UNIT-2

Set Theory: Introduction, Operations on Binary Sets, Principle of Inclusion and Exclusion, Applications of Principle of Inclusion and Exclusion.

Set Theory

- Set theory is the most fundamental concept used in various disciplines particularly in modern mathematics.
- Knowledge of set theory is essential to establish the concept of relations, ordering, mappings and transformations from which the matrix idea follows.
- Set and mathematical logic are used in the design of computers and electric circuits.

Set: Any collection of well-defined objects is called a set. The objects can be numbers, people, letters etc. Any object belonging to a set is called a member or element of that set.

Eg:

- Set of vowels in alphabets.
- Set of all positive integers less than 100.
- A collection of rocks.
- The set of people living on earth.

If $x \in A$ which is read as	If $x \notin A$ which is read as
"x is an element of A"	"x is an not an element of A"
"x belongs to A"	"x does not belongs to A"
"x is in A"	"x is not in A"

Set can be specified in two ways.

1) ***Tabular form***: A set can be defined by listing its members are separated and enclosed in braces.
   Eg: A set A containing members 1, 2,3,4,5 is generally written as {1, 2, 3, 4, 5}
2) ***Set builder form:*** A set can be defined by stating the characteristics that its elements must satisfy.
   Eg: Let X be the set of even integers. This can be represented as
   $X = \{x \mid x \text{ is an even integer}\}$
   Read as "X is the set of integer x such that x is even".
   Examples:
   - X = {a,e,i,o,u}
   - Y = {1,2,3,4,5,6}
   - Z = {x|x is a person living on earth

Imporant terms

1. ***Null set:*** A set that does not contain any elements is called null set or empty set. It is denoted by ϕ

   Eg: a) $X = \{x \mid x \text{ is an integer and } 2 < x < 3\}$
   There is no integers between 2 and 3.
   b) $X = \{x \mid x^2 = 9 \text{ and x is even}\}$
   No even integer satisfy $x^2 = 9$

2. ***Universal set:*** A set is said to be a universal set if it contains every set under discussion .It is also known as universe of discourse and is denoted as **U** or **E**.
   Eg: a) A set of postgraduate students, the universe set can be taken as the set of all students.
   b) In dice throwing experiment the all possible values are U= {1, 2, 3, 4, 5, 6}.

3. ***Subsets or set inclusion:*** Let X and Y be two sets. If every element of X is an element of Y, then X is said to be a subset of Y and written as $X \subseteq Y$.
   Eg: X = {a,b,c} Y = {a,b} Z = {a,c} and W = {c}
   - $\{a,b\} \subseteq \{a,b,c\} \Rightarrow Y \subseteq X$
   - $\{a,c\} \subseteq \{a,b,c\} \Rightarrow Z \subseteq X$
   - $\{c\} \subseteq \{a,c\} \Rightarrow W \subseteq Z$
   - $\{C\} \subseteq \{a,b,c\} \Rightarrow W \subseteq X$

   Note:-
   - ✓ Every set is a subset of itself. i.e, for any set A, $A \subseteq A$ i.e, Set inclusion is reflexive.
   - ✓ A empty set is a subset of every set. Set inclusion is transitive, that is for any three sets A, B and C if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

4. ***Proper subset:*** Let X and Y be two sets. X is said to be a proper subset of Y if X is a subset of Y and there is atleast one element in Y that is not in X**.** It is denoted as $X \subset Y$.
   - It is not reflexive.
   - It is transitive.

   Eg: X = {1,3,5} Y = {1,2,3,4,5}
   $X \subseteq Y$ but $X \neq Y$
   X is proper subset of Y.

5. ***Equal sets:*** Two sets X and Y are said to be equal if $X \subseteq Y$ and $Y \subseteq X$ i.e, X = Y if both X and Y have same elements.
   Eg:
   i. X = { 1,2,3,4,5} Y = {3,5,1,2,4} Are they equal?
   The elements of X and Y are same, so they are equal sets.
   ii. X = { 1,2,3} Y = {1,1,2,3} Are they equal?
   $X \subseteq Y$ and $Y \subseteq X$
   $\therefore X = Y$
   iii. X = { {1,2},3} Y = {1,2,3} Are they equal?
   Since members of sets are not equal.
   $\therefore X \neq Y$

6. ***Finite set:*** If number of elements in set are finite or countable then X is said to be finite set.
   Eg: a) $X = \{x \mid x \text{ is an integer and } 5 \le x \le 12\}$
   {5,6,7,8,9,10,11,12} Countable, finite
   b) All countries in the world.
   $X = \{x \mid x \text{ is a country in the world}\}$ Countable, finite

7. ***Infinite set:*** If number of elements in set are infinte or not countable then X is said to be infinite set.
   Eg*:* {2,4,,8...} $X = \{2^x \mid x \text{ is a } +\text{ve integer}\}$ Uncountable, infinite

8. ***Power set:*** The collection of all subsets of a set X is called the power set of X and is denoted by P(x) , 2^X .

