posterior probabilities, p-value and paradoxes

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extrasensory perception

$$n = 104{,}490{,}000$$

$$\bar{y} = \frac{52263471}{104490000} = 0.500177$$

$$H_1 : \mu = 0.5$$

$$H_2 : \mu \neq 0.5$$

$\mathrm{Z} = 3.61$ with p-value 0.000302

$$BF[H_1 : H_2] = 14.91$$

$$P(\mathrm{H}_1 \mid \text{data}) = 0.9371$$

priors and Bayes factor: known σ^2

$$\mu = .5 \mid H_1 \quad \Leftrightarrow \quad \bar{Y} \mid \sigma^2, H_1 \sim \mathsf{N}(.5, \sigma^2/n)$$

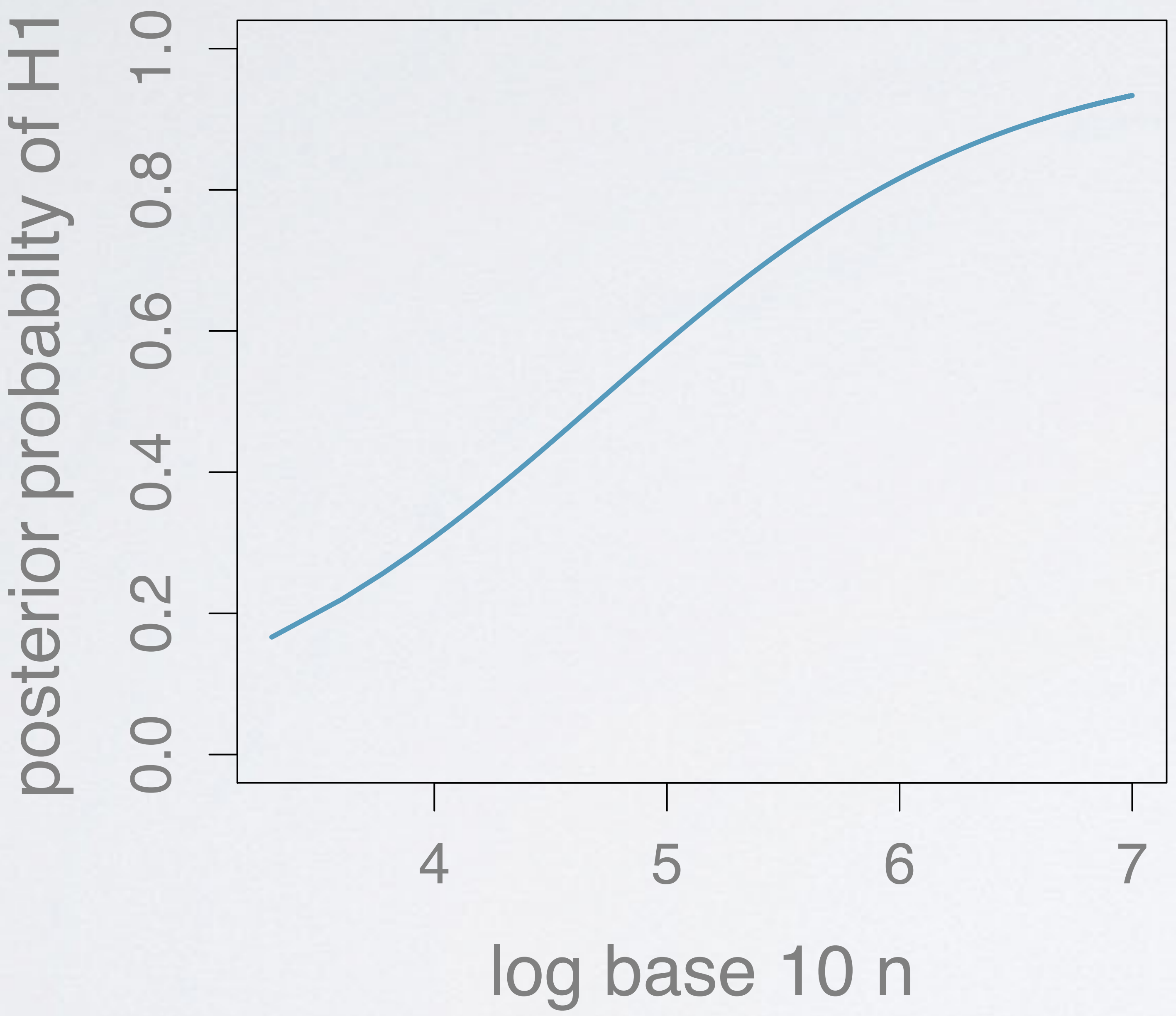
$$\mu \mid \sigma^2, H_2 \sim \mathsf{N}(.5, \sigma^2/n_0) \quad \Leftrightarrow \quad \bar{Y} \mid \sigma^2, H_2 \sim \mathsf{N}\left(.5, \sigma^2\left(\tfrac{1}{n} + \tfrac{1}{n_0}\right)\right)$$

