TECHNICAL UNIVERSITY OF CLUJ-NAPOCA
FACULTY OF AUTOMOTIVE, MECHATRONICS AND MECHANICAL ENGINEERING
DEPARTMENT OF MECHATRONICS AND MACHINE DYNAMICS

SEMESTER PROJECT

SINGLE-STAGE INDUSTRIAL GEARBOX

at

ELEMENTS OF MECHANICAL ENGINEERING

Sem. **II**, Year **2020-2021**

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Chapter 1. Introduction

1.1 Project theme

Design a single stage speed reducer gearbox based on a spur gear (an elementary gearing) that can be used for an industrial mixer. The conceptual design of the mechanical power transmission is presented in Fig. 1.1. It consists of a three-phase induction motor (1), a single stage spur gear reducer (2), two coupling element (3) and the industrial process (4). The material used for the gears is OLC 45 quality carbon steel and for the shafts OL 50 carbon steel. Each student will use their given input data to design the mechanical power transmission.

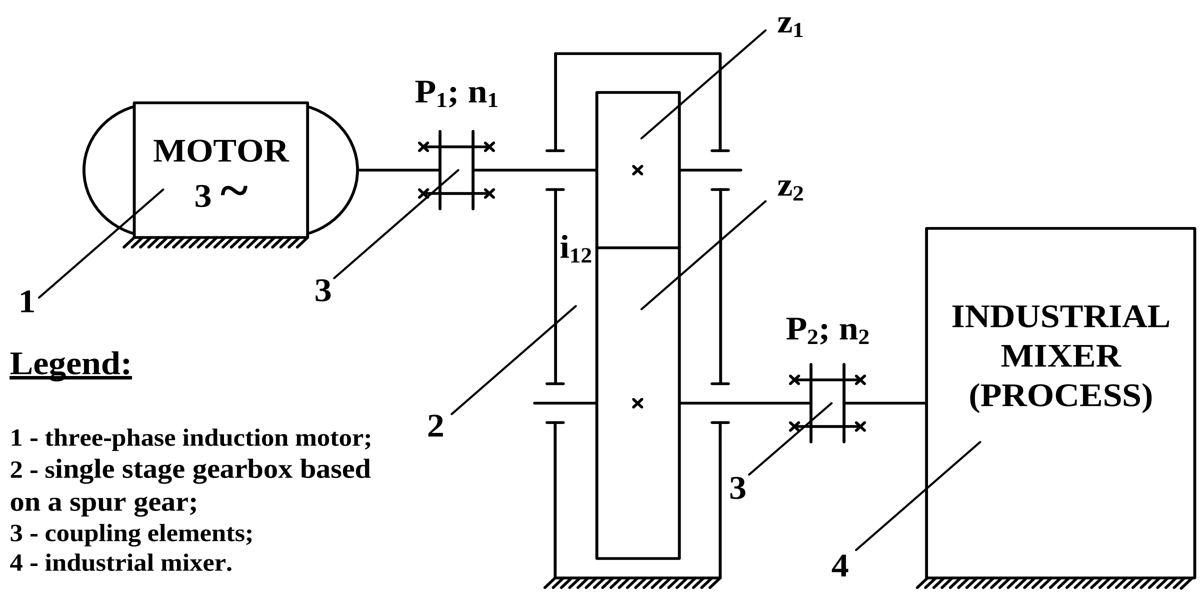


Fig. 1.1. *Kinematic diagram of the mechanical power transmission.*

<u>Input data:</u>

n = 1 student number
P_2 = 10.4 [kW] – output power
n_s = 1500 [RPM] – synchronous speed of the motor
i_{12} = 2.6 (transmission ratio)
z_1 = 25 (number of teeth of the input gear)
L_h = 20000 [hours] - number of running hours

1.2 Gear speed reducers

Gear speed reducers are mechanical devices used to reduce the speed of an actuator (usually engines) safely and efficiently. They are present in various mechanisms like: cars, bikes (every vehicle which has a gearbox), industrial and household machineries and also they are an object of interest regarding the research in the robotics and mechatronics which almost certainly will, if they have not already, become most important engineering fields of research.

Depending on how they are implemented there are several categories of speed reducers: Parallel axis gear speed reducer, Bevel gear speed reducer, Hypoid speed reducer, Worm speed Reducer and Helical Speed Reducer depending on their structure, on the elements that composed the speed reducer such as: helical gears, spur gears, bevel gears, worm gear and wheels (for more detail about them check [9], some of them are also detailed in 1.3).

Gear speed reducers are part of transmission systems which are vital in any vehicles powered by engines and they have the role to transmit and adapt the power of the engine to the drive wheels. They are also widely used in municipal waterworks, locks, and flood control systems. They are extremely important and we interact with them very often without even noticing.

