or $\left(40-\dfrac{1.96}{20}, 40+\dfrac{1.96}{20}\right)$ or $(40 - 0.98,\ 40 + 0.98)$

i.e., (39.02, 40.98)

Example 7 : An ambulance service claims that it takes on the average less than minutes to reach its destination in emergency calls. A sample of 36 calls has a mean of minutes and the variance of 16 minutes. Test the claim at 0.05 level significance.

[JNTU 2005, (H) May 2012 (Set No. 4), (K) May 2013 (Set No.

Solution : Given $n = 36$, $\bar{x} = 11$, $\mu = 10$ and $\sigma = \sqrt{16} = 4$

1. **Null Hypothesis** $H_0 : \mu = 10$
2. **Alternative Hypothesis** $H_1 : \mu < 10$
3. **Level of significance :** $\alpha = 0.05$
4. **The test statistic is,** $Z = \dfrac{\bar{x}-\mu}{\dfrac{\sigma}{\sqrt{n}}} = \dfrac{11-10}{4/\sqrt{36}} = \dfrac{6}{4} = 1.5$

Tabulated value of Z at 5% level of significance is 1.645.

Hence calculated $Z <$ tabulated Z

$\therefore$ We accept the null hypothesis H_0.

Example 8 : It is claimed that a random sample of 49 tyres has a mean life of 152 km. This sample was drawn from a population whose mean is 15150 kms and a standard deviation of 1200 km. Test the significance at 0.05 level.

[JNTU 2005, 2006S, (K) May 2013 (Set No.

Solution : Given $n = 49$, $\bar{x} = 15200$, $\mu = 15150$ and $\sigma = 1200$

1. **Null Hypothesis** $H_0 : \mu = 15150$
2. **Alternative Hypothesis** $H_1 : \mu \neq 15150$
3. **Level of significance :** $\alpha = 0.05$
4. **Critical region :** Accept the null hypothesis if $-1.96 < Z < 1.96$
5. **The test statistic is,** $Z = \dfrac{\bar{x}-\mu}{\dfrac{\sigma}{\sqrt{n}}} = \dfrac{15200-15150}{1200/\sqrt{49}} = 0.2917$

Since $|Z| < 1.96$ therefore, we accept the null hypothesis.

Example 9 : An insurance agent has claimed that the average age of policy holders who issue through him is less than the average for all agents which is 30.5 years. A random sample of 100 policy holders who had issued through him gave the following age distribution

Age	16-20	21-25	26-30	31-35	36-40
No. of persons	12	22	20	30	16

4. **The test statistic is** $z = \dfrac{p-P}{\sqrt{\dfrac{PQ}{n}}} = \dfrac{0.544-0.5}{\sqrt{\dfrac{(0.5)(0.5)}{125}}}$

$$= \frac{0.044}{0.045} = 0.9839$$

Since calculated value of $|Z|$ is less than 1.645, we accept the Null Hypothesis H_0 at 5% level of significance.

Example 7 : Experience had shown that 20% of a manufactured product is of the top quality. In one day's production of 400 articles only 50 are of top quality. Test the hypothesis at 0.05 level. **[JNTU 2005 S, 2008 S (Set No. 2), (K) May 2013 (Set No. 4)]**

Solution : We have $n = 400$, $x = 50$ and $p = \dfrac{x}{n} = \dfrac{50}{400} = 0.125$

1. **Null Hypothesis** $H_0 : P = 0.2$
2. **Alternative Hypothesis** $H_1 : P \neq 0.2$
3. **Level of significance** : $\alpha = 0.05$ $P \not< 20\%$ left
4. **The test statistic is** $z = \dfrac{p-P}{\sqrt{\dfrac{PQ}{n}}}$

i.e., $z = \dfrac{0.125-0.2}{\sqrt{\dfrac{(0.2(0.8)}{400}}} = \dfrac{-0.075}{0.02} = -3.75$

Since $|z| = 3.75 > 1.96$, we reject the Null Hypothesis H_0 at 5% level of significance.

i.e., $P = 20\%$ is not correct.