Bayes factor $BF[H_1 : H_2] = \left(\frac{n+n_0}{n_0}\right)^{1/2} \exp\left\{-\frac{1}{2}\frac{n}{n+n_0}Z^2\right\}$

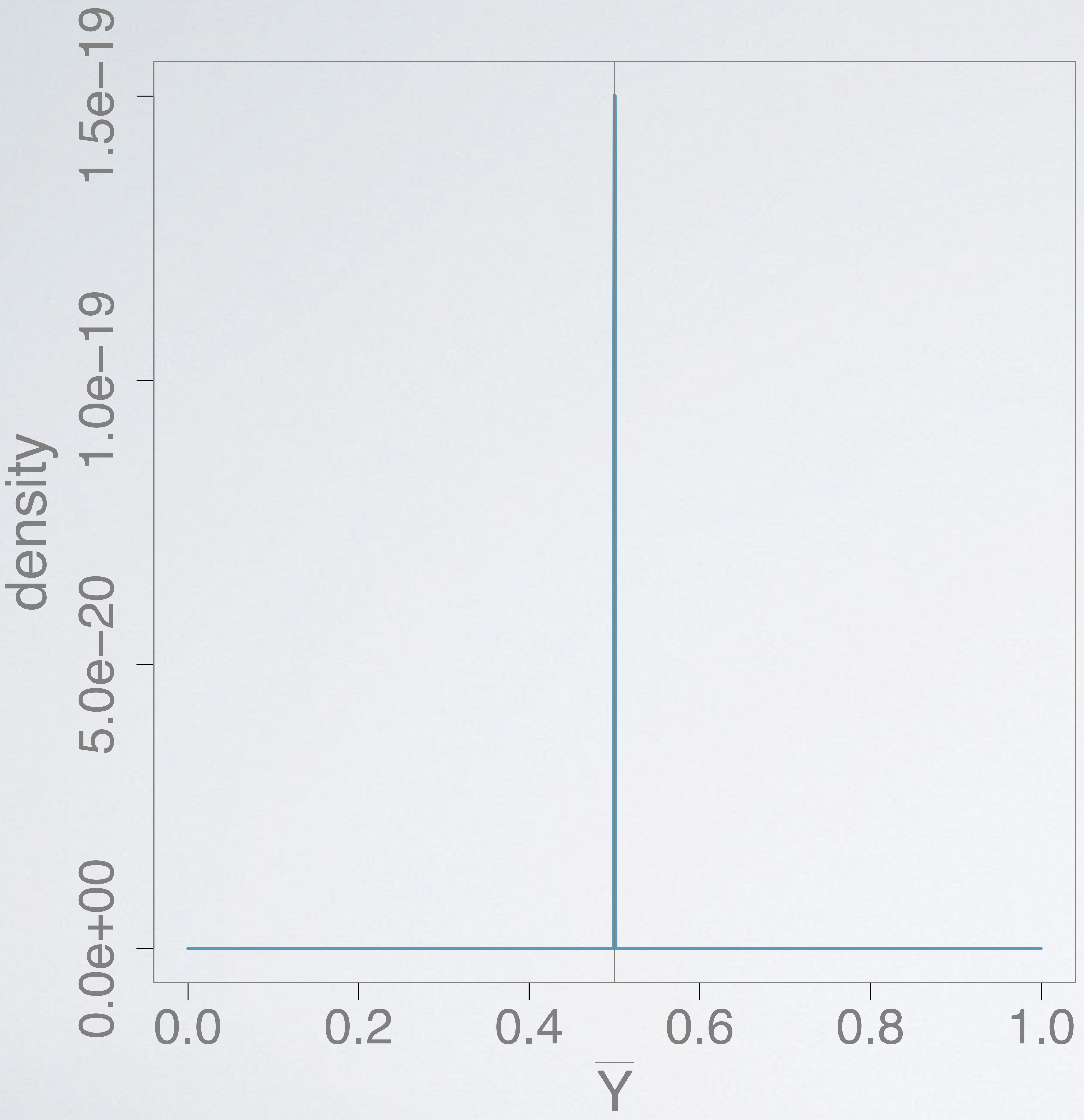
$$\text{Z-score} \quad \mathrm{Z} = \frac{(\bar{Y}-.5)^2}{\sigma/\sqrt{n}}$$