Eg: Find the power set of X = {1,2,3}

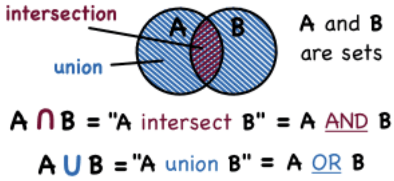
P(X) = { φ ,{1}, {2}, {3}, {1,2}, {1,3}, {2,3} ,{1,2,3}}

If set having n distinct elements then it an 2^n subsets.

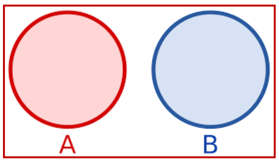
3 elements → 2^3 subsets

= 8 subsets

9. ***Venn diagrams:*** Venn diagrams are used to represent sets in simple manner. They are used to denote relationship between 2 or more sets.



10. ***Disjoint set:-*** Two sets X and Y are said to be disjoint sets if no element between the sets, that is no element of X is in Y and no element of Y is in X.

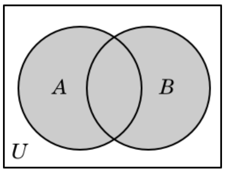


Eg: 1) X = {a,b,c,d,e} Y = {1,2,3,4,5}

2) X = {x|x is a +ve integer} Y = {y|y is a –ve integer}

∴ X and Y are disjoint sets.

Operations on sets:-



1) ***Union of two sets:*** The union of two sets is defined as the set of all the elements that are members of set A, set B or both and is denoted by $A \cup B$ read as A union B.

$$A \cup B = \{x \mid x \in \text{A or } x \in \text{B}\}$$

Eg: A = {1,2,3,4,5,a,b} B = {a,b,c,d,e}

$A \cup B$ = {1,2,3,4,5,a,b} $\cup$ {a,b,c,d,e}

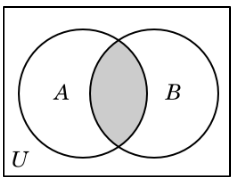
= {1,2,3,4,5,a,b,c,d,e}

Properties:- Let A,B,C be three sets then

- ✓ $A \cup A = A$
- ✓ $A \cup \phi = A$
- ✓ $A \cup U = U$
- ✓ $A \cup A^1 = U$
- ✓ $A \subseteq A \cup B$ and $B \subseteq A \cup B$
- ✓ $A \cup B = B \cup A$
- ✓ $(A \cup B) \cup C = A \cup (B \cup C)$

2) ***Intersection of two sets:*** The intersection of any two sets A and B is the set containing of all the elements that belong to both A and B is denoted by $A \cap B$ read as A intersection B.

$$A \cap B = \{x \mid x \in \text{A and } x \in \text{B}\}$$



Eg: A = {1,2,3,4,5,a,b} B = {a,b,c,d,e}

$A \cap B$ = {1,2,3,4,5,a,b} $\cap$ {a,b,c,d,e}

= {a,b}

Properties:- Let A,B,C be three sets then

- ✓ $A \cap A = A$

- ✓ $A \cap \phi = \phi$
- ✓ $A \cap U = A$
- ✓ $A \cap B \subseteq$ A and $A \cap B \subseteq B$
- ✓ $A \cap B = B \cap A$
- ✓ $(A \cap B) \cap C = A \cap (B \cap C)$

3) ***Mutually disjoint sets:*** A collection of sets is called a disjoint collection if every pair of sets in the collection is disjoint. The elements of a disjoint collection are called mutually disjoint.

Eg: A = {{a,b}, {c}} B = {{a}, {b,c}} C = {{a,b,c}}

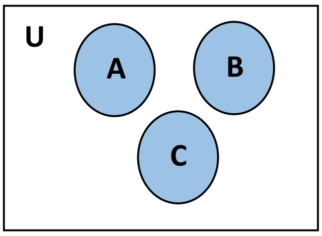
Show that A,B,C are mutually disjoint.