Example 8 : A social worker believes that fewer than 25% of the couples in a certain area have ever used any form of birth control. A random sample of 120 couples was contacted. Twenty of them said that they have used. Test the belief of the social worker at 0.05 level. **[JNTU 2005 (Set No. 1)]**

Solution : We have

$n = 120$, $x = 20$, $p = \dfrac{x}{n} = \dfrac{20}{120} = \dfrac{1}{6}$ and $P = 0.25$, $Q = 1 - P = 0.75$ $P = 0.25$

1. **Null Hypothesis** $H_0 : P = 0.25$
2. **Alternative Hypothesis** $H_1 : P < 0.25$ **(left tailed test)** $H_1: P < 0.25$
3. **Level of significance,** $\alpha = 0.05$
4. **The test statistic is**

Solution : Given ... 76.7

and σ = S.D of population = 8.6

1. **Null Hypothesis** H_0 : $\mu = 73.2$
2. **Alternative Hypothesis** H_1 : $\mu > 73.2$ (Right-tailed test)
3. **Level of significance** : α = 99% or probability is 0.01)
4. **The test statistic** is $z = \dfrac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{76.7-73.2}{\frac{8.6}{\sqrt{4}}} = \dfrac{3.5}{4.3} = 0.814$

Tabulated value of z at 99% level of significance is 2.33.

Hence calculated z $\leq$ tabulated z

$\therefore$ The null hypothesis H_0 is accepted. That is, $\bar{x}$ and μ do not differ significantly.

Example 2 : A sample of 64 students have a mean weight of 70 kgs. Can this be regarded as a sample from a population with mean weight 56 kgs and standard deviation 25 kgs. **[JNTU 2006, (A) Nov. 2010 (Set No. 2)]**

Solution :

Given $\bar{x}$ = mean of the sample = 70 kgs

μ = mean of the population = 56 kgs

σ = S.D of population = 25 kgs

and n = sample size = 64

1. **Null Hypothesis** H_0 : A sample of 64 students with mean weight of 70 kgs can be regarded as a sample from a population with mean weight 56 kgs and standard deviation 25 kgs.
2. **Alternative Hypothesis** H_1 : Sample cannot be regarded as one coming from the population.
3. **Level of significance** : $\alpha = 0.05$ (assumption)
4. **The test statistic** is $z = \dfrac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{70-56}{\left(\frac{25}{\sqrt{64}}\right)} = 4.48$

5. The null hypothesis H_0 is rejected, since $|Z| > 1.645$

Note : The null hypothesis can be rejected even at 1%

Example 3 : An oceanographer wants to check whether th
certain region is 57.4 fathoms, as had previously been recorded. W
0.05 level of significance , if readings taken at 40 random locations
a mean of 59.1 fathoms with a standard deviation of 5.2 fathoms.

Solution : Given $n = 40$, $\overline{x} = 59.1$ and $\sigma = 5.2$

1. **Null Hypothesis** $H_0 : \mu = 57.4$
2. **Alternative hypothesis** $H_1 : \mu \neq 57.4$
3. **Level of significance :** $\alpha = 0.05$
4. **The test statistic** is $Z = \dfrac{\overline{x} - \mu}{\sigma} = \dfrac{59.1 - 57.4}{5.2/\sqrt{40}} = 2.067$

$S.D : S = h\sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2} = 5 \cdot \sqrt{\frac{164}{100} - \left(\frac{16}{100}\right)^2}$ [$\because$ h = 5]

$= 6.35$

1. **Null Hypothesis H_0** : The sample is drawn from a population with mean μ i.e. $\overline{x}$ and μ do not differ significantly where μ = 30.5 years.

2. **Alternative Hypothesis H_1** : μ < 30.5 years (left tail test)

Now, $\overline{x} = 28.8$, $S = 6.35$, $\mu = 30.5$ years and $n = 100$

3. **The test statistic** is, $Z = \frac{\overline{x} - \mu}{S/\sqrt{n}} = \frac{28.8 - 30.5}{6.35/\sqrt{100}} = -2.677$

$\therefore \quad |Z| = 2.68.$

Tabulated value of Z at 5% level of significance is 1.645 (left tail test).

Here calculated $Z >$ tabulated Z.

$\therefore$ The null hypothesis H_0 is rejected.

i.e., $\overline{x}$ and μ differ significantly.

i.e., the sample is not drawn from a population with mean μ = 30.5 years

Example 10 : The mean life time of a sample of 100 light tubes produced by a company found to be 1560 hrs with a population S.D of 90 hrs. Test the hypothesis for $\alpha = 0.05$ that the mean life time of the tubes produced by the company is 1580 hrs.

