search using recursive function */

#include<stdio.h>
#include<conio.h>
int l_search(int[],int,int);
void main()
{
 int a[10],i,n,key,x;
 clrscr();
 printf("Enter Integer Array size :");
 scanf("%d",&n);
 printf("\n Enter elements \n");
 for(i=0;i<n;i++)
   scanf("%d",&a[i]);
 printf("\n Enter Key element to search :");
   scanf("%d",&key);
 x=l_search(a,key,n-1);
 if(x==-1)
   printf("unsuccessful search");
 else
  printf("key element found at:%d",x);
 getch();
}
int l_search(int a[],int key,int i)
{

  if(i<0)
    return -1;
  if(key==a[i])
    return i;
  return l_search(a,key,i-1);

}
```

- The linear search can be applied on sorted or unsorted list of elements i.e. this method required no ordering of elements in the list that's why it is some times referred to as ' brute force' method.
- For large size lists, the performance of linear search, how ever makes it a poor search strategy . The time it takes to perform linear search grows proportationally with the size of the list.

Analysis:

- If there are N items in the list, then in the 'worst case' (i.e. where there is no key element in the list or key element is the last element in the list) N comparisons are required.
- The 'best case' , in which the first comparison returns a match, it requires a single(1)

comparison.

- The average case roughly requires 'N/1' comparisons to search the element.

Time Complexity	Best Case	Best Case
	O(1)	O(N)

Binary Search

The binary search method is a classical method and it is one of the most efficient searching techniques, which requires the list to be sorted.

To search for an element in the list the binary search procedure splits the list and locates the moddle element of the list. It is then compared with the search element. If the search element is matched with the middle element, then the search is complete. Otherwise, the key element may be in the upperhalf or lower half. If the key element is less than the middle element, the first part(upperhalf/ left half) of the list is searched else the second part(lower half/ right half) of the list is searched. The process is continued until the key element is found or the portion of the sublist to be searched is empty.

In this method,

- the terms ub and lb are used to represent last and first positions of an array. The mid position is found out as , mid = (lb+ub)/2.
- Now, compare the key element with the element in the mid position then, any one the following cases may arise:

case 1: If the key element is equal to the element present in the mid position, then the search option is complete.

Case 2: if the key element is less than the element present in the mid position, then there is no need to check elements in the second(right) half of array i.e. right half can be excluded. Now assign mid-1 to ub.

Case 3: If the key element is greater than the element present in the mid position, then there is no need to check the elements in the first(left) half of array i.e. left half can be excluded. Now assign mid+1 to lb.

- This process is executed repeatedly on the non-excluded portion of the array until the key element is found or if we eliminate all elements from search then we can say the element is not present in the given list.

Note: Since, in this method, we eliminate elements by half each time, this method is faster when compared to sequential search.

Algorithm: Binary_Search(A,N,Key)

```
//  A is an array consisting of N elements in ascending order.
// Key is the element to be searched
{
      lb <-- 0;  ub <-- N-1;
      while ( lb<=ub)   do
      {
          mid <-- (lb + ub)/2;
          if(key = A[mid]) then
               break;
          else
               if( A[mid] > key) then
                     ub <-- mid-1;
             else
                     lb <-- mid+1;
          }
          if(lb>ub)
            print ' unsuccessful search';
          else
             print 'element found at', mid;
}
```

/* Program for Binary Search (non - recursive) */

```
#include<stdio.h>
#include<stdlib.h>
#include<conio.h>
int binarysearch(int a[],int key,int low,int high)
{
  int mid;
  while(low<=high)
  {
    mid=(low+high)/2;
    if(a[mid]==key)
       return mid;
    if(a[mid]>key)
      high=mid-1;
    else
      low=mid+1;
  }
  return -1;
}
void main()
{
 int a[10],i,n,key,x;
 clrscr();
 printf ("Enter Integer Array size :");
 scanf("%d",&n);
 printf("\n Enter elements in Sorted order\n");
 for(i=0;i<n;i++)
   scanf("%d",&a[i]);
 printf("\n Enter Key element to search :");
 scanf("%d",&key);
 x=binarysearch(a,key,0,n-1);
 if(x==-1)
   printf("the key element not found");
 else
   printf("the key element found at:%d",x);
 getch();
}
```

/* Program for Binary Search (recursive) */