$A \cap B = \varphi$

$B \cap C = \varphi$

$A \cap C = \varphi$

∴ A,B,C are mutually disjoint.

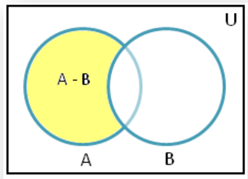


4) ***Difference of two sets:*** The difference of any two sets A and B is the set of elements that belongs to A but not B. It is denoted by A-B and read as ‘A difference B’. A-B is also denoted by A|B or A~B. It is also called the relative complement of B in A.

Eg: A = {1,2,3,4,5,6} B = {3,5,7,9}

A-B = {1,2,4,6}

B-A = {7,9}



Difference of two sets
A - B

Properties:- Let A,B be two sets then

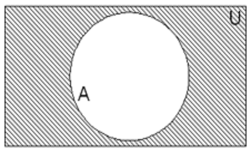
- ✓ $A^1 = U - A$
- ✓ $A - B = A \cap B^1$
- ✓ $A - A = \varphi$
- ✓ $A - \varphi = A$
- ✓ $A - B = B - A$ iff A = B
- ✓ $A - B = A$ iff $A \cap B = \varphi$
- ✓ $A - B = \varphi$ iff $A \subseteq B$

5) ***Complement of a set:*** Let U be the universal set for any set A, U-A is called the absolute complement of A. The absolute complement of a set A is called a complement of A and is denoted by ~A, A′ and A^c.

Eg: U = Z^+ and A = {1,3,5,...} then A^c

U = {1,2,3,4,5,....}   A = {1,3,5,...}

A^c = {2,4,6,8,10,..}

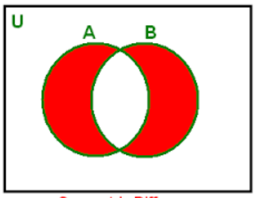


Properties:- Let A,B be two sets then

- ✓ $A \cup A^1 = U$
- ✓ $A \cap A^1 = \varphi$
- ✓ $U^1 = \varphi$
- ✓ $\varphi^1 = U$
- ✓ $(A^1)^1 = A$

- ✓ $(A \cup B)^1 = A^1 \cap B^1$
- ✓ $(A \cap B)^1 = A^1 \cup B^1$

6) ***Symmetric difference of two sets:*** Let A and B be any two sets. The symmetric difference of A and B is the set consisting of all elements that belong to A or B, but not to both A and B.



Symmetric Difference Between A And B

It is denoted by $A \oplus B,\ A + B.$

$$A + B = \{x \mid x \in \text{A and } x \notin \text{B}\} \text{ or } \{x \mid x \notin \text{A and } x \in \text{B}\}$$
$$= (A - B) \cup (B - A)$$

Properties:- Let A,B be two sets then

- ✓ $A + A = \varphi$
- ✓ $A + \varphi = A$
- ✓ $A + B = B + A$
- ✓ $A + B = (A \cup B) - (A \cap B)$

Eg: A = {2,3,4} B = {1,2} Find A+B

$$A + B = (A - B) \cup (B - A)$$
= ({2,3,4} - {1,2}) $\cup$ ({1,2} – {2,3,4}
= ({3,4} $\cup$ {1}) = {1,3,4}

7) **Cartesian product:-** Cartesian product of A and B, denoted A × B, is the set whose members are all possible ordered pairs (a, b) where a is a member of A and b is a member of B.
Eg: The cartesian product of {1, 2} and {red, white} is {(1,red), (1,white), (2,red), (2, white)}.

TABLE 1 Set Identities.

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

Principle of Inclusion and Exclusion:-

When the two tasks are done at the same time, to count the number of ways in which any one of the tasks can be done, we add the number of ways of doing each of the two tasks and then subtract the number of ways of doing both tasks. This technique is called the principle of inclusion and exclusion.

Principle:- Let A and B be any two finite sets. Then the number of elements in the union of the two sets A and B is the sum of the number of elements in the sets minus the no. of elements in their intersection that is $|A \cup B| = |A| + |B| - |A \cap B|$

Eg:- In a class of 50 students, 20 students play football , 16 play hockey. It is found that 10 students play both. Find number of students play neither.