[JNTU (A) Dec. 2009 (Set No. 1)]

Solution : Given $\overline{x}$ = Mean of the sample = 1560 hrs

μ = Mean of the population = 1580 hrs

n = Sample size = 100

σ = Standard deviation = 90 hrs

1. **Null Hypothesis** $H_0 : \mu = 1580$

2. **Alternative Hypothesis** $H_1 : \mu \neq 1580$

3. **Level of significance :** $\alpha = 0.05$

4. **The test statistic is** $z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} = \frac{1560 - 1580}{90/\sqrt{100}} = \frac{-20}{9}$

Hence, we reject the Null Hypothesis H_0 at 5% level of significance and conclude th

the Australians are taller than Englishmen.

Example 9 : The mean life of a sample of 10 electric bulbs (or motors) was found be 1456 hours with S.D. of 423 hours. A second sample of 17 bulbs (motors) chosen from different batch showed a mean life of 1280 hours with S.D. of 398 hours. Is there a signific difference between the means of two batches? **[JNTU (K) 2009, Nov. 2012 (Set No. 1**

Solution : It is given that

n_1 = Sample size of first batch = 10

n_2 = Sample size of second batch = 17

$\bar{x}_1$ = Mean life of first batch = 1456

$\bar{x}_2$ = Mean life of second batch = 1280

σ_1 = Standard deviation of first batch = 423

σ_2 = Standard deviation of second batch = 398

1. Null Hypothesis $H_0 : \mu_1 = \mu_2$

2. Alternative Hypothesis $H_1 : \mu_1 \neq \mu_2$

3. Level of significance : $\alpha = 0.05$

4. The test statistic is $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} = \dfrac{1456 - 1280}{\sqrt{\dfrac{(423)^2}{10} + \dfrac{(398)^2}{17}}}$

$$= \frac{176}{\sqrt{17892.9 + 9317.88}} = \frac{176}{164.96} = 1.067$$

Since $z < z_\alpha = 1.96$, we accept the null hypothesis H_0 *i.e.,* no, there is no differe between the mean life of electric bulbs of two batches.

(*i*) ...cept the Null Hypothesis

(*ii*) Since $|z| < 2.58$, we accept the Null Hypothesis H_0 at 0.5% level of significance.

(*iii*) Since, $|z| < 2.06$, we accept H_0 at 4% level of significance.

(*iv*) Since, $|z| < 1.96$, we accept H_0 at 5% level of significance.

(*v*) Since, $|z| < 1.645$, we accept H_0 at 10% level of significance.

(*vi*) Since, $|z| > 1.44$, we reject H_0 at 15% level of significance.

Example 13 : In 64 randomly selected hours of production, the mean and the standard deviation of the number of acceptance pieces produced by an automatic stamping machine are $\bar{x} = 1.038$ and $\sigma = .146$

At the .05 level of significance does this enable us to reject the null hypothesis $\mu = 1.000$ against the alternative hypothesis $\mu > 1.000$?

[JNTU (H) Nov. 2010 (Set No. 4)]

Solution : Let the Null Hypothesis be $H_0 : \mu = 1.000$

Then the Alternative Hypothesis is $H_1 : \mu > 1.000$

Here $\quad \bar{x}$ = Mean of the sample $= 1.038$

μ = Mean of the population $= 1.000$

σ = S.D. of the population $= 0.146$

and $\quad n$ = Sample size $= 64$

$\therefore$ The test statistic is $z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

$$= \frac{1.038 - 1.000}{0.146 / \sqrt{64}} = \frac{0.038}{0.146 / 8}$$

$$= 2.082$$

Thus we see that $z = 2.082 > 1.645$

Hence, we reject the Null Hypothesis H_0 at 5% level of significance and conclude that the mean of the population $\mu > 1.000$

$$z = \frac{p-P}{\sqrt{\frac{PQ}{n}}} = \frac{\frac{1}{6}-0.25}{\sqrt{\frac{(0.25(0.75)}{120}}} = \frac{-0.0833}{0.0395} = -2.107$$

Since $|z| = 2.107 < 2.33 = z_{0.05}$, we accept the null hypothesis H_0. That is, the claim or belief of social worker is true.

