$$\ddot{x} + a(t)\dot{x} + b(t)x = f(t) \tag{1}$$

$a(t)$, $b(t)$, $f(t)$, $t_0 \le t \le t_1$ $x(t)$

$$\begin{aligned} \dot{x}(t_0) - \alpha_0 x(t_0) = \alpha_1, \\ \dot{x}(t_1) - \beta_0 x(t_1) = \beta_1, \end{aligned} \tag{2}$$

$\alpha_0, \alpha_1, \beta_0, \beta_1$, $x(t)$, $\dot{x}(t)$, $t = t_0$, $t = t_1$,

Пример. This is example

$$\ddot{x} + x = 0 \tag{3}$$

$$x = c_1 \cos(t) + c_2 \sin(t) \tag{4}$$

$$\dot{x}(0) = 0, \qquad \dot{x}(1) = 1; \tag{5}$$

$$\dot{x}(0) = 0, \qquad \dot{x}(\pi) = 1; \tag{6}$$

$$\dot{x}(0) = 0, \qquad \dot{x}(\pi) = 0; \tag{7}$$

$$c_1 = -\frac{1}{\sin(t)}, \qquad c_2 = 0,$$

$$1 = -c_1 \sin(\pi) = 0$$

$$x = c_1 \cos(t)$$

c_1,

Solving boundary problem using Green's function

$$\ddot{x} + a(t)\dot{x} + b(t)x = f(t) \tag{8}$$

$$\begin{aligned} \alpha_0 \dot{x}(t_0) + \alpha_1 x(t_0) = 0 \quad (\alpha_0^2 + \alpha_1^2 > 0) \\ \beta_0 \dot{x}(t_1) + \beta_1 x(t_1) = 0 \quad (\beta_0^2 + \beta_1^2 > 0) \end{aligned} \tag{9}$$

$x = x_1(t)$,

$$\ddot{x} + a(t)\dot{x} + b(t)x = 0 \tag{10}$$

$$\alpha_0 \dot{x_1}(t_0) + \alpha_1 x_1(t_0) = 0$$

$x = x_2(t)$

$$\beta_0 \dot{x_2}(t_1) + \beta_1 x_2(t_1) = 0$$

$x_1(t)$, c, $x = cx_1(t)$, $x_2(t)$

$$\begin{aligned} \beta_0 \dot{x_1}(t_1) + \beta_1 x_1(t_1) \neq 0 \\ \alpha_0 \dot{x_2}(t_0) + \alpha_1 x_2(t_0) \neq 0 \end{aligned} \tag{11}$$

$x = x_1(t)$, $x = x_2(t)$, $x(t)$,

$$x(t) = c_1(t)x_1(t) + c_2(t)x_2(t) \tag{12}$$

$\dot{c}_1(t)$, $\dot{c}_2(t)$,

$$\begin{aligned} \dot{c}_1(t)x_1(t) + \dot{c}_2(t)x_2(t) = 0 \\ \dot{c}_1(t)\dot{x}_1(t) + \dot{c}_2(t)\dot{x}_2(t) = f(t) \end{aligned}$$

$$\begin{aligned} \dot{c}_1(t) &= -\frac{x_2(t)f(t)}{w(t)}, \\ \dot{c}_2(t) &= \frac{x_1(t)f(t)}{w(t)}, \end{aligned} \tag{13}$$

where

$$w(t) = \begin{vmatrix} x_1(t) & x_2(t) \\ \dot{x}_1(t) & \dot{x}_2(t) \end{vmatrix} \neq 0$$

$x_1(t)$, $x_2(t)$,

$$\begin{aligned} c_1(t) &= -\int_{t_1}^{t} \frac{x_2(s)f(s)}{w(s)} ds + \gamma_1 = \int_{t}^{t_1} \frac{x_2(s)f(s)}{w(s)} ds + \gamma_1, \\ c_2(t) &= \int_{t_0}^{t} \frac{x_1(s)f(s)}{w(s)} ds + \gamma_2 \end{aligned}$$

where γ_1, γ_2, $c_1(t)$, $c_2(t)$

$$x(t) = x_1(t) \int_{t}^{t_1} \frac{x_2(s)f(s)}{w(s)} ds + x_2(t) \int_{t_0}^{t} \frac{x_1(s)f(s)}{w(s)} ds + \gamma_1 x_1(t) + \gamma_2 x_2(t) \tag{14}$$

t,

$$\dot{x}(t) = \dot{x}_1(t) \int_{t}^{t_1} \frac{x_2(s)f(s)}{w(s)} ds + \dot{x}_2(t) \int_{t_0}^{t} \frac{x_1(s)f(s)}{w(s)} ds + \gamma_1 \dot{x}_1(t) + \gamma_2 \dot{x}_2(t) \tag{15}$$

$(\alpha_0 \dot{x}_1(t_0) + \alpha_1 x_1(t_0) = 0)$

$$0=(\alpha_0\dot{x}_1(t_0)+\alpha_1 x_1(t_0))\int_t^{t_1}\frac{x_2(s)f(s)}{w(s)}ds+\gamma_1(\alpha_0\dot{x}_1(t_0)+\alpha_1 x_1(t_0))+$$
$$+\gamma_2(\alpha_0\dot{x}_2(t_0)+\alpha_1 x_2(t_0))=\gamma_2(\alpha_0\dot{x}_2(t_0)+\alpha_1 x_2(t_0))$$

$\gamma_2=0,\ \gamma_1=0,$

$$x(t)=x_1(t)\int_t^{t_1}\frac{x_2(s)f(s)}{w(s)}ds+x_2(t)\int_{t_0}^{t}\frac{x_1(s)f(s)}{w(s)}ds$$

or

$$x(t)=\int_{t_0}^{t_1}G(t,s)f(s)ds \tag{16}$$

$$G(t,s)=\begin{cases}\frac{x_1(s)x_2(t)}{w(s)}, & t_0\le s\le t,\\ \frac{x_1(t)x_2(s)}{w(s)}, & t\le s\le t_1,\\ \frac{x_1(s)x_2(t)}{w(s)}, & t_0\le t\le s,\\ \frac{x_1(t)x_2(s)}{w(s)}, & s\le t\le t_1,\end{cases} \tag{17}$$

$G(t,s),\ f(t),\ x_1(t),\ x_2(t),\ G(t,s),$

1. $t\neq s\ G(t,s);$
2. $t=t_0\ t=t_1\ G(t,s);$
3. $t=s\ G(t,s);$
4. $t=s\ G'_t(t,s):$
$$G'_t|_{t=s+0}-G'_t|_{t=s-0}=1$$

$$G'_t=\begin{cases}\frac{\dot{x}_1(t)x_2(s)}{w(s)}, & t_0\le t\le s,\\ \frac{x_1(s)\dot{x}_2(t)}{w(s)}, & s\le t\le t_1,\end{cases}$$

$$G'_t|_{t=s+0}-G'_t|_{t=s-0}=\frac{x_1(s)\dot{x}_2(s)-\dot{x}_1(s)x_2(s)}{w(s)}=1.$$

$G(t,s),\ f(t),$