Lindley's paradox: fixed Z, large n

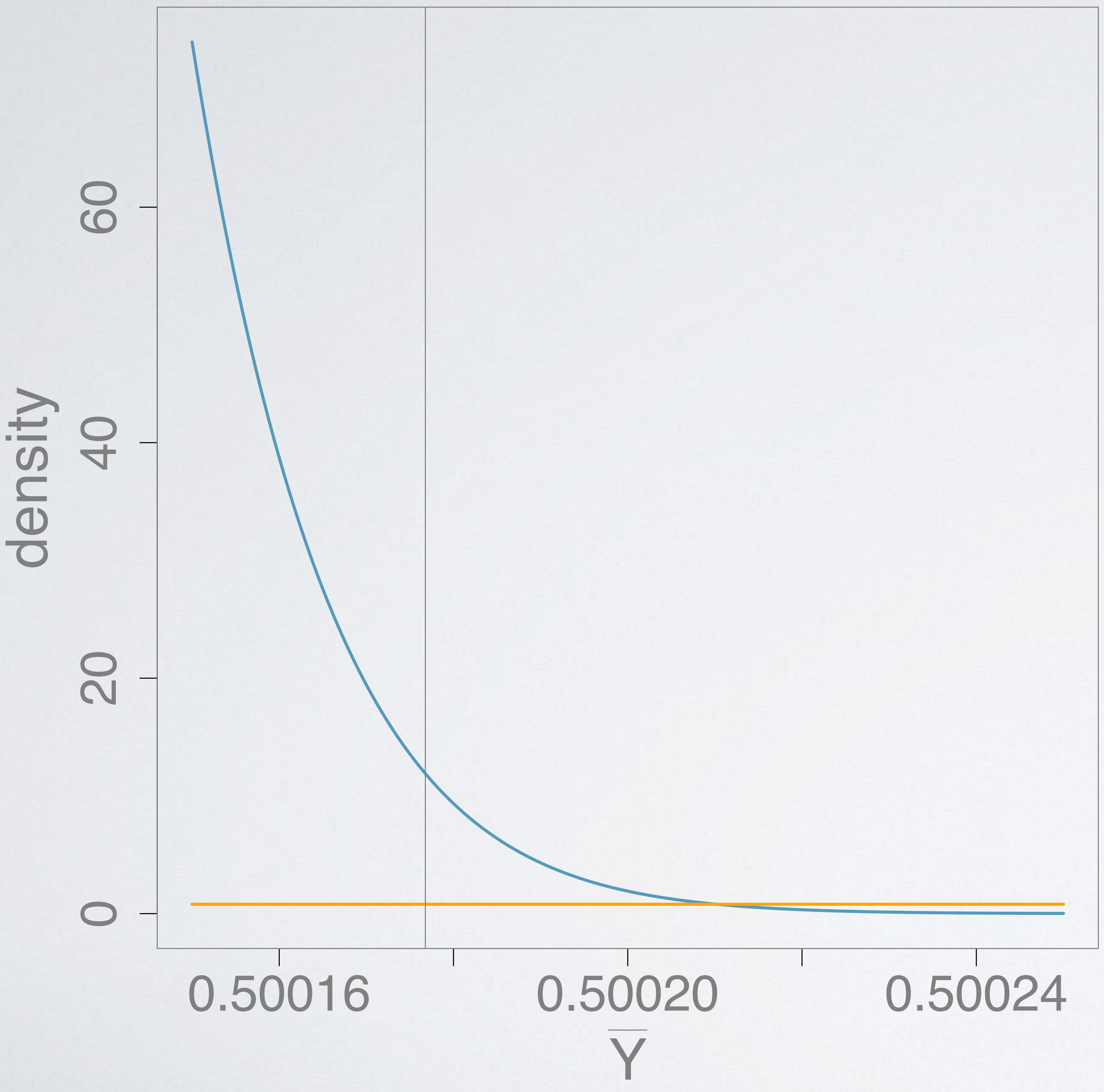
$$BF[H_1 : H_2] = \left(\frac{n+n_0}{n_0}\right)^{1/2} \exp\left\{-\frac{1}{2}\frac{n}{n+n_0}Z^2\right\}$$