The transmissions and speed reducers are devices that obtained the required rotational speed. However, they have one or more important role. That is that they obtain a rotational force (torque) proportional to the deceleration ratio. For example, if we have the rotational speed, that rotational force (torque) will double. This is because the mechanism of changing and decelerating speed uses the principle of leverage (Ex. [9]).

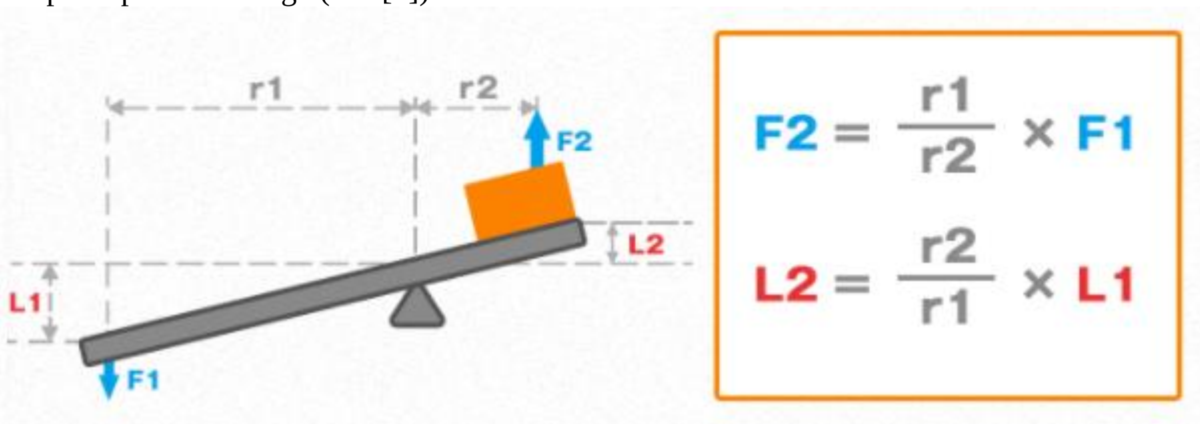


Fig. 1.2. *Leverage Principle [9]*

The above figure(see Fig.1.2) shows the principle of leverage. The formula in the above figure demonstrates that a large force called F2 can be obtained with the force of F1 (Ex. [9]), noticing the same procedure for transmission(see Fig.1.3).

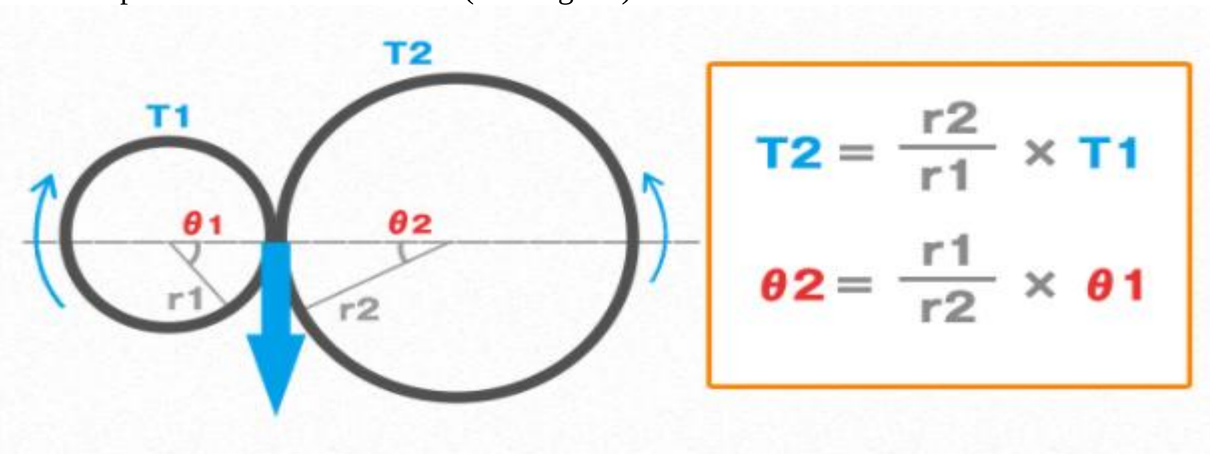


Fig. 1.3. *Rotating Body [9]*

Historically, the first ideas of speed reducers and the concept of gearboxes belonged to Muslim polymath named Ismail al-Jazari. In his book called "The Book of Knowledge of Ingenious Mechanical Devices" he talked about arrangements of wheels and mechanical devices, automatic controls, valves and another mechanisms and concepts we study today in engineering. After a long break of 6 centuries, the concept of speed reducer was mentioned again by a British manufacturing and engineering company called Watt & Boulton Engine which designed a gearbox featuring two gears and a rotational speed governor in 1817. Later in 1904, the Sturtevant brothers invented what they called a "horseless carriage gearbox", which was actually an early automatic power transmission speed reducer.

