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AIML

P&S Assignment

* 20% of items produced from a factory are defective. Find the probability that in a sample of 5 chosen at random:

(i) None is Defective

(ii) One is Defective

(iii) $P(1<x<4)$

sol:

Given,

$n=5$

$P = 20\% = 20/100 = 2/10$

$\therefore q = 1-P = 1-\frac{2}{10} = 8/10$

$$\boxed{P(x=r) = nC_r \, p^r q^{n-r}}$$

(i) None of defective

$$P(x=0) = 5C_0 \left(\frac{2}{10}\right)^0 \left(\frac{8}{10}\right)^{5-0}$$

$$= \left(\frac{8}{10}\right)^5 = 0.3276$$

(ii) One is defective

$$P(x=1) = 5C_1 \left(\frac{2}{10}\right)^1 \left(\frac{8}{10}\right)^{5-1}$$

$$= 5\left(\frac{1}{5}\right)\left(\frac{8}{10}\right)^4$$

$$= 0.4096$$

(iii) $P(1<x<4) = P(x=2) + P(x=3)$

(iii) $P(1<x<4)$

$$= 5c_2\left(\frac{1}{5}\right)^2\left(\frac{8}{10}\right)^3 + 5c_3\left(\frac{1}{5}\right)^3\left(\frac{8}{10}\right)^2$$

$$= 10\left(\frac{1}{25}\right)\left(\frac{64}{125}\right) + 10\left(\frac{1}{125}\right)\left(\frac{16}{25}\right)$$

$$= 0.256$$

* A hospital switch board recieves an average of 4 Emergency calls in a 10 minute interval. what is the probability that:

(i) There are atmost 2 Emergency calls in 10 minutes interval.

(ii) There are Exactly 3 emergency calls in 10 minutes interval.

soln

Given

Mean $(\lambda) = 4$

$$\boxed{P(X=x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}}$$

(i) Atmost 2 calls

$$\Rightarrow P(X \le 2) = P(x=0) + P(x=1) + P(x=2)$$

$$= \frac{e^{-4}4^0}{0!} + \frac{e^{-4}4^1}{1!} + \frac{e^{-4}4^2}{2!}$$

$$= e^{-4}[1+4+8]$$

$$= e^{-4}(13)$$

$$= 0.2381$$

(ii) Exactly '3' calls

$$P(x=3) = \frac{e^{-4} \cdot 4^3}{3!}$$

$$= e^{-4}\left(\frac{64}{6}\right)$$

$$= 0.19536$$

* 2% of the items of a factory are defective. The items are packed in boxes. What is the probability that there will be :

(i) 2 Defective items

(ii). Atleast three defective items in a box of 100 items.

sol?

Given,

$n = 200$

$p = 2\% = 2/100 = 1/50 = 0.02$

$q = 1-p = 1 - \frac{1}{50} = 49/50 = 0.98$

(i) 2 defective items

$$P(x=2) = 100_{c_2} (0.02)^2 (0.98)^{98}$$

$$= 0.27341$$

(ii) Atleast three - $P(x \geq 3)$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - [\, 100_{c_0} (0.02)^0 (0.98)^{100} + 100_{c_1} (0.02)^1 (0.98)^{99} +$$

$$ {}^{100}C_2 (0.02)^2 (0.98)^{98} \Big] $$

$$ = 1 - [0.1326 + 0.2706 + 0.27341] $$

$$ = 0.3233. $$

* If x is poisson variate such that $3P(x=4) = \frac{1}{2}[P(x=2)] + P(x=0)$

Find :

(i) The mean of x

(ii) $P(x <= 2)$.

Given $3P(x=4) = \frac{1}{2}(P(x=2)) + P(x=0)$

$$ \frac{3e^{-\lambda} \cdot \lambda^4}{4!} = \frac{1}{2}\left[\frac{e^{-\lambda} \cdot \lambda^2}{2!}\right] + \frac{e^{-\lambda} \cdot \lambda^0}{0!} $$

$$ \frac{3e^{-\lambda} \cdot \lambda^4}{4!} = e^{-\lambda}\left[\frac{\lambda^2}{4} + 1\right] $$

$$ \frac{3\lambda^4}{4!} = \frac{\lambda^2}{4} + 1 $$

$$ \frac{\lambda^4}{8} - \frac{\lambda^2}{4} - 1 = 0 $$

$$ \lambda^4 - 2\lambda^2 - 8 = 0 $$

$$ \lambda = 2, -2, \sqrt{2}i, -\sqrt{2}i $$

(i)
Consider $\lambda = 2$

(ii) $P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$

$$= \frac{e^{-2} 2^0}{0!} + \frac{e^{-2} 2^1}{1!} + \frac{e^{-2} 2^2}{2!}$$

$$= e^{-2}[1+2+2]$$

$$= 5(e^{-2})$$

$$= \underline{\underline{0.6766}}$$

* In a Normal distribution 7% of items are under 35 and 89% are under 63. Determine the Mean and Variance of the distribution.

sol^n: let μ be the mean &

σ be the standard deviation of the normal curve.

Given 7% of the items are under 35. i.e; $50-7 = 43\%$

$$P(x<35) = 7\% = 0.07$$

89% are under 63 i.e; $89-50 = 39\%$

$$P(x<63) = 89\% = 0.89$$

for Area 0.43, $Z = 1.48$

$\because$ 35 is less than μ

$$Z = -1.48$$

for Area 0.39, $Z = 1.23$

$\therefore Z = \frac{x-\mu}{\sigma} \Rightarrow \frac{35-\mu}{\sigma} = -1.48$ & $\frac{63-\mu}{\sigma} = 1.23$

$35 - \mu = 1.48\sigma$

$-1.48\sigma + \mu - 35 = 0$ —①

$63 - \mu = 1.23\sigma$

$1.23\sigma + \mu - 63 = 0$ —②

solve ①, ②

$\sigma = -10.33$

$\mu = -50.291$

$\therefore$ Mean $= -50.291$

Variance $(\sigma^2) = 106.7089.$

* If the mass of 300 students are normally distributed with mean 68kgs and S.D 3kgs. How many students have mass

(i) Greater than 72 kgs

(ii) Less than (or) equal to 64 kgs

(iii) Between 65 and 71 kgs Inclusive.

sol?

Given,

Mean $(\mu) = 68$

$n = 300$

SD $(\sigma) = 3$

(i) Greater than 72 $\quad z_1 = \frac{x - \mu}{\sigma} = \frac{72 - 68}{3} = \cancel{6.18}\ 1.33$

$P(x > 72) = P(z > 1.33)$

$= 0.5 - 0.4082$

$= 0.0918$

(ii) less than (or) equal to 64 $\quad z_1 = \frac{x-\mu}{\sigma} = \frac{64-68}{3} = -1.33$

$P(x \leq 64) = P(z \leq -1.33)$

$= 0.5 + A(z_1)$

$= 0.5 + 0.4082$

$= 0.9082$

(iii) Between 65 & 71 kgs

$z_1 = \frac{65-68}{3} = -1$, $z_2 = \frac{71-68}{3} = 3/3 = 1$

$P(65 \leq x \leq 71) = P(z_1 \leq z \leq z_2)$

$= P(-1.0 \leq z \leq 1.0)$ $\quad$ Here $z_1 < 0$, $z_2 > 0$

$= A(z_2) + A(z_1)$

$= 0.3413 + 0.3413$

$= 0.6826$