comparing two proportions using Bayes factors and posterior probabilities

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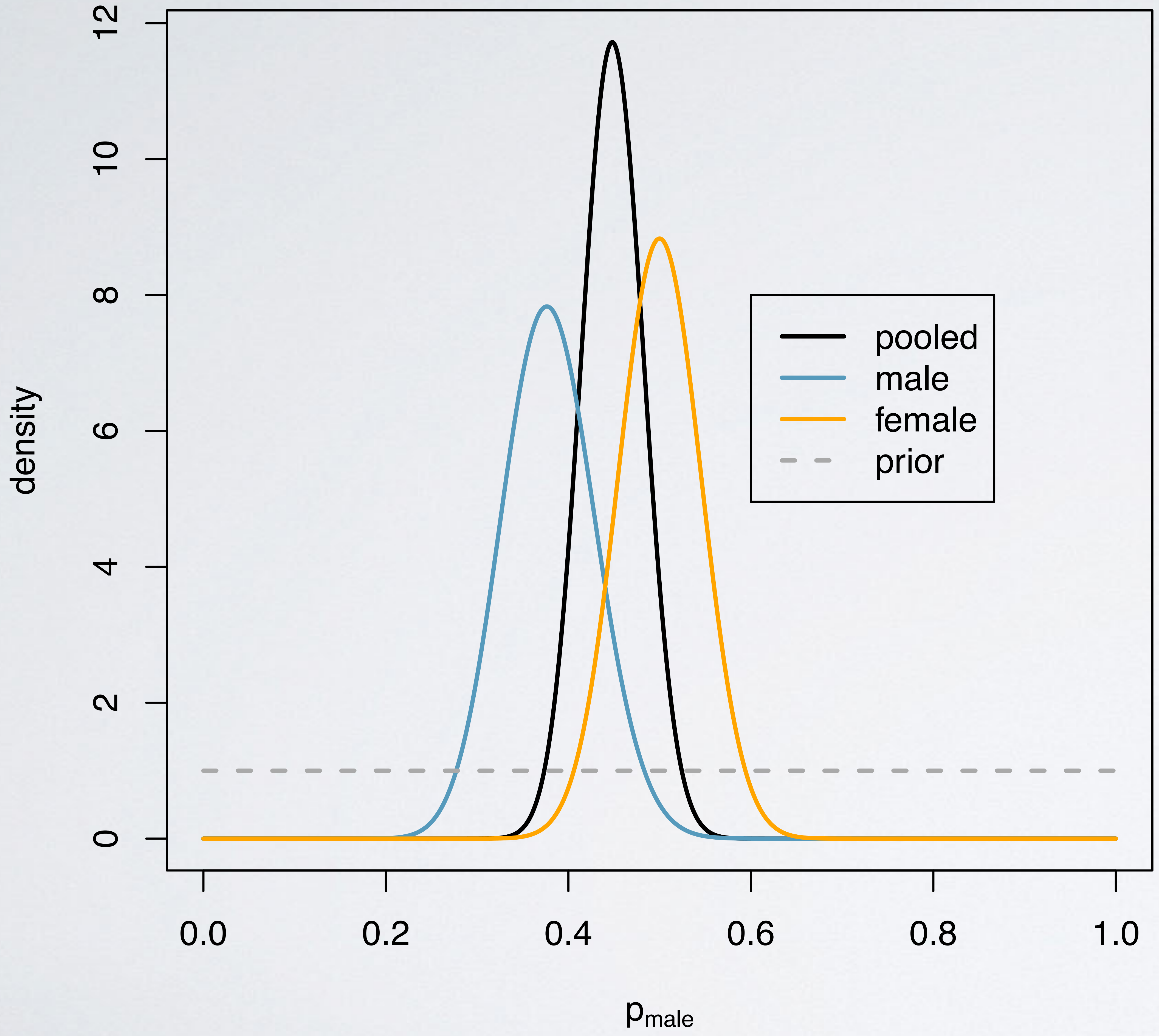
recap

$$H_1\text{: } p_{male} = p_{female}$$

$$H_2\text{: } p_{male} \neq p_{female}$$

Source: https://commons.wikimedia.org/wiki/File:Bully_Free_Zone.jpg

pooled prior and posterior distributions under H_1



posterior probability of H1

$$p(H_1 \mid \text{data}) = \frac{p(\text{data}|H_1)p(H_1)}{p(\text{data}|H_1)p(H_1)+p(\text{data}|H_2)p(H_2)} = \frac{PO[H_1:H_2]}{PO[H_1:H_2]+1)}$$

- posterior odds = Bayes factor × prior odds

$$PO[H_1 : H_2] = BF[H_1 : H_2] \times O[H_1 : H_2]$$

- Bayes factor to compare H1 to H2

$$BF[H_1 : H_2] = \frac{p(\text{data}|H_1)}{p(\text{data}|H_2)}$$

- prior odds $O[H_1 : H_2] = \frac{p(H_1)}{1-p(H_1)}$

Bayes factor for comparing two proportions

$$BF[H_1 : H_2] = \frac{B(a_m + a_f + R_m + R_f, b_m + b_f + n_m + n_f - (R_m + R_f))}{B(a_m + a_f, b_m + b_f)} \div$$

$$\frac{B(a_m + R_m, b_m + n_m - R_m)}{B(a_m, b_m)} \times \frac{B(a_f + R_f, b_f + n_f - R_f)}{B(a_f, b_f)}$$

- B(a, b) is a special function called the Beta function

SurveyUSA example

- Bayes factor for comparing H_1 to H_2 is 2.93
- slight evidence in favor of H_1 but not "worth a bare mention"
- posterior probability of H_1 with equal prior odds

$$P(H_1 \mid \text{data}) = \frac{2.93}{1+2.93} = 0.75$$

- p-value $\approx$ 0.08

summary

- default prior distribution for testing
- posterior distributions under H_2 and H_1
- Bayes factors and posterior probabilities
- different prior distributions for p_{male}, p_{female} and p and $p(H_1)$
- inference with paired or matched normal samples