During the first part of 20 century, the manufacturers were focusing on developing speed reducers for automotive industry, but with the evolution of technology, they become an integrated part of many applications, they are produced in a wide range of sizes and they are constantly improved in terms of: precision, energy efficiency, versatility and many others. For extra details check [10], [11].

The production process is pretty complex and it can be divided in three main parts: making the gears and shafts using cutting and forming processes, creating the gear housings which have to protect the gears against corrosion and another types of contamination and finally the assembling process which brings everything into one functional mechanism which will look like in figure 1.4 (see Fig.1.4). The most common materials used in manufacturing speed reducers are steel (higher or lower quality), plastic and sometimes even wood.

Fig. 1.4 *Gear speed reducer [12]*

An important aspect to point out as a conclusion is that, if you are an engineer or maybe you are just a student which has a project to do for university and you want to buy a speed reducer, you have to take a very close look at specifications before sending the order. The most important specifications are the one related to environment, operating conditions and motor type. For more details check [13], where you will find out also what risks are if the specifications and rules made by the manufacturer are not respected.

1.3 Power transmission design analysis and justification

Some of the most known and spread mechanical devices worldwide, met in most of the mechanisms that we see every day (cars, industrial machines, household items, robots, and even bikes) are speed reducers and speed amplifiers. When building a speed reducer, the first thing needed is an interdisciplinary team with experience in this domain, a team which is able to analyze and design according to the project requirements a speed reducer which is optimal when considering the following criteria: safety of the people which interact with the mechanism, cost (not too high to be unaffordable but not so low that it can damage the quality of the product by using poor materials), size, reliability and low noise.

In order to be able to quantify a technical solution according to these criteria we need to take a closer look to the possible designs which can be used to implement such a device. Four possible solutions could be: belt drive design, chain drive design, single-stage gear-reducer design and belt drive with single-stage gear-reducer design (for a closer look to technical drawings and performances of each design check **[**8]**)** .

The design chosen for our device is single-stage gear-reducer because it has some important features like the high level of safety for operators, the rotating components being enclosed. Another important advantage is the high reliability due to the fact that precision metallic parts are used in a sealed housing(to prevent the contaminating of the metal), belts and chains are also good but they are usually more expensive and their design may also include more moving parts which is seen as a possible risk of failure. The maintenance cost and size are expected to be reduced because of design's relative simple implementation.

Another possible alternative could be the belt drive with single-stage gear-reducer design because using it we could provide a variable speed operation which could be useful for our application (industrial mixer). Varying the speed of the mixer is possible by using different belt drive ratio. Nevertheless, we can vary speed using our single-stage gear-reducer too, by using a variable speed electric drive motor.

Having the design, we need to focus on what type of gear-type reducer to use. Here are some possible types we can use (for more details check [8]):

<u>Single-reduction spur gears</u>, with high efficiency (> 95%), only radial loads are met so the selection of bearings is significantly simplified and spur gears are relatively cheap to produce. Also shafts position would be parallel, easy to align with the motor and the drive shaft for the mixer.

<u>Single-reduction helical gears</u>, similar with single-reduction spur gears which come to some advantages like smaller size of the mechanism and also some disadvantages like the apparition of axial thrust loads which will cause changes in bearings selection and in housing mechanism.

<u>Bevel gears</u>, more difficult to design and assemble.

<u>Worm and wormgear</u> drive present many disadvantages like: lower efficiency, heat generation and the need of a stronger engine.

Considering all these facts, the final decision is **single-reduction spur gear reducer design** because it works well for our industrial mixer application and the costs are likely to be the lowest.

Chapter 2. Selecting the Actuator

2.1 Selecting the AC motor

2.1.1 Synchronous speed calculation

$$n_s = \frac{60 \times f}{p}[RPM] \quad n_s = 1500[RPM] \tag{1}$$

where:
f = motor supply's frequency in hertz
p = number of **pairs** of magnetic poles

2.1.2 Calculate the slip

$$s = \frac{n_s - n_n}{n_s} = \frac{1500 - 1440}{1500} = 0{,}04 = 4\% = slip \tag{2}$$

$where\;\; s - slip, n_s - synchronous\ speed\ [RPM],\ n_n - rotor\ nominal\ speed\ [RPM]$
$n_n < n_s$

2.1.3 Calculate the required motor power [kW] = Pm (needed to transmit P2)

Actuator power

$$P_m = \frac{P_2}{\eta}[kW] = \frac{10.4}{0.93} = 11.18[kW], \quad P_2 = 10.4[kW] \tag{3}$$

$where\;\; P_m - required\ motor\ power\ [kW]$
$P_2 - output\ power\ [kW]$
$\eta - mechanical\ transmission\ efficiency$

Mechanical transmission efficiency

$$\eta = \eta_g \times \eta_b^2 \times \eta_l$$
$$= 0.96 \times (0.99)^2 \times 0.99 = 0.93 - Mechanical\ transmission\ efficiency \tag{4}$$