predictive distributions



predictive distributions: zoomed view



Bartlett's paradox

$$BF[H_1 : H_2] = \left(\frac{n+n_0}{n_0}\right)^{1/2} \exp\left\{-\frac{1}{2}\frac{n}{n+n_0}Z^2\right\}$$

n_0 goes to zero

$BF[H_1 : H_2]$ goes to infinity

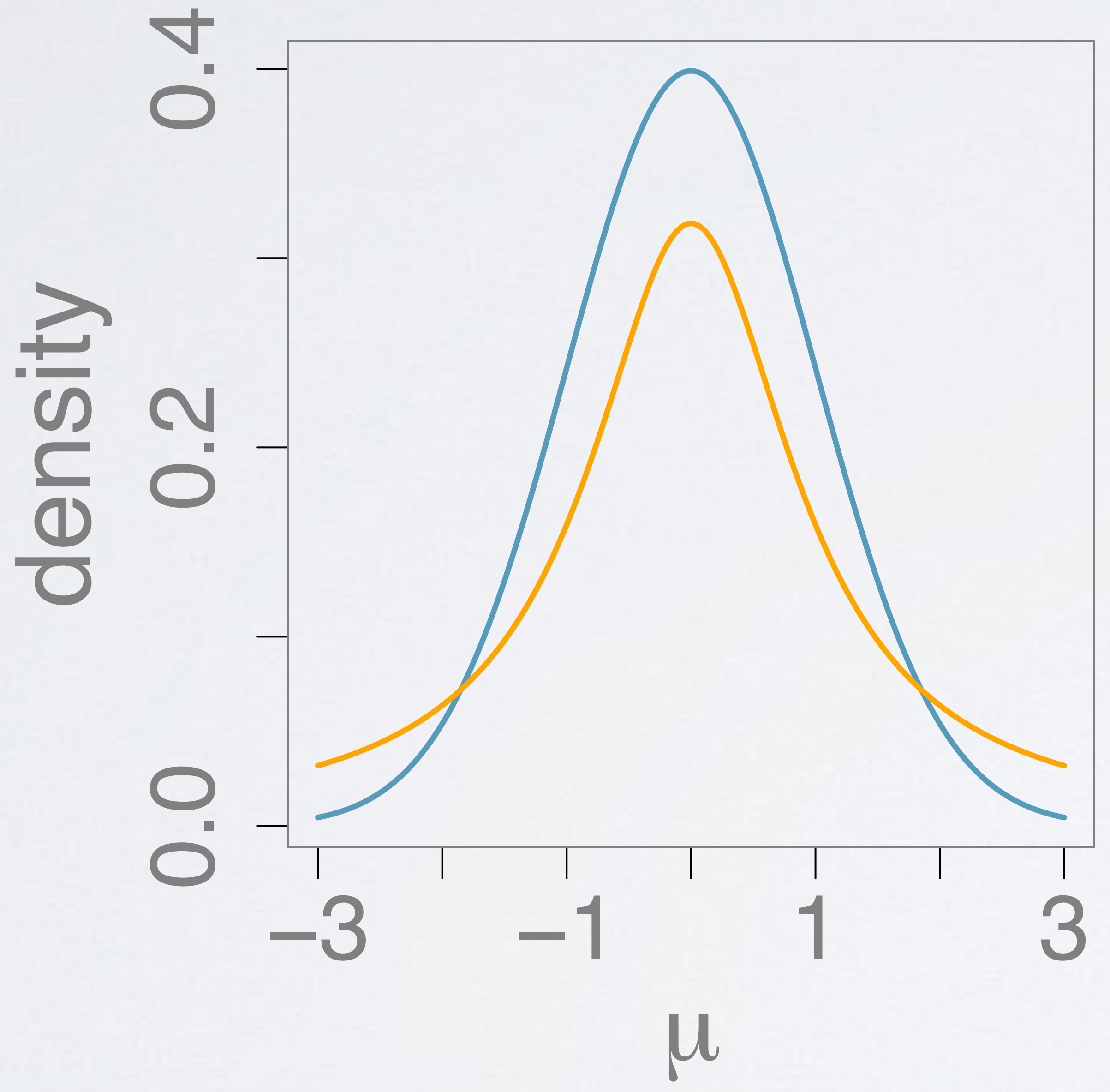
$P(\mathrm{H}_1 \mid \mathrm{data})$ goes to 1