```
#include<stdio.h>
#include<conio.h>
int binarysearch(int a[],int key,int low,int high)
{
  int mid;
  if (low>high)
        return -1;
  else
  {
    mid=(low+high)/2;
    if(a[mid==key)
      return mid;
    else
      if(a[mid]>key)
        return  binarysearch(a,key,lb,mid-1);
    else
      return binarysearch(a,key,mid+1,ub);
  }
}
void main()
{
 int a[10],i,n,key,x;
 clrscr();
 printf ("Enter Integer Array size :");
 scanf("%d",&n);
 printf("\n Enter elements in Sorted order\n");

 for(i=0;i<n;i++)
   scanf("%d",&a[i]);

 printf("\n Enter Key element to search :");
   scanf("%d",&key);

 x=binarysearch(a,key,0,n-1);
 if(x==-1)
   printf("the key element not found");
 else
   printf("the key element found at:%d",x);
 getch();
}
```

SORTING

"Sorting refers to the operation of rearranging data in either in ascending or descending order".

* the data may be numerical data or character data.

The sorting methods are classified into two types. They are ' Internal Sorting' and 'External Sorting'. In internal sorting, all the data elements to be sorted are present in the main memory. External sorting techniques are applied to large data sets which reside on secondary storage devices and cannot be completely fit in main memory.

There are many sorting methods available. But no method for sorting is best in all cases. The factors to be considered while choosing a sorting technique are,

- Programming time of the sorting technique
- Execution time of the sorting technique
- No. of comparisons required for sorting the list.
- Main or secondary memory space needed for sorting technique.

Different applications require different sorting methods. The different sorting methods are,

- Bubble Sort (or) Exchange Sort
- Selection Sort
- Quick Sort (or) Partition Exchange Sort
- Insertion Sort
- Merge Sort

Note: A sorting method is said to be stable when it has the minimum no. of swaps.