Example 9 : A manufacturer claims that only 4% of his products are defective. A random sample of 500 were taken among which 100 were defective. Test the hypothesis at 0.05 level. **[JNTU 2005 (Set No. ...]**

0.4%

Solution : We have

$$x = 100,\ n = 500,\ p = \frac{x}{n} = 0.2 \text{ and } P = 0.04,\ Q = 1 - P = 0.96$$

1. **Null Hypothesis H_0** : P = 0.04
2. **Alternative Hypothesis H_1** : $P > 0.04$ (Right tailed test)
3. **Level of significance** : $\alpha = 0.05$
4. **The test statistic** is

$$z = \frac{p-P}{\sqrt{\frac{PQ}{n}}} = \frac{0.2-0.04}{\sqrt{\frac{(0.04)(0.96)}{500}}} = \frac{-0.16}{0.00876} = -18.26$$

Since $|z| = 18.26 > 1.645 = z_{0.05}$, we reject the null hypothesis H_0.

Example 10 : In a sample of 500 from a village in Rajasthan, 280 are found to be wheat eaters and the rest rice eaters. Can we assume that the both articles are equally popular. **[JNTU (K) 2009, May 2012 (Set No. ...]**

Solution : Given $n = 500$

p = sample proportion of wheat eaters $= \frac{280}{500} = 0.56$

... of wheat eaters $= \frac{1}{[illegible]} = 0.5$

Rejection Rule for $H_0 : P_1 = P_2$:

(*i*) If $|z| > 1.96$, then reject H_0 at 5% level of significance.

(*ii*) If $|z| > 2.58$, then reject H_0 at 1% level of significance.

(*iii*) If $|z| > 1.645$, then reject H_0 at 10% level of significance.

SOLVED EXAMPLES

Example 1 : Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal are same, at 5% level. **[JNTU (H) May 2011 (Set No. 2)]**

Solution : Given sample sizes, $n_1 = 400$, $n_2 = 600$

Proportion of men, $p_1 = \dfrac{200}{400} = 0.5$

Proportion of women, $p_2 = \dfrac{325}{600} = 0.541$

1. **Null hypothesis H_0 :** Assume that there is no significant difference between the option of men and women as far as proposal of flyover is concerned.

   i.e., $H_0 : p_1 = p_2 = p$

2. **Alternative hypothesis H_1 :** $p_1 \neq p_2$ (two tailed)

3. **The test statistic** is $z = \dfrac{p_1 - p_2}{\sqrt{pq\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$ [Method of pooling]

$$\text{where } p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$= \frac{400 \times \dfrac{200}{400} + 600 \times \dfrac{325}{600}}{400+600} = \frac{525}{1000} = 0.525$$

$$\text{and } q = 1 - p = 1 - 0.525 = 0.475$$

$$\therefore \quad z = \frac{0.5 - 0.541}{\sqrt{0.525 \times 0.425\left(\dfrac{1}{400}+\dfrac{1}{600}\right)}} = \frac{-0.041}{0.032} = -1.28$$

$|z| = 1.28$

Since $|z| < 1.96$, we accept the null hypothesis H_0 at 5% level of significance *i.e.,* there is no difference of opinion between men and women as far as proposal of flyover is concerned.

Example 2 : A manufacturer of electronic equipment subjects samples of two competing brands of transistors to an accelerated performance test. If 45 of 180 transistors of the first kind and 34 of 120 transistors of the second kind fail the test, what can he conclude at the level of significance a = 0.05 about the difference between the corresponding sample proportions.

[JNTU 2003S, 2004 (Set No.

Solution : We have $n_1 = 180$, $x_1 = 45$, $x_2 = 34$, $n_2 = 120$

Since $|z| < 1.96$, we accept the null hypothesis H_0 at 5% level of significance. *i.e.*, 8% difference in the sale of two brands of cigarettes is a valid claim.