Sol:- No. of students play football (A) = 20

No. of students play hockey (B) = 16

No. of students play both $|A \cap B|$= 10

No. of students play either football or hockey

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$= 20 + 16 - 10$$
$$\therefore\ |A \cup B|\ |A \cup B| = 26$$

No. of students play neither football nor hockey = 50-26 = 24.

Eg:- Among 60 students, 45 passed in I sem examinations and 30 passed in the II sem examination. If 12 did not pass in either sem, how many passed in both semesters?

Sol:- No. of students passed in I sem (A) = 45

No. of students passed in II sem (B) = 30

No. of students not passed in either sem =$(A \cup B)'$ = $A' \cap B' = 12$

The students passed in first or second or both

$$|A \cup B| = U - (A \cup B)'$$
$$|A \cup B| = 60 - 12 = 48$$
$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$48 = 45 + 30 - |A \cap B|$$
$$|A \cap B| = 45 + 30 - 48$$
$$\therefore\ |A \cap B| = 27$$ passed in both sem.

Eg:- How many positive integers not exceeding 2000 are divisible by 7 or 11.

Sol:- Let A be set of +ve integers not exceeding 2000 that are divisible by 7

$$|A| = \left|\frac{2000}{7}\right| = 285$$

Let B be set of +ve integers not exceeding 2000 that are divisible by 11

$$|B| = \left|\frac{2000}{11}\right| = 181$$

$|A \cap B|$ be set of +ve integers not exceeding 2000 that are divisible by 7 and 11.

$$|A \cap B| = \left|\frac{2000}{7*11}\right| = 25$$

$|A \cup B|$ be set of +ve integers not exceeding 2000 that are divisible by A or B i.e, 7 or 11

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$= 285 + 181 - 25$

$\therefore |A \cup B| = 441$

Note:- $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

Eg-1: 1,232 students have taken a course in Spanish, 879 in French & 114 in Russian.100 students have taken a course in both Spanish and French,25 in both Spanish and Russian,12 in both French and Russian.2092 students have taken course in atleast one of Spanish, French or Russian. How many students have taken a course in all the three languages?

Sol: Let the no. of students who have taken course in Spanish be $|S| = 1232$

No. of students who have taken course in French be $|F| = 879$

No. of students who have taken course in Russian be $|R| = 114$

No. of students who have taken course in both Spanish and French be $|S \cap F| = 100$

No. of students who have taken course in both Spanish and Russian be $|S \cap R| = 25$

No. of students who have taken course in both French and Russian be $|F \cap R| = 12$

No. of students who have taken course in atleast one of Spanish, French and Russian be $|S \cup F \cup R| = 2092$

No. of students who have taken course in all the three languages be $|S \cap F \cap R|$

$S \cup F \cup R| = |S| + |F| + |R| - |S \cap F| - |S \cap R| - |F \cap R| + |S \cap F \cap R|$

$2092 = 1232 + 879 + 114 - 100 - 25 - 12 + |S \cap F \cap R|$

$\therefore S \cap F \cap R| = 4$

Eg-2: Out of 260 people 64 watch sports channel, 58 watch film channel and 94 watch news channel. 28 people watch both sports and film channel,26 watch both sports and news channel,22 watch both film and news channel.14 people watch all the three channels.

Find: i) No. of people who are interested in watching any of the three channels.

ii) No. of people who are interested in watching sports channel only.

iii) No. of people who are interested in watching film channel only.

iv) No. of people who are interested in watching news channel only.

Sol: Total No of people U=260

No. of people who watch sports channel $|S| = 64$

No. of people who watch film channel $|F| = 58$

No. of people who watch news channel $|N| = 94$

No. of people who watch both sports and film channels $|S \cap F| = 28$

No. of people who watch both sports and news channels $|S \cap N| = 26$

No. of people who watch both film and news channels $|F \cap N| = 22$

No. of people who watch all the three channels $|S \cap F \cap N| = 14$

i) No. of people who are interested in watching any of the three channels

$|S \cup F \cup N| = |S| + |F| + |N| - |S \cap F| - |S \cap N| - |F \cap N| + |S \cap F \cap N|$