- Stable sorting methods are - Bubble Sort, Selection Sort & Quick Sort
- Unstable Sorting methods are – Insertion Sort, Merge Sort

Bubble Sort (or) Exchange Sort

The simplest and the most widely used sorting technique is ' Bubble Sort'.

This method compares the two adjacent(consecutive) elements of the list. If thesy are not in order, th two elemtns will be interchanged. If they are in order, the two elements remains the same. This process continues (n-1) times for sorting an array of size n. There ends one pass. During the first pass the largest/smallest will be moved to Nth position. The second pass will be continued with first (N-1) elements. Repeat this process till all the elemtns are in sorted order.

* Since, each pass places one element into its proper position, a list of N elements requires (N-1) passes.

The procedure for bubble sort to sort 'N' elements in ascending order is,

Step 1: First compare two elements at time starting with the first tow elements.

Step 2: If the first element is larger than second element then exchange the two elements

Step 3: Go down one element and compare that element to the element that follows it.

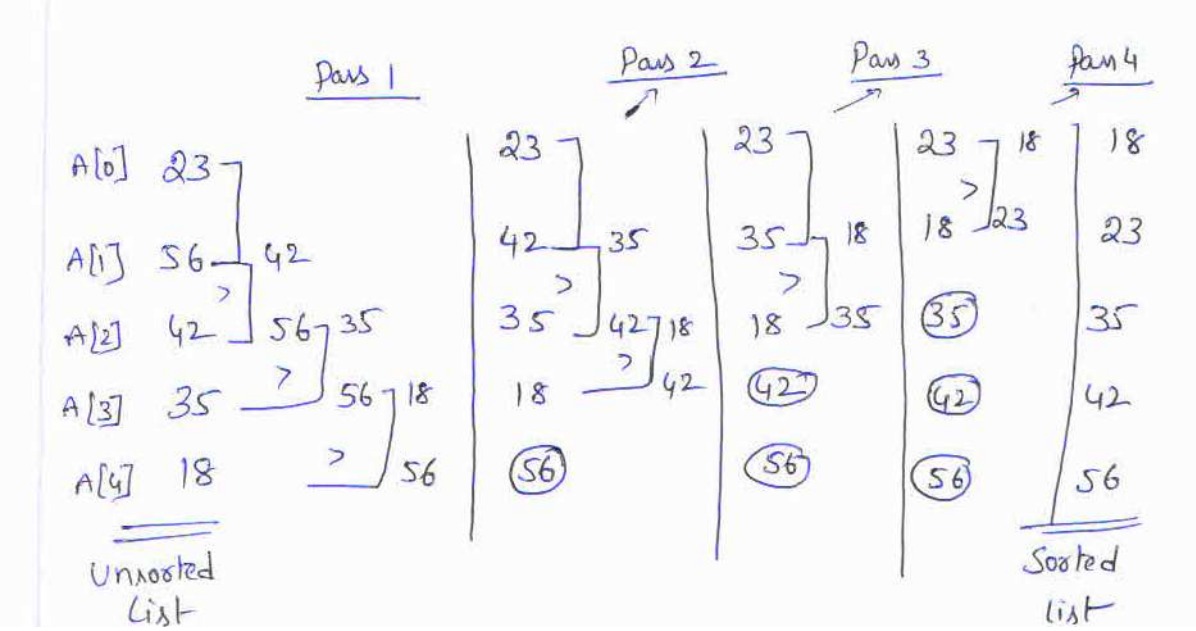
Step 4: Continue this process till the end of list. (During this process the largest element moved to Nth position)

Step 5: Repeat steps 1 thru 4 (N-1) times by leaving the last element of sorting list each time.

For example, consider the list of 5 elements,

A[0]	A[1]	A[2]	A[3]	a[4]
23	56	42	35	18

The following comparisons are made in each pass:



" The first largest element is placed in proper position i.e. at A[N-1] after the first pass" . In general " Kth largest element is placed in its proper position i.e. at A[n-k] after pass 'k' ".

The complete set of passes in bubble sort method is,

Algorithm: Bubble_Sort(A,N)