Example 5 : In two large populations, there are 30%, and 25% respectively of fair haired people. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations. **[JNTU (A) Nov. 2010, (K) Nov. 2009, May 2013 (Set No. 1)]**

$p = \frac{30}{100}$

Solution :

Given $n_1 = 1200, \ n_2 = 900$

$P_1 =$ Proportion of fair haired people in the first population $= \dfrac{30}{100} = 0.3$

$P_2 =$ Proportion of fair haired people in the second population $= \dfrac{25}{100} = 0.25$

1. **Null Hypothesis H_0** : Assume that the sample proportions are equal *i.e.*, the difference in population proportions is likely to be hidden in sampling *i.e.*,
   $H_0 : P_1 = P_2$
2. **Alternative Hypothesis H_1 : $P_1 \neq P_2$**
3. **The test statistic** is, $z = \dfrac{P_1 - P_2}{\sqrt{\dfrac{P_1 Q_1}{n_1} + \dfrac{P_2 Q_2}{n_2}}}$

$P = \dfrac{0.3 \times 100 + 0.25 \times 100}{100 + 100}$

$= \dfrac{0.3 - 0.25}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

0.275

$1 - 0.275$

0.791

where $Q_1 = 1 - P_1 = 1 - 0.3 = 0.7$

$Q_2 = 1 - P_2 = 1 - 0.25 = 0.75$

$$\therefore \quad z = \frac{0.3 - 0.25}{\sqrt{\dfrac{0.3 \times 0.7}{1200} + \dfrac{0.25 \times 0.75}{900}}} = \frac{0.05}{0.0195} = 2.56$$

i.e., $z = 2.56$

Since $z > 1.96$, therefore we reject the Null Hypothesis H_0 at 5% level of significance (Two-tailed test) *i.e.*, the sample proportions are not equal. Thus we conclude that the difference in population proportions is unlikely that the real difference will be hidden.

Example 6 : A company wanted to introduce a new plan of work and a survey was conducted for this purpose. Out of sample of 500 workers in one group 62% favoured the new plan and another group of sample of 400 workers 41% were against the new plan. Is there any significant difference between the two groups in their attitude towards the new plan at 5% level of significance? **[JNTU (A) Dec. 2009 (Set No. 1)]**

0.62 and $p_2 = 1 - 0.41 = 0.59$

15. Th... supplied by a manufacturer is 1800 with a $S.D.$ 100. By a new technique in the manufacturing process, it is claimed that the breaking strength of the cables have increased. In order to test this claim, a sample of 50 cables is tested. It is found that the mean breaking strength is 1850. Can we support the claim at 1% level of significance.

16. An investigation of the relative merits of two kinds of flash light batteries showed that a random sample of 100 batteries of brand A tested on the average 36.5 hrs with a $S.D.$ of 1.8 hrs while a random sample of 80 batteries of brand B tested on the average 36.8 hrs with a $S.D.$ of 1.5 hrs. Use a level of significance of 0.05 to test whether the observed difference between the average life times is significant.

17. Given the following information relating to two places A and B. Test whether there is any significant difference between their mean wages :

	A	***B***
Mean wages (Rs.)	47	49
***S.D.* (Rs.)**	28	40
Number of workers	1000	1500

18. The mean consumption of food grains among 400 sampled middle class consumers is 380 gms per day per person with a $S.D.$ of 120 gms. A similar sample survey of 600 working class consumers gave a mean of 410 gms with a $S.D.$ of 80 gms. Are we justified in saying that the two classes consume the same quantity of food grains. Use 5% level of significance.

10. 0.34, 0.36.

11. $H_0: \mu = 4$ ft, $H_1: \mu \neq 4$ ft, $Z = 3.33$ significant.

12. $|Z| = 2.5$, significant at 5% level of significance.

14. 291.64, 296.35.

15. $H_0: \mu = 1800, \sigma = 100, H_1: \mu_1 > 1800, Z = 3.53$, Ac

16. $H_0: \overline{x}_1 = \overline{x}_2$; $|Z| = 1.21$, Accept H_0

17. $H_0: \overline{x}_1 = \overline{x}_2$, $|Z| < 1.96$, Accept H_0

18. $H_0: \overline{x}_1 = \overline{x}_2$, $H_1: \overline{x}_1 \neq \overline{x}_2$, $|Z| = 4.39$ significant.

19. $H_0: p = \frac{1}{3}$, $H_1: P < 0.5$; $z = -9.128$, H_0 rejected.