$= 64 + 58 + 94 - 28 - 26 - 22 + 14$

$= 154$

ii) No. of people who are not interested in watching any of the three channels

$|S \cap F \cap N|' = U - |S \cup F \cup N|$

$= 260 - 154$

$= 106$

ii) No. of people who are interested in only sports and film channels

$= |S \cap F| - |S \cap F \cap N|$
$= 28 - 14$
$= 14$

No. of people who are interested in only sports and news channels

$= |S \cap N| - |S \cap F \cap N|$
$= 26-14$,
$= 12$

No. of people who are interested in **sports channel only** $= |S| -14-12-14$
$= 64-40$
$= 24$

iii) No. of people who are interested in only sports and film channels

$= |S \cap F| - |S \cap F \cap N|$
$= 28 - 14$
$= 14$

No. of people who are interested in only film and news channels

$= |F \cap N| - |S \cap F \cap N|$
$= 22-14$
$= 8$

No. of people who are interested in **film channel only** $= |F| -14-8-14$
$= 58-36$
$= 22$

iv) No. of people who are interested in only news and film channels

$= |F \cap N| - |S \cap F \cap N|$
$= 22 - 14$
$= 8$

No. of people who are interested in only news and sports channels

$= |S \cap N| - |S \cap F \cap N|$
$= 26-14$
$= 12$

No. of people who are interested in **news channel only** $= |N| -8-12-14$
$= 94-34$
$= 60$

Applications of Principle of inclusion and exclusion

There is an alternative form of the principle of inclusion-exclusion that is useful in counting problems. In particular, this form can be used to solve problems that ask for the number of elements in a set that have none of n properties $P_1, P_2, \ldots, P_n$. Let A_i be the subset containing the elements that have property P_i . Let's denote the number of elements with all the properties $P_{i1}, P_{i2}, \ldots, P_{ik}$ by $N(P_{i1} P_{i2} \ldots P_{ik})$. Writing these quantities in terms of sets, we have $|A_{i1} \cap A_{i2} \cap \cdots \cap A_{ik}| = N(P_{i1} P_{i2} \ldots P_{ik})$. Let's denote the number of elements with none of the properties $P_{i1}, P_{i2}, \ldots, P_{ik}$ by
$N(P'_{i_1} P'_{i_2} \ldots P'_{i_k})$.

Suppose the number of elements in the set is N. Then it follows that

$$N(P'_{i_1} P'_{i_2} \ldots P'_{i_k}) = N - |A_{i_1} \cup A_{i_2} \cup \cdots \cup A_{i_k}|.$$

From the inclusion-exclusion principle, we see that

$$N(P'_{i_1}P'_{i_2}\dots P'_{i_k}) = N - \sum_{1\le i\le n} N(P_i) + \sum_{1\le i<j\le n} N(P_iP_j) - \sum_{1\le i<j<k\le n} N(P_iP_jP_k) + \cdots + (-1)^n N(P_1P_2\dots P_n).$$

Eg-1:-*How many of the integers 1, 2, 3,..., 2009 are not divisible by any of the numbers 4,5,6?*

Sol: Let N = {1,2,3,...,2009},N=2009

Let P_1 be the property that an integer is divisible by 4 $\therefore$ P_1 = {4,8,12,...,2008},$N(P_1) = \left\lfloor\frac{2009}{4}\right\rfloor = 502$.

Let P_2 be the property that an integer is divisible by 5 $\therefore$ P_2 = {5,10,15,...,2005}, $N(P_2) = \left\lfloor\frac{2009}{5}\right\rfloor = 401$.

Let P_3 be the property that an integer is divisible by 6 $\therefore$ P_3 = {6,12,18,...,2004}, $N(P_3) = \left\lfloor\frac{2009}{6}\right\rfloor = 334$.

The least common multiple of some numbers a1,a2,...,an is the smallest positive number which is divisible by all of a1,a2,...,an. It is written as lcm {a1,a2,...,an}.

$N(P_1 P_2)$ i.e $P1\cap P2$ contains exactly numbers divisible by 4 and 5. These are precisely the numbers {20,40,60,...,2000} divisible by lcm{4,5} = 20.

Therefore $N(P_1 P_2) = \left\lfloor\frac{2009}{lcm(4,5)}\right\rfloor = \left\lfloor\frac{2009}{20}\right\rfloor = 100$.

We get lcm {4,6} = 12, so $N(P_1 P_3) = |P1 \cap P3| = \left\lfloor\frac{2009}{12}\right\rfloor = 167$.

And lcm {5,6} = 30 implies $N(P_2 P_3) = |P2 \cap P3| = \left\lfloor\frac{2009}{30}\right\rfloor = 66$.

Finally lcm{4,5,6} = 60, and $N(P_1 P_2 P_3) = |P1 \cap P2 \cap P3| = \left\lfloor\frac{2009}{60}\right\rfloor = 33$.