Cauchy prior

$$\mu \mid \sigma^2, n_0, H_2 \sim \mathsf{N}(\mu_0, \frac{\sigma^2}{n_0})$$

$$n_0 \mid H_2 \sim \mathsf{G}(1/2, 1/2)$$

$$\mu \mid \sigma^2, H_2 \sim \mathsf{C}(\mu_0, \sigma^2)$$



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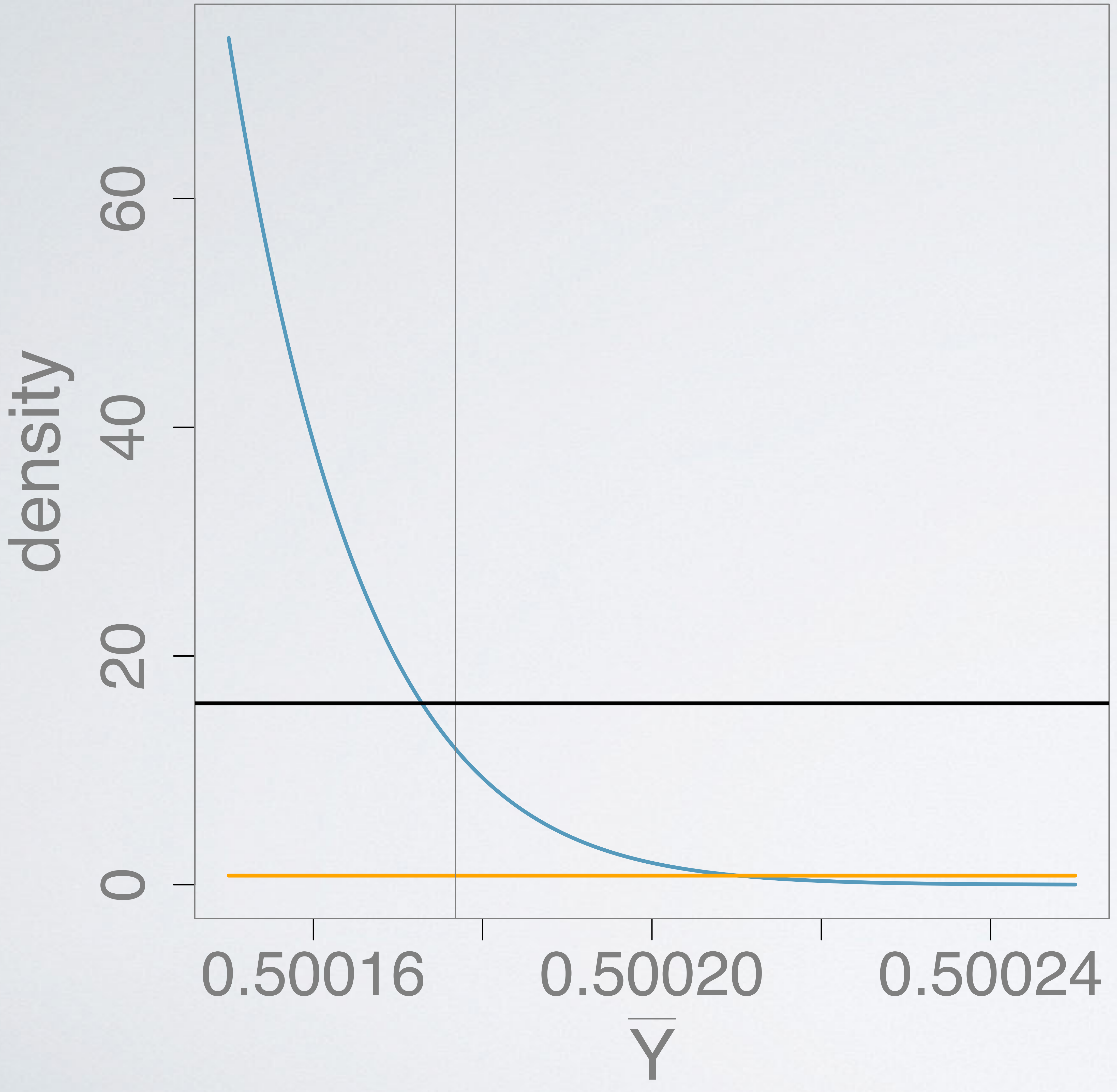

summary

- use robust priors and subjective information
- test point null hypotheses $\mu = \mu_0$?
- ESP versus bias in random number generators