21. $|z| = 2.42$

19. In a city 250 men out of 750 were found to be smokers. Does this info support the conclusion that the majority of men in this city are smok

[JNTU 2005S, 2006S, 2008S (Se

20. Test the significance of the difference between the means of the samp the following data :

	Size of sample	Mean	S.D
Sample A	100	61	4
Sample B	200	63	6

[JNTU (K) Dec. 2013 (Se

(**Hint :** Refer Solved Example 4 in Test of Hypothesis I)

21. In a random sample of 100 tube lights produced by companny A, the m time of tube light is 1190 hours with standard deviation of 90 hours. A random sample of 75 tube lights from company B the mean life time hours with standard deviation of 120 hours. Is there a difference betw mean life times of the two brands of tube lights at a significance level o

[JNTU (K) Mar. 2014 (Se

22. 500 articles from a factory are examined and found to be 2% defective

Problems related to Student's t Test (When S.D of the sample is not given directly)

SOLVED EXAMPLES

Example 1 : A random sample of 10 boys had the following I.Q's : 70, 120, 110, 101, 88, 83, 95, 98, 107 and 100.

(a) Do these data support the assumption of a population mean I.Q of 100 ?

(b) Find a reasonable range in which most of the mean I.Q. values of samples of 10 boys lie. **[JNTU 2008, (A) Apr. 2012, (H) May 2011 (Set No. 4), (K) May 2013 (Set No. 2)]**

Solution : (a) Here S.D. and mean of the sample is not given directly.

We have to determine these S.D. and mean as follows.

Mean, $\bar{x} = \frac{\sum x}{n} = \frac{972}{10} = 97.2$

x	$x - \bar{x}$	$(x - \bar{x})^2$
70	–27.2	739.84
120	22.8	519.84
110	12.8	163.84
101	3.8	14.44
88	–9.2	84.64
83	–14.2	201.64
95	–2.2	4.84
98	0.8	0.64
107	9.8	96.04
100	2.8	7.84
972		1833.60

We know that $S^2 = \frac{1}{n-1} \sum_i (x_i - \bar{x})^2 = \frac{1833.60}{9}$

$\therefore$ Standard deviation, $S = \sqrt{203.73} = 14.27$

1. **Null Hypothesis H_0 :** The data support the assumption of a population mean I.Q of 100 in the population.
2. **Alternative Hypothesis H_1 : $\mu \neq 100$**
3. **Level of significance, $\alpha = 0.05$**
4. **The test statistic is** $t = \frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{97.2 - 100}{14.27/\sqrt{10-1}} = -0.62$

$\therefore \quad |t| = 0.62$ *i.e.*, Calculated value of $t = 0.62$

Tabulated value of t for $(10-1)$ d.f. *i.e.*, 9 d.f. at 5% level of significan 2.26 (two-tailed list).

Since calculated value of $t <$ tabulated value of t, we accept the null hypo H_0. *i.e.*, the data support the assumption of mean I.Q of 100 in the population

(*b*) The 95% confidence limits are given by $\bar{x} \pm t_{0.05} \cdot S/\sqrt{n}$

$$= 97.2 \pm 2.26 \times 4.512 = 97.2 \pm 10.198 = 107.4 \text{ and } 87$$

$\therefore$ The 95% confidence limits within which the mean I.Q values of sampl 10 boys will lie is (87, 107.40).

Example 2 : The heights of 10 males of a given locality are found to be 70, 67, 6 61, 68, 70,64,64,66 inches. Is it reasonable to believe that the average height is greater th inches? Test at 5% significance level assuming that for 9 degrees of freedom ($t = 1.8$ $\alpha = 0.05$).

[JNTU (H) May 2011 (Set

<u>**Solution :**</u> Calculation for sample mean and $S.D.$

$$\text{Mean, } \bar{x} = \frac{\sum x}{n} = \frac{660}{10} = 66$$

x	$x-\bar{x}$	$(x-\bar{x})^2$
70	4	16
67	1	1
62	−4	16
68	2	4
61	−5	25
68	2	4
70	4	16
64	−2	4
64	−2	4
66		0

Example 8 : Two independent samples of 8 and 7 items respectively had the foll values.

Sample I	11	11	13	11	15	9	12	14
Sample II	9	11	10	13	9	8	10	–

Is the difference between the means of samples significant ?