$where\;\; \eta_g - spur\ gear\ efficiency = 0.96 \div 0.98$
$\eta_b - one\ pair\ of\ bearings\ efficiency = 0.99 \div 0.995$
$\eta_l - lubrication\ efficiency = 0.99$

! The systems which use spur gears are efficient ($\eta > 0.9$)

2.1.4 The required values $->$ The AC motor values (we choose the right engine $P_m < P_n$)

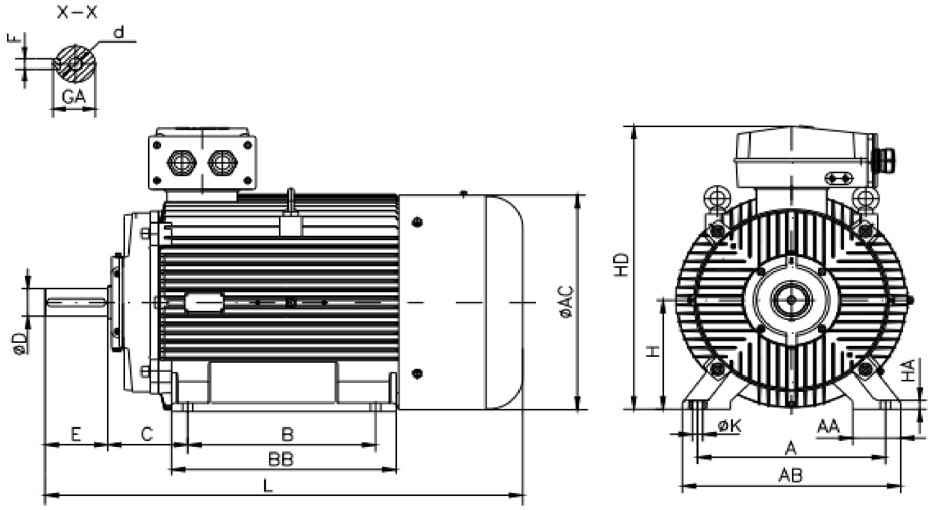
$$P_m = 11.18[kW] \;\; n_s = 1500[RPM] => \; P_n = 15[kW] \;\; n_n = 1440[RPM] \tag{5}$$

I choose the ASU 160L-4 AC motor with the following properties:
$P_n = 15\ kW - nominal\ power\ of\ the\ motor$
$n_n = 1440 - rotor\ nominal\ speed[RPM] < n_s = 1500[RPM]$
$\eta = 0{,}90 = 90\% - mechanical\ transmission\ efficiency$

2.1.5 The AC engine dimensions

(see Fig.2.1)

Motoare asincrone seria ASU - IM 1001 (IMB3)



Gabarit	A	B	C	H	K	D nom.		D tol.	E		F h9		GA		d		AA	AB	BB	HA	AC	HD	L	
						2p=2	2p>2		2p=2	2p>2	2p=2	2p>2	2p=2	2p>2	2p=2	2p>2							2p=2	2p>2
160M	254	210	108	160	14.5	42		k6	110		12		45		M16			310	298	14	315	404	630	
160L	254	254	108	160	14.5	42		k6	110		12		45		M16		84	310	298	14	315	404	630	
180M	279	241	121	180	14.5	48		k6	110		14		51,5		M16		72	350	295	18	355	438	668	
180L	279	279	121	180	14.5	48		k6	110		14		51,5		M16		72	350	335	18	355	438	708	
200L	318	305	133	200	18.5	55		m6	110		16		59		M20		70	380	367	17	395	507	780	
225S	356	286	149	225	18.5	-	60	m6	-	140	-	18	-	64	M20		80	425	358	22	445	552	-	840
225M	356	311	149	225	18.5	55	60	m6	110	140	16	18	59	64	M20		80	425	383	22	445	552	835	865
250M	406	349	168	250	24	60	65	m6	140	140	18	18	64	69	M20		95	490	440	20	494	607	925	925
280S	457	368	190	280	24	65	75	m6	140	140	18	20	69	79,5	M20		125	540	523	20	494	677.5	986	
280M	457	419	190	280	24	65	75	m6	140	140	18	20	69	79,5	M20		125	540	523	20	494	677.5	986	
315S	508	406	216	315	28	65	80	m6	140	170	18	22	69	85	M20		130	590	580	25	554	819	1109	1139
315M	508	457	216	315	28	65	80	m6	140	170	18	22	69	85	M20		130	590	580	25	554	819	1109	1139
315MX/LX	508	457	216	315	28	65	80	m6	140	170	18	22	69	85	M20		130	590	607	25	623	819	1232	1292
315LY/LZ	508	508	216	315	28	65	80	m6	140	170	18	22	69	85	M20		130	590	607	25	623	819	1232	1292
355Ma	610	560	254	355	28	70	100	m6	140	210	20	28	74.5	106	M20	M24	110	714	695	32	698	920	1370	1480
355Mb	610	560	254	355	28	70	100	m6	140	210	20	28	74.5	106	M20	M24	110	714	695	32	698	920	1370	1480
355L	610	630	254	355	28	70	100	m6	140	210	20	28	74.5	106	M20	M24	110	714	765	32	698	920	1450	1560

