comparing two paired means using Bayes factors

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zinc in drinking water

location	surface	bottom	difference
1	0.43	0.415	0.015
2	0.266	0.238	0.028
3	0.567	0.39	0.177
4	0.531	0.41	0.121
5	0.707	0.605	0.102
…	…	…	…
10	0.723	0.612	0.111

hypotheses

parameter of interest $\quad \mu_B - \mu_S \equiv \mu_{diff}$

no differences $\quad H_1 : \mu_B = \mu_S \Leftrightarrow \mu_{diff} = 0$
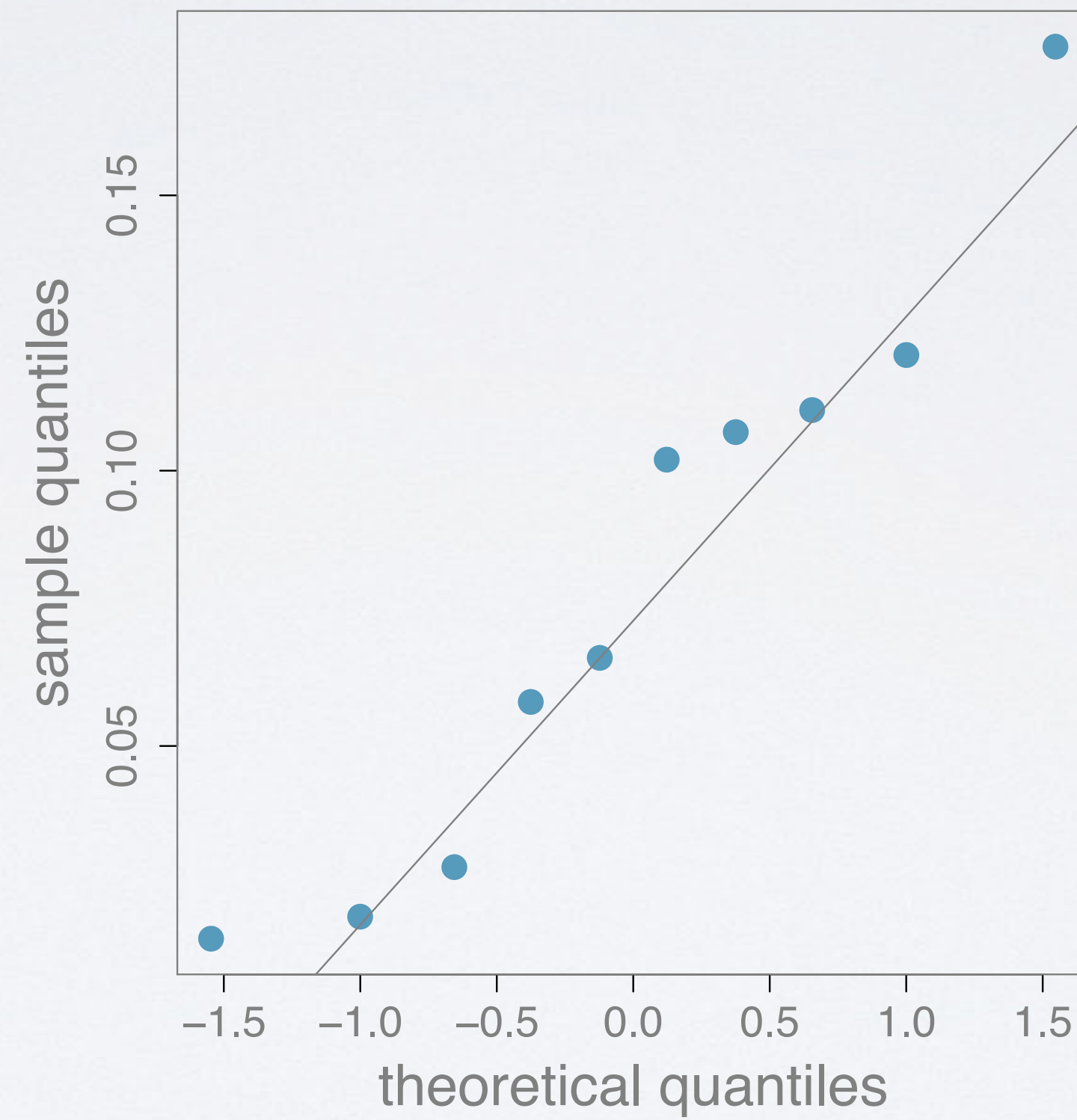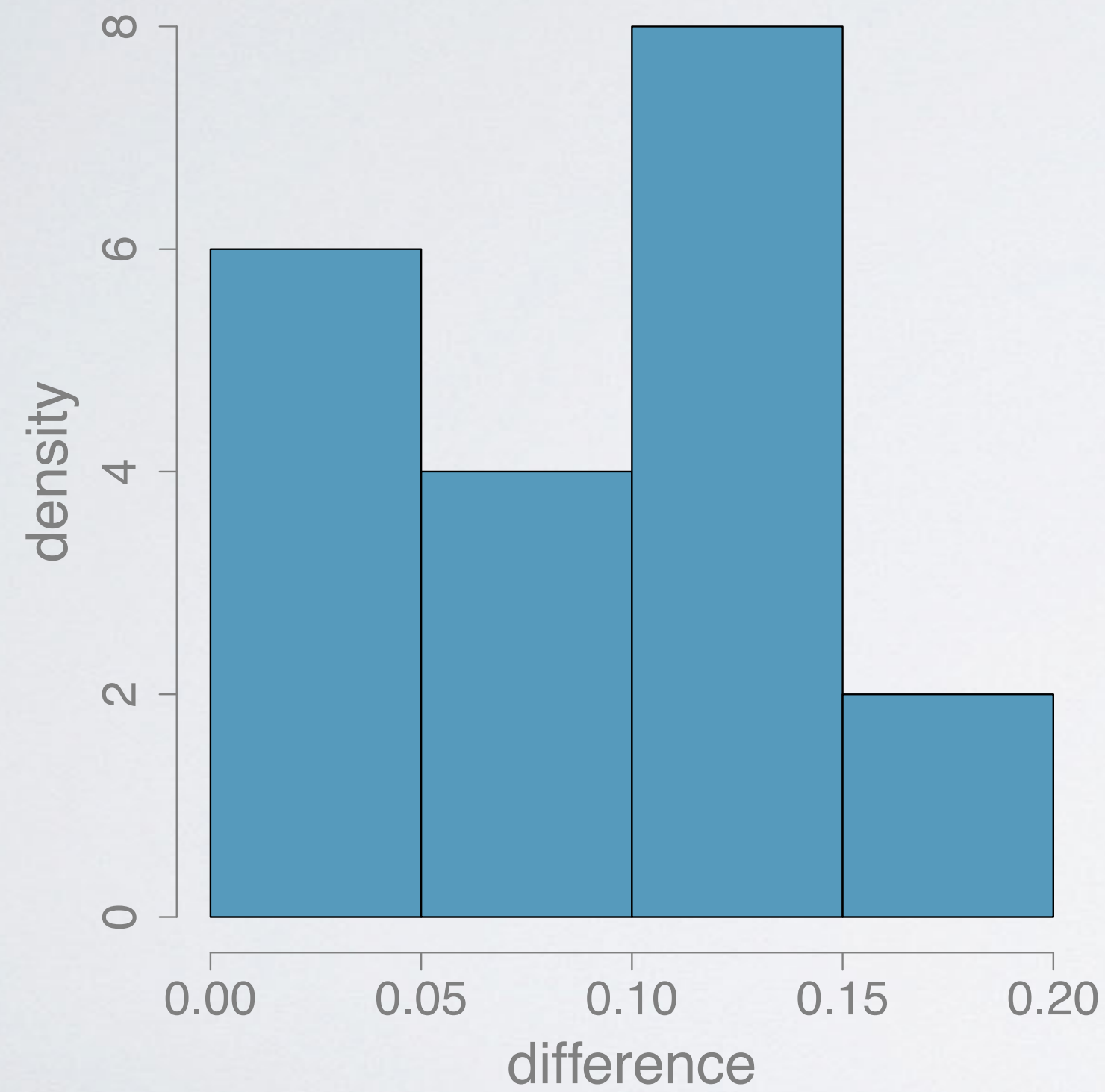
means are different $\quad H_2 : \mu_B \neq \mu_S \Leftrightarrow \mu_{diff} \neq 0$