[JNTU 2005S (Set

Solution : Given $n_1 = 8$, $n_2 = 7$ and

$$\bar{x} = \frac{1}{8}(11 + 11 + 13 + 11 + 15 + 9 + 12 + 4) = \frac{96}{8} = 12$$

$$\bar{y} = \frac{1}{7}(9 + 11 + 10 + 13 + 9 + 8 + 10) = \frac{70}{7} = 10$$

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$(y-\bar{y})$	$(y-\bar{y})^2$
11	–1	1	9	–1	1
11	–1	1	11	1	1
13	1	1	10	0	0
11	–1	1	13	3	9
15	3	9	9	–1	1
9	–3	9	8	–2	4
12	0	0	10	0	0
14	2	4			
96		26	70		16

$$\text{Now } S^2 = \frac{1}{n_1+n_2-2}\left[\Sigma(x_i-\bar{x})^2 + \Sigma(x_i-\bar{y})^2\right]$$

$$= \frac{1}{8+7-2}(26+16) = \frac{42}{13} = 3.23$$

$$\therefore\ S = 1.8$$

1. **Null Hypothesis** $H_0 : \mu_1 = \mu_2$
2. **Alternative Hypothesis** $H_1 : \mu_1 \neq \mu_2$ (two tailed test)
3. **Level of significance :** $\alpha = 0.05$
4. **The test statistic is** $t = \dfrac{\bar{x}-\bar{y}}{S.\sqrt{\dfrac{1}{n_1}+\dfrac{1}{n_2}}} = \dfrac{12-10}{(1.8)\sqrt{\dfrac{1}{8}+\dfrac{1}{7}}} = \dfrac{2}{0.9316} = 2.15$

Tabulated value of t for $8 + 7 - 2 = 13$ d.f. at 5% level of significance is 2.1

Since calculated t < tabulated t, we accept H_0 *i.e.*, the difference between th means of samples is not significant.

Since Calculated $t >$ Tabulated t, we reject the null hypothesis H_0 and conclude that both horses A and B do not have the same running capacity.

Example 6 : To examine the hypothesis that the husbands are more intelligent than the wives, an investigator took a sample of 10 couples and administered them a test which measures the I.Q. The results are as follows :

Husbands	117	105	97	105	123	109	86	78	103	107
Wives	106	98	87	104	116	95	90	69	108	85

Test the hypothesis with a reasonable test at the level of significance of 0.05.

[JNTU 2004, 2005, 2007S, (A) Nov. 2010, (K) May 2013 (Set No. 3)

Solution : We have $n_1 = 10$, $n_2 = 10$ and

$$\bar{x} = \frac{1}{10}(117 + 105 + 97 + 105 + 123 + 109 + 86 + 78 + 103 + 107)$$

$$= \frac{1}{10}(1030) = 103$$

$$\bar{y} = \frac{1}{10}(106 + 98 + 87 + 104 + 116 + 95 + 90 + 69 + 108 + 85)$$

$$= \frac{1}{10}(958) = 95.8$$

Now we compute the standard deviations of both samples

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$(y-\bar{y})$	$(y-\bar{y})^2$
117	14	196	106	10.2	104.04
105	2	4	98	2.2	4.84
97	–6	36	87	–8.8	77.44
105	2	4	104	8.2	67.24
123	20	400	116	20.2	408.04
109	6	36	95	–0.8	0.64
86	–17	289	90	–5.8	33.64
78	–25	625	69	–26.8	718.24
103	0	0	108	12.2	148.84
107	4	16	85	–10.8	116.64
1030		1606	958		1679.6

Now $S^2 = \frac{1}{n_1 + n_2 - 2}[\Sigma(x_i - \bar{x})^2 + \Sigma(y_i - \bar{y})^2]$

$= \frac{1}{18}[1606 + 1679.6] = \frac{1}{18}(3285.6) = 182.53$

$\therefore S = 13.51$

1. **Null Hypothesis $H_0 : \mu_1 = \mu_2$** (i.e., no difference in I.Q.)
2. **Alternative Hypothesis $H_1 : \mu_1 > \mu_2$**

(*i.e.*, husbands are more intelligent than wives) (one tailed test, right)

3. **Level of significance : $\alpha = 0.05$**
4. **The test statistic is** $t = \frac{\bar{x} - \bar{y}}{S.\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{103 - 95.8}{(13.51)\sqrt{\frac{1}{10} + \frac{1}{10}}} = 1.19168$

Since $t_{cal} = 1.19168 < t_{tab} = 1.734$, we accept the null hypothesis H_0 i.e., There is no difference in I.Q's.

Example.7 : Find the maximum difference that we can expect with probability 0.95 between the means of samples of sizes 10 and 12 from a normal population if their standard