What is a set ?

A set is a collection of objects

History of Set Theory

The founders of set theory are Georg Cantor, Bertrand Russell,
Ernst Zermelo,
Abraham Fraenkel

Russel's Paradox

$$let\ R = \{x | x \notin x\}$$

In other words: a set that contains sets that dont contain themselves causes paradox

Axioms of Set Theory

1. Axiom of extensionality

$$\forall x \forall y [\forall z (z \in x \iff z \in y) \implies x = y]$$

2. Axiom of regularity

$$\forall x[\exists a(a \in x) \implies \exists y]$$

3. Axiom of subsets

$$\forall z \forall w_1 \forall w_2 \ldots \forall w_n \exists y \forall x [x \in y \iff ((x \in z) \land \varphi(x, w_1, w_2, \ldots, w_n, z))]$$

4. Axiom of pairing

$$\forall x \forall y \exists z ((x \in z) \wedge (y \in z))$$

5. Axiom of union

$$\forall F \exists A \forall Y \forall x [(x \in Y \wedge Y \in F) \implies x \in A]$$

6. Axiom schema of replacement

$$\forall A \forall w_1 \forall w_2 \ldots \forall w_n [\forall x (x \in A \implies \exists! y \varphi) \implies \exists B \forall x (x \in A \implies \exists y (y \in B \land \varphi))]$$

7. Axiom of infinity

$$\exists X[\exists e(\forall z \neg(z \in e) \wedge e \in X) \wedge \forall y(y \in X \implies S(y) \in X)]$$

$$where\ S(w) = w \cup \{w\}$$

8. Axiom of power set

$$\forall x \exists y (z \subset x \implies z \in y)$$

9. Axiom of choice

$$\forall X \exists R \left(R well - orders X \right)$$

Applications of Set theory