```
// A is an array of N elements
{
        for i:= 1 to N-1 do
        {
          for j:=0 to N-i-1  do
             if( A[j] > A[j+1] ) then
                  swap( A[j],A[j+1]);
        }
}
```

```
/*  Program for BUBBLE SORT   */
#include<stdio.h>
#include<stdlib.h>
#include<conio.h>
void Bubble_Sort( int[], int)
void main()
{
        int a[10],n,i;
        clrscr();
        printf("\n Enter Array size :");
        scanf("%d",&n);
        printf("\n Enter elements \n");
        for (i=0;i<n;i++)
        scanf("%d",&a[i]);
        bubblesort(a,n);
        printf("\n Array elements after sorting:
        \n");
        for (i=0;i<n;i++)
        printf("%d  ",a[i]);
        getch();
}
```

```
void Bubble_Sort(int a[],int n)
{
   int i,j,temp;
   for(i=1;i<n;i++)
   {
    for(j=0;j<n-i;j++)
    {
     if(a[j]>a[j+1])
     {
       temp=a[j];
       a[j]=a[j+1];
       a[j+1]=temp;
     }
    }
  }
}
```

Analysis (or) Time Complexity for Bubble Sort:

- To sort N elements Bubble Sort requires (N-1) passes.
  - Pass 1 requires (n-1) comparisons
  - Pass 2 requires (n-2) comparisons
  - ..
  - ..
  - Pass k requires (n-k) comparisons
  - ..
  - ..
  - Pass (n-1) performs 1 comparison
- The total no. of comparisons = 1+2+3+ ..... + n-1

= n(n-1)/2 = (n2-n)/2
= O(n2)

Selection Sort

Selection Sort is the easiest method to sort list of elements.

In selection sort method first find the smallest(largest) element in the list and swap it with the first element. Then find the second smallest(largest) element in the list and swap it with the second element of the list. The process of searching and swapping the next smallest is repeated until all the elements in the list have been sorted in ascending (descending) order.

i.e the Selection Sort searches all of the elements in a list until it finds the smallest element. It swaps

this with the first element in the list.

Next it finds the smallest of the remaining elements and swaps it with the second element and so on. The procedure to sort 'n' elements in ascending order is,

Pass 1: Find the position (minpos) of the smallest element in the list of 'n' elements. Then interchange A[minpos] & A[0]. i.e. A[0] is sorted means it is in proper place.

Pass 2: Find position (minpos) of the smallest element in the sublist of (n-1) elements then interchange A[minpos] & A[1]. Now A[0] & A[1] are sorted i.e. A[0] ≤ A[1].

..

..

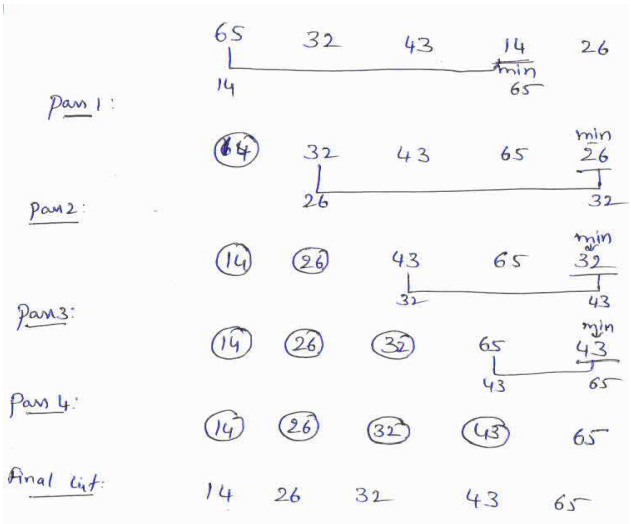
pass (n-1) : Find the position (minpos) of the smallest element in A[n-2] & A[n-1] then interchange A[minpos] & A[n-2]. Now A[0], A[1],...., A[n-2] are sorted means A[n-1] is also in proper place.

Thus " the list of n elements are sorted after (n-1) passes".

For example, suppose an array A contains 5 elements: 65 32 43 14 26

In first pass, the smallest element is searched by making of 4 comparisons and the smallest element is 14. Then it is swapped with the first element i.e. 65. After the first pass the elements are : 14 32 43 65 26.

The complete set of passes in selection sort are:



Algorithm: Selection_Sort(A, n)

```
// A is an array of n elements
{
        for i:= 0 to n-2 do
        {
          min := A[i];  minpos := i;
          for j:= i+1 to n-1 do
                if(a[j]< min) then
                {   min := a[j] ;  minpos := j;    }
          swap(A[i], A[minpos]);
```

```
        }
}
```

/* Program for Selection Sort */

```
#include<stdio.h>
#include<stdlib.h>
#include<conio.h>
void Selection_Sort(int[], int);
void main()
{
  int a[10],n,i;
  clrscr();
  printf("\n Enter Array size :");
  scanf("%d",&n);
  printf("\n Enter elements \n");
  for (i=0;i<n;i++)
  scanf("%d",&a[i]);
  Selection_Sort(a,n);
  printf("\n Array elements after sorting: \n");
  for (i=0;i<n;i++)
  printf("%d  ",a[i]);
 }
```

