the gamma-Poisson conjugate families

Dr. David Banks
Duke University

Poisson random variable

probability mass function for Poisson distribution

λ mean
variance

$$\mathrm{P}[X = k] = \frac{\lambda^k}{k!} \exp(-\lambda) \text{ for } k = 0, 1, \ldots$$

$k! = k \times (k\text{-}1) \times \ldots \times 1$

$\lambda > 0$


1875 - 1894
15 cavalry units
200 casualties
λ average number
per year, per unit
https://pixabay.com/en/monument-golden-rider-dresden-547813/

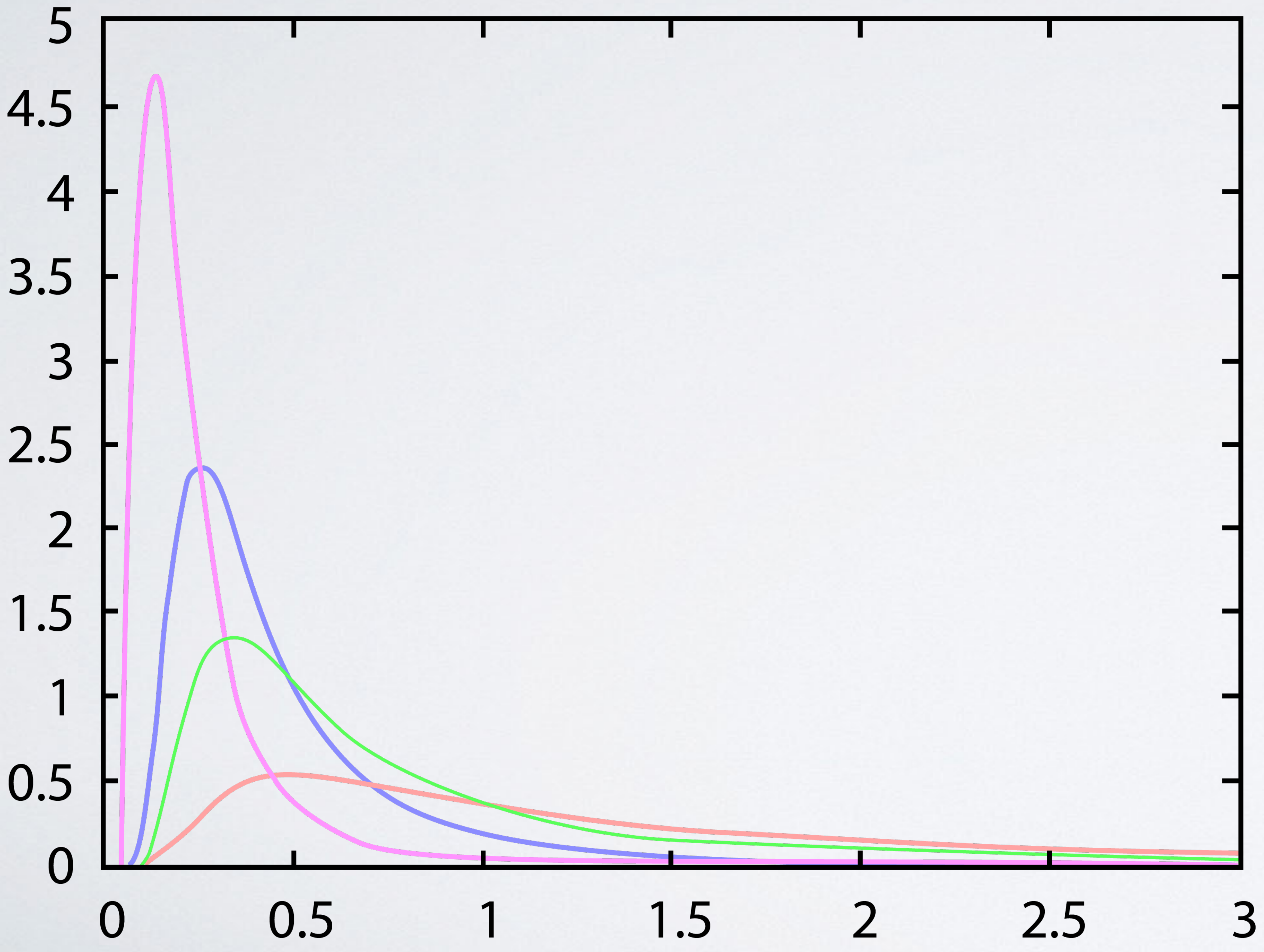
the Bayesian general

$$\lambda = 0.75$$

std. dev. = 1

gamma distribution

continuous non-negative random variable



parameters for the pdf for the gamma

$$k = 9/16 \quad \theta = 4/3$$

$$\text{mean} = k\theta \quad = 0.75$$

$$\text{std. dev.} = \theta\sqrt{k} = 1$$

new parameters

$$x_1, x_2, \ldots x_n$$

$$k^* = k + \sum x_i$$

$$\theta^* = \frac{\theta}{(n\theta + 1)}$$

Prussian cavalry dataset

- n = 300 observations
- 200 casualties

$$
\begin{aligned}
k^* &= k + \textstyle\sum_{i=1}^{n} x_i \\
&= (9/16) + 200 \\
&= 200.5625
\end{aligned}
\qquad
\begin{aligned}
\theta^* &= \frac{\theta}{n\theta+1} \\
&= \frac{4/3}{300\times(4/3)+1} \\
&= 0.0033
\end{aligned}
$$

has the general changed his mind?

	λ	**uncertainty**
before	0.75	1
after	0.67	0.047

summary

1. we learned about the **Poisson distribution**
2. we know the gamma-Poisson families are **conjugate**
3. we were given the formula for updating the **gamma parameters** after seeing data
4. we reanalyzed a **classic data set**