Fig. 2.1.*ASU 160L-4 AC motor dimensions [1]*

2.2 Power transmission kinematics

2.2.1 Determining the number of teeth of the output gear

$z_2 = i_{12} \times z_1 = 2.6 \times 25 = 65 - number\ of\ teeth\ for\ the\ second\ gear(output\ gear)$ (6)

$where\ \ i_{12} - transmission\ ratio$

$z_1 - number\ of\ teeth\ for\ the\ first\ gear(input\ gear)$

$-if\ z_2\ is\ a\ whole\ number\ u_{12} = i_{12} = 2.6 - transmission\ ratio$

$-therefore, the\ real\ transmission\ ratio\ u_{12}\ with\ the\ new\ \ z_2\ value\ can\ be\ calculated\ as(Ex.[2]):$

$$u_{12} = \frac{z_2}{z_1} = \frac{65}{25} = 2.6 \qquad (7)$$

$-in\ this\ situation\ the\ following\ condition\ must\ be\ meet(Ex.[2]):$

$$-2.5\% \leq \frac{u_{12}-i_{12}}{i_{12}} \leq +2.5\% \rightarrow \ \varepsilon_{u12} = \pm 2.5\%\ \ is\ ok, because\ u_{12} will\ be\ used\ further! \qquad (8)$$

2.2.2 The Input/Output shaft speed

$$n_1 = n_n[RPM] => n_1 = 1440[RPM] \rightarrow Input\ (engine\ speed) \quad (9)$$
$$n_2 = \frac{n_1}{i_{12}} = \frac{1440}{2.6}[RPM] = 553.84[RPM] \rightarrow Output\ \ , where\ i_{12} - transmission\ ratio \quad (10)$$

2.2.3 Power transmitted by input shaft

$$P_1 = P_m[kW] \ \rightarrow \ \ P_1 = 11.18[kW] - Power\ transmitted\ by\ shaft\ 1(engine\ power) \quad (11)$$

2.2.4 The Input/Output shaft Torque

$$T_1 = 95500 \times \frac{P_1}{n_1} = 741.45[daN * cm] \quad (12)$$

$$T_2 = 95500 \times \frac{P_2}{n_2} = \ 1793.29[\mathrm{daN} * \mathrm{cm}] \quad (13)$$

Chapter 3. Spur Gear Design

3.1 Gear tooth strength analysis and verification

3.1.1 Strategy

Size the spur gears according to contact stress and then check if the dimensions obtained respects the bending stress conditions too. We will use as material OLC45 high quality steel.

3.1.2 Pitch line velocity (derived from angular velocity and radius of the spoor gear)

Pinion peripheral speed or pitch line velocity is associated with angular velocity and radius of the spoor gear.

$$v = 0.1 \times \sqrt[4]{n_1^2 \times \frac{P_1}{u_{12}}} = 5.87[m/s] \quad (14)$$

$$where\ \ P_1 = 11.18 \times 1.34 = 14.98\ HP(HorsePower) \quad (15)$$

$$P_1 - transmitted\ power\ in\ horse\ power\ [HP]$$

$$(1kW = 1.34HP)$$

$$n_1 - Input\ (engine\ speed)$$

$$u_{12} - transmission\ ratio$$

3.1.3 Equivalent load

$$k = k_c \times k_d = 1.2 \rightarrow Load\ factor,$$

$$where{:}\ \ k_c - load\ distribution\ factor, k_c = 1 \div 1.22$$

$$k_d - load\ dynamic\ factor, k_d = 1.2 \quad (16)$$

The dynamic load factor was chosen considering: precision class, angular velocity, tooth shape and Hardness Brinell using the catalog from [3] table 9.2.

We use OLC45 with NB=197 ≤350(HB-Hardness Brinell)