```
void Selection_Sort(int a[], int n)
{
 int i,j,min,temp;
 for(i=0;i<n-1; i++)
 {
  min=i;
  for(j=i+1; j<n; j++)
  {
   if(a[j]<a[min])
        min=j;
  }
  temp=a[i];
  a[i]=a[min];
  a[min]=temp;
 }
}
```

Analysis of Selection Sort:

Analysis (or) Time Complexity for Bubble Sort:

- To sort N elements Bubble Sort requires (N-1) passes.
  - Pass 1 requires (n-1) comparisons
  - Pass 2 requires (n-2) comparisons
  - ..
  - ..
  - Pass k requires (n-k) comparisons
  - ..
  - ..
  - Pass (n-1) performs 1 comparison
- The total no. of comparisons = 1+2+3+ ..... + n-1

= n(n-1)/2 = (n2-n)/2

= O(n2)

Insertion Sort

The main idea of Insertion Sort is to consider each element at a time, into the appropriate position relative to the sequence of previously ordered elements, such that the resulting sequence is also ordered.

In each pass an element is compared with its predecessors and if it is not in the right position, it is

placed in the right place among the elements being compared.

Suppose an array 'A' with 'n' elements A[0],A[1],....A[n-1]. The insertion sort algorithm scans A from A[0] to A[n-1], inserting each element A[k] into proper position in the previously sorted subarray A[0], A[1], ... , A[k-1].

The procedure is, consider the first element, for only one element no sort is required because it is trivially sorted. So the procedure starts with second element.

Pass 1: A[1] is inserted either before or after A[0] so that A[0] & A[1] are sorted.

Pass 2: A[2] is inserted into proper position by comparing with A[0],A[1] so that A[0],A[1] & A[2] are sorted.

..

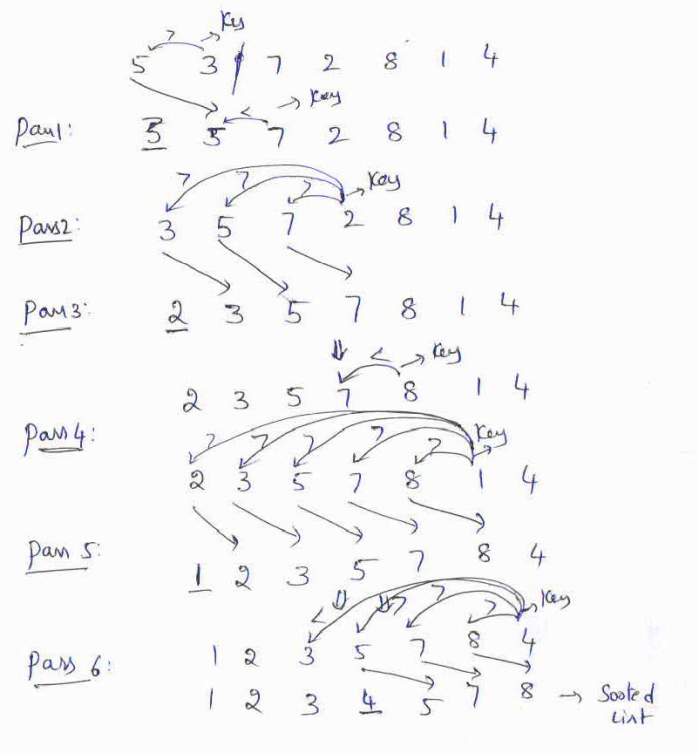
..

Pass n-1: A[n-1] is inserted into its proper positioning A[0], A[1],...., A[n-2] so that all the n elements are sorted

* Simply, in Pass k, A[k] is compared with its previous elements (A[0],A[1],...., A[k-1]) and inserted after the element which is less than or equal to the A[k].

Example: consider the list of elements - 5 3 7 2 8 1 4

To sort this list in ascending order using insertion method, the passes are:



Algorithm: Insertion_Sort(A,n)

```
// A is an array of n elements
{
        for i:= 1 to n-1 do
        {
           key:=A[i];  j := i-1;
           while ((j ≥ 0) and (A[j] > key)) do
           {
                 A[j+1] := A[j];  j := j-1;
           }
           a[j+1] := key;
        }
}
```

/* Program for INSERTION SORT */

```
#include<stdio.h>
#include<stdlib.h>
#include<conio.h>
void Insertion_Sort(int[], int);
void main()
{
int a[10],n,i;
clrscr();
printf("\n Enter Array size :");
scanf("%d",&n);
printf("\n Enter elements \n");
for (i=0;i<n;i++)
scanf("%d",&a[i]);
Insertion_Sort(a,n);
printf("\n Array elements after sorting: \n");
for (i=0;i<n;i++)
printf("%d  ",a[i]);
getch();
}
```

```
void Insertion_Sort(int a[],int n)
{
  int i,j,key;
  for(i=1;i<n;i++)
  {
        key=a[i];j=i-1;
        while((j>=0)&&(a[j]>key))
        {
           a[j+1]=a[j];
           j--;
        }
        a[j+1]=key;
  }
}
```

Analysis:

It is easiest sorting method. If list is already sorted, only one comparison is made on each pass so that (n-1) passes requires (n-1) comparisons. Hence the time complexity is O(n) i.e. Best case.

In worst case i.e if the list is sorted in reverse order (unsorted) then the total no. Of comparisons are ,

$1 + 2 + 3 + ... + (n-1) = n(n-1)/2 = (n2-n)/2 = O(n^2)$.

So, the Worst Case time complexity is : $O(n^2)$

and the Best Case time complexity is : O(n)