Now number of positive integers n, where $1\le n\le 2009$ that are not divisible by 4,5 and 6 is

$\therefore N(P'_1P'_2P'_3) = |P'_1 \cap P'_2 \cap P'_3| = N - N(P_1 P_2 P_3)$

$= N-[N(P_1)+ N(P_2) +N(P_3)-N(P_1 P_2)- N(P_1 P_3)- N(P_2 P_3)+N(N(P_1P_2 P_3)]$

$= 2009-502-401-334+100+167+66-33$

$= 1072.$

Eg-2: Determine the number of primes not exceeding 100 and not divisible by 2,3,5 or 7.

Sol: Let P_1 be the property that an integer is divisible by 2.

Let P_2 be the property that an integer is divisible by 3.

Let P_3 be the property that an integer is divisible by 5.

Let P_4 be the property that an integer is divisible by 7.

Thus the number of positive integers not exceeding 100 that are not divisible by 2,3,5,7 is

$N(P'_1 P'_2 P'_3 P'_4) = N-N(P_1 P_2 P_3 P_4)$

$=N-[N(P_1)+ N(P_2)+ N(P_3)+ N(P_4)-N(P_1 P_2)-N(P_1 P_3)- N(P_1 P_4)-N(P_2 P_3)- N(P_2 P_4)- N(P_3 P_4)$
$+N(P_1P_2 P_3)+ +N(P_1P_2 P_4) +N(P_1P_3 P_4) +N(P_2P_3 P_4) - N(P_1P_2P_3 P_4)]$

$=99 - \left\lfloor\frac{100}{2}\right\rfloor - \left\lfloor\frac{100}{3}\right\rfloor - \left\lfloor\frac{100}{5}\right\rfloor - \left\lfloor\frac{100}{7}\right\rfloor + \left\lfloor\frac{100}{2.3}\right\rfloor + \left\lfloor\frac{100}{2.5}\right\rfloor + \left\lfloor\frac{100}{2.7}\right\rfloor + \left\lfloor\frac{100}{3.5}\right\rfloor +$
$\left\lfloor\frac{100}{3.7}\right\rfloor + \left\lfloor\frac{100}{5.7}\right\rfloor - \left\lfloor\frac{100}{2.3.5}\right\rfloor - \left\lfloor\frac{100}{2.3.7}\right\rfloor - \left\lfloor\frac{100}{25.7}\right\rfloor - \left\lfloor\frac{100}{3.5.7}\right\rfloor + \left\lfloor\frac{100}{2.3.5.7}\right\rfloor$

$= 99-50-33-20-14+16+10+7+6+4+2-3-2-1-0+0$

$=21$

Thus the number of integers not exceeding 100 that are divisible by none of 2,3,5,7 is 21.Hence,the number of prime numbers not exceeding 100 is

$= 4+ N(P'_1 P'_2 P'_3 P'_4)$

$=4+21$

=25

Eg-3: Determine the number of positive integers n, where 1≤n≤2000 and n is not divisible by 2,3 or 5 but is divisible by 7.

Sol: Let P_1 be the property that an integer is divisible by 2 , P_2 be the property that an integer is divisible by 3, P_3 be the property that an integer is divisible by 5.

Now number of positive integers n, where 1≤n≤2000 that are not divisible by 2,3 and 5 is

$N(P'_1 P'_2 P'_3) = N-N(P_1 P_2 P_3)$

$= N-[N(P_1)+ N(P_2) +N(P_3)-N(P_1 P_2)- N(P_1 P_3)- N(P_2 P_3)+N(N(P_1P_2 P_3)]$

$=2000 - \left\lfloor\frac{2000}{2}\right\rfloor - \left\lfloor\frac{2000}{3}\right\rfloor - \left\lfloor\frac{2000}{5}\right\rfloor + \left\lfloor\frac{2000}{2.3}\right\rfloor + \left\lfloor\frac{2000}{2.5}\right\rfloor + \left\lfloor\frac{2000}{3.5}\right\rfloor - \left\lfloor\frac{2000}{2.3.5}\right\rfloor$

=2000-1000-666-400+333+200+133-66

=2000-1466

= 534

Hence the number of positive integers n, 1≤n≤2000 that are not divisible by 2,3,5 but are divisible by 7 is $\left\lfloor\frac{534}{7}\right\rfloor = 76$