3.1.4 Equivalent torque

$$T_{e_1} = k \times T_1 = 889.74[daN * cm] \quad (17)$$

3.1.5 Contact stress (dimensioning central/axial distance A)

$$\psi_A = \frac{A}{B} = 0.4\ (medium\ speed\ reducer),$$

$$\psi_A - axial\ coefficient\ of\ the\ gear, \psi_A = 0.3 \div 0.6 \quad (18)$$

$$\sigma_{a_k} = 26 \times HB = 26 \times 197 = 5122[daN/cm^2]\ \text{(for OLC 45)} \quad (19)$$

$$where\ \alpha_{a_k} - allowable\ contact\ stress$$

$$HB - Hardness\ Brinell\ according\ to\ table\ from\ [3]table\ 9.4.$$

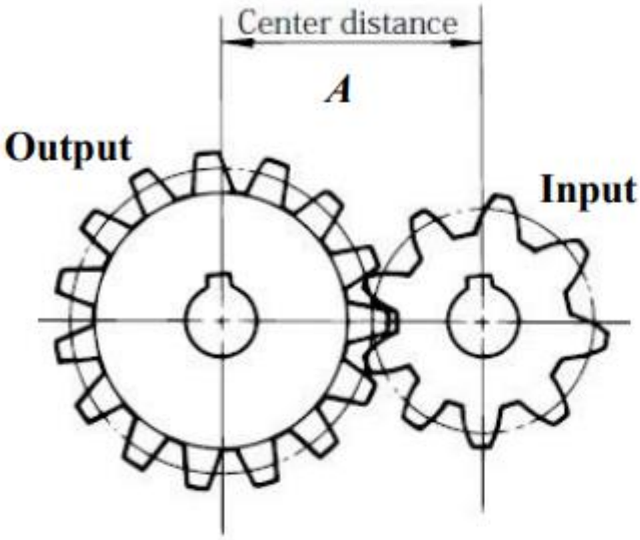
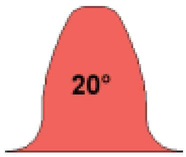


Fig. 3.1. *Axial distance A [3]*

Axial distance A(see Fig. 3.1) influences directly the contact stress, using $\alpha(pressure\ angle) = 20^{\circ}$(see Fig.3.2)

$$A_{min} = (u_{12} + 1) \times \sqrt[3]{\frac{T_{e_1}}{\psi_A \times u_{12} \times sin2\alpha} \times \left(\frac{865}{\sigma_{ak}}\right)^2}\ [cm] = 10.76[cm] = 107.6[mm] \qquad (20)$$



$\alpha = 20^{\circ}$

Fig. 3.2. *Tooth Shape [3]*

$A_w(standardized) = 108[mm]$ $choosing\ a\ rounded\ \ value\ > A_{min}$

3.1.6 Circular pitch module(m)

- the distance between 2 successive teeth (must be equal for both spur gears)

$z_1 = 25 \rightarrow number\ of\ teeth\ for\ the\ first\ gear$
$z_2 = z_1 \times i_{12} = 25 \times 2.6 = 65\ teeth$
$\rightarrow number\ of\ teeth\ for\ the\ second\ gear$ (using relation (6))

$$m_{min} = \frac{2A_W}{z_1+z_2}\frac{cos\alpha}{cos\alpha_0}[mm], where\ \alpha_0 = 20^{\circ} \rightarrow m_{min} = 2.4[mm] \qquad (21)$$

m $\geq m_{min}$
$m(standardized) = 2.5\ mm \rightarrow A\ changes!$

$$A = \frac{m \times (z_1+z_2)}{2} = \frac{2.5 \times (25+65)}{2} = 112.5[mm] \rightarrow Final\ axial\ distance \qquad (22)$$

3.1.7 Modular coefficient of the gear

$$\psi_m = \frac{A \times \psi_A}{m} = 18 \in (8, 40), where\ \psi_A = 0.4\ - axial\ coefficient\ of\ the\ gear \qquad (23)$$

3.1.8 Bending stress (verification)

$$\begin{cases} \sigma = \frac{2T_{e_1}}{\pi \times m^3 \times z_1 \times \psi_m \times C_f \times cos\alpha_0} \leq \sigma_{ai} \left[\frac{daN}{cm^2}\right] \to \sigma = \frac{2\times 889.74}{3.14\times(0.25)^3\times 25\times 18\times 0.1355\times 0.94} = 661.51 \left[\frac{daN}{cm^2}\right] \\ m_b = \sqrt[3]{\frac{0.68\times T_{e_1}}{z_1\times\psi_m\times C_f\times\sigma_{ai}}} \leq m\ [mm] \to m_b = \sqrt[3]{\frac{0.68\times 889.74}{25\times 18\times 0.1355\times 1327.16}} = 0.19[cm] \leq m = 0.25[cm] \end{cases} \quad (24)$$

3.1.9 Allowable bending stress

$$\sigma_{ai} = \frac{\sigma_0}{k_\sigma \times C} = \frac{4300}{1.8\times 1.8} = 1327.16 \left[\frac{daN}{cm^2}\right] \quad (25)$$

$where\ k_\sigma - stress\ concentration\ coefficient, k_\sigma = 1.2 \div 2$

$C - safety\ coefficient,\ C = 1.5 \div 2$

For additional information check [3] table 4.1.