sub-hypotheses $\quad H_3 : \mu_B > \mu_S \Leftrightarrow \mu_{diff} > 0$

$$H_4 : \mu_B < \mu_S \Leftrightarrow \mu_{diff} < 0$$

assumptions for likelihood

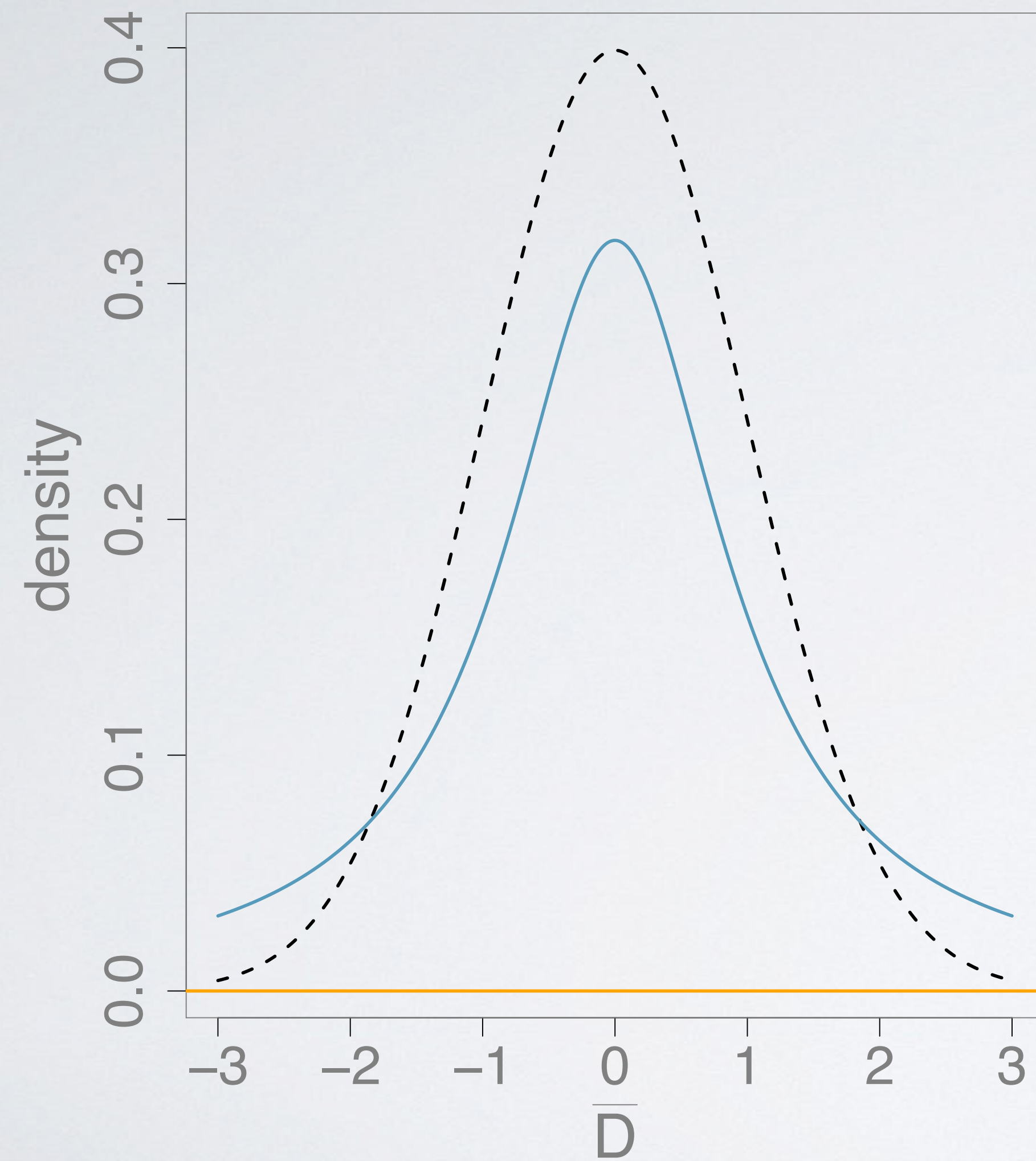$$D_i \overset{\text{iid}}{\sim} N(\mu_{diff}, \sigma^2)$$

priors with unknown σ^2

$$\mu_{diff} \mid \sigma^2, H_2 \sim N(0, \sigma^2/n_0)$$

$$1/\sigma^2 \mid H_2 \sim Ga\left(\nu_0/2, \sigma_0^2 \nu_0/2\right)$$

$$\Leftrightarrow \quad \bar{D} \mid H_2 \sim \mathrm{t}_{\nu_0}\left(0, \sigma_0^2\left(\tfrac{1}{n} + \tfrac{1}{n_0}\right)\right)$$

priors with unknown σ^2

Bayes factor with unknown σ² and reference prior

a limiting case with $\nu_0 \to 0$

$$BF[H_1 : H_2] = \left(\frac{n+n_0}{n_0}\right)^{1/2} \left(\frac{t^2 \frac{n_0}{n+n_0}+\nu}{t^2+\nu}\right)^{\frac{\nu+1}{2}}$$

▸ t-statistic $t = \frac{|\bar{D}|}{s/\sqrt{n}}$

▸ sample standard deviation s

▸ degrees of freedom $\nu = n - 1$

zinc concentration example

▸ p-value ≈ 0.00089

▸ Bayes factor for comparing H_1 to H_2 is 0.0154

▸ evidence against H_1

▸ evidence in favor of H_2 is 64.86

▸ posterior probability of H_1 with equal prior odds

$$P(H_1 \mid \text{data}) = \frac{0.0154}{1 + 0.0154} = 0.0152$$