$\sigma \leq \sigma_{ai} \to Verification\ successful!$

$m_b \leq m \to Verification\ successful!$

3.2 Final geometrical elements of the gears

3.2.1 Geometrical elements of the gears

The diameters of pitch circles

$D_{d_1} = m \times z_1 = 2.5 \times 25 = 62.5\ [mm]$ (26)

$D_{d_2} = m \times z_2 = 2.5 \times 65 = 162.5\ [mm]$ (27)

The diameters of outside circles

$D_{e_1} = 2R_{e_1} = m(z_1 + 2 \times f_0) = 2.5(25 + 2) = 2.5 \times 27 = 67.5[mm]$ (28)

$D_{e_2} = 2R_{e_2} = m(z_2 + 2 \times f_0) = 2.5(65 + 2) = 2.5 \times 67 = 167.5[mm]$ (29)

Whole depth

$h = m(2f_0 + \omega_0) = 2{,}5(2 + 0.25) = 2.5 \times 2.25 = 5.62[mm]$ (30)

The diameters of root circles

$D_{i_1} = 2R_{i_1} = m(z_1 - 2f_0 - 2\omega_0) = 2.5(25 - 2 - 2 \times 0.25) = 56.25[mm]$ (31)

$D_{i_2} = 2R_{i_2} = m(z_2 - 2f_0 - 2\omega_0) = 2.5(65 - 2 - 2 \times 0.25) = 156.25[mm]$ (32)

$where\ \ f_0 = 1 - reference\ head\ height,\ \omega_0 = 0.25 - reference\ foot\ height$

Gear face width

$B = A \times \psi_A = 45[mm]$ (33)

Output gear (round up the obtained value)

$B_2 = A \times \psi_A = 45[mm]$ (34)

Input gear (round up the obtained value)

$B_1 = B_2 + m = 47.5[mm]$ (35)

3.2.2 Contact ratio

The radii of the base circles of the two gears

$R_{b_1} = R_{d_1} cos\alpha_0 = \frac{m \times z_1}{2} cos\alpha_0 = 29.37[mm]$ (36)

$R_{b_2} = R_{d_2} cos\alpha_0 = \frac{m \times z_2}{2} cos\alpha_0 = 76.37[mm]$ (37)

$$\varepsilon = \frac{\sqrt{R_{e_1}^2 - R_{b_1}^2} + \sqrt{R_{e_2}^2 - R_{b_2}^2} - A \times sin\alpha}{\pi \times m \times cos\alpha_0} = 1.72 > 1.2 \rightarrow works\ OK! \quad (38)$$

3.2.3 Span measurement

$For\ z_1 = 25 \rightarrow\ n_1 = 3(number\ of\ teeth\ between\ 18 - 26)$

$N_1 = z_1 \times \frac{\alpha_0}{180^{\circ}} + 0.5 = 3.27$ (39)

$L_{n_1} = m[(N_1 - 0.5)\pi + z_1 inv\ \alpha_0]cos\ \alpha_0 = 21.24\ [mm]$ (40)

$For\ z_2 = 65 \rightarrow\ n_2 = 8(number\ of\ teeth\ between\ 72 - 80)$

$N_2 = z_2 \times \frac{\alpha_0}{180^{\circ}} + 0.5 = 7.72$ (41)

$L_{n_2} = m[(N_2 - 0.5)\pi + z_2 inv\ \alpha_0]cos\ \alpha_0 = 55.41\ [mm]$ (42)

$where\ inv\ \alpha_0 = inv\ 20^{\circ} = 0.014904$

Chapter 4. Output Shaft Design

Shafts are subjected to torsion and bending stress! (see Fig. 4.1)

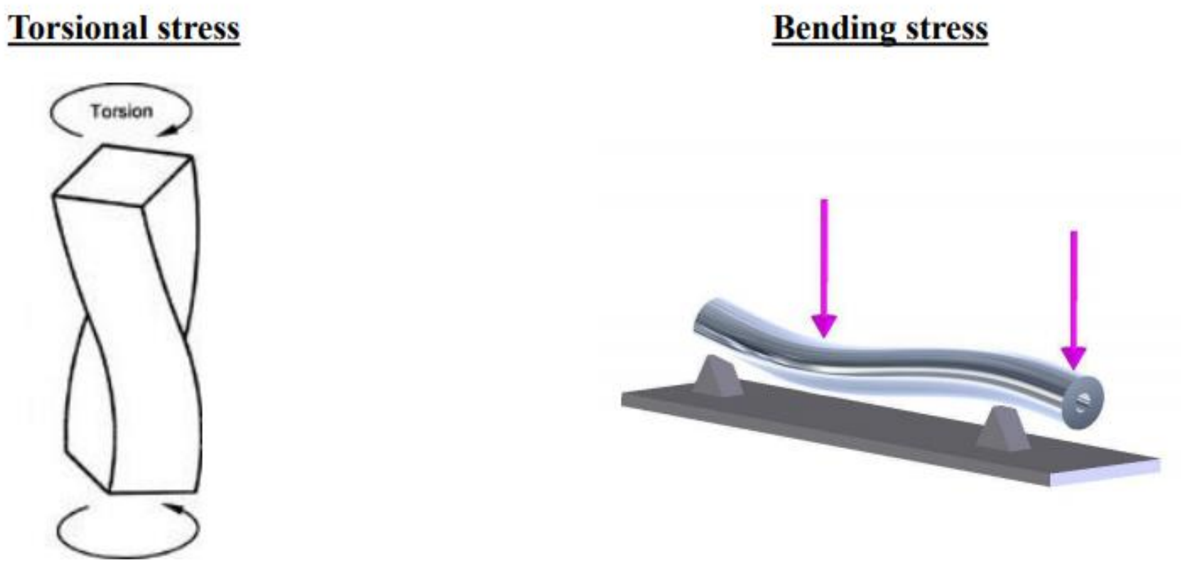


Fig. 4.1. *Torsion and Bending stress [4]*

4.1 Pre-dimensioning

4.1.1 Preliminary diameter

Preliminary diameter dp is determined at torsional stress because at this point the length of the shaft is not known.(Ex.[4])

$$d_{p\,min} = \sqrt[3]{\frac{16\times T_2}{\pi\times\tau_{at}}}\,[cm] = 3.93[cm] = 39.3[mm] \tag{43}$$

$$where\ \ \tau_{at} = 120 \div 250[daN/cm^2] - Torsional\ shear\ stress$$

$$d_p \geq d_{p_{min}} \rightarrow d_p = 40[mm] - preliminary\ diameter \tag{44}$$

4.1.2 Preliminary length of the output shaft

$$b = B_2[mm] = 45[mm](face\ width) - the\ width\ of\ the\ second\ (output) gear \tag{45}$$

$$l = b + 2l_1 + l_2 = 45 + 20 + 20 = 85[mm] \tag{46}$$

$$where\ l_1 = 10[mm]\quad l_2 = 0.5 \times d_p = 20[mm]$$

$$particular\ lengths\ which\ must\ be\ considered. For\ more\ details\ check\ [4]$$

4.1.3 Forces acting on a spur gear mesh

$\eta \approx 1$ (Mechanical transmission efficiency)
$F_{t1} \approx F_{t2}$ (Tangential forces)
$F_{r1} \approx F_{r2}$ (Radial forces)

The spur gears mechanism is efficient in transmitting tangential and radial forces.
The forces occur between 2 teeth on a specific line as can be seen in the picture. (see Fig.4.2)

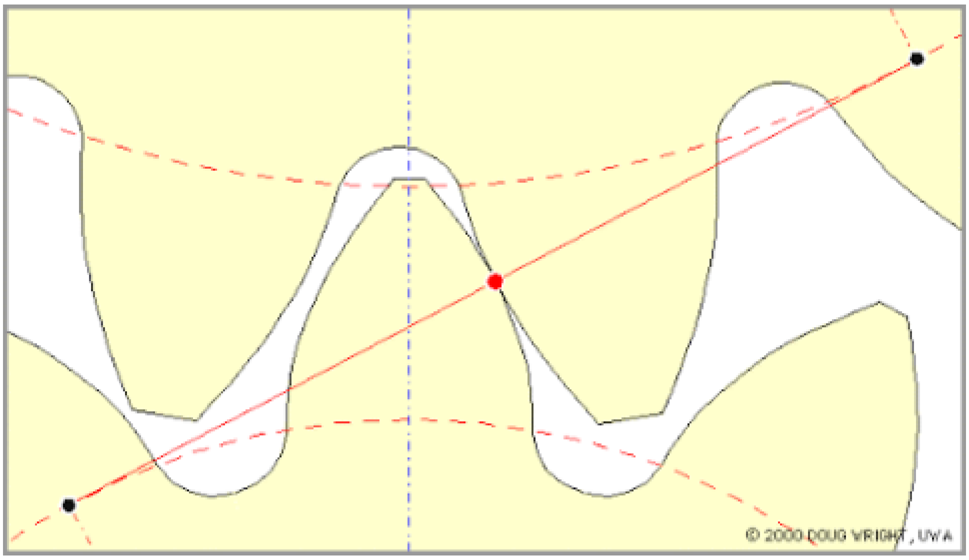


Fig. 4.2. *Where forces act [4]*

4.1.4 Tangential force (acts on vertical plane, on point number 6)

$D_{d_2} = m \times z_2 = 2.5 \times 65 = 162.5\ [mm] -$ pitch diameter (using relation (25))

$$F_{t_2} = \frac{2T_2}{D_{d_2}}[daN] = 220.71[daN] \qquad (47)$$

4.1.5 Radial force (acts on horizontal plane)

$$F_{r_2} = F_{t_2} tan\alpha_0 [daN] = 220.71 \times 0.363 = 80.11[daN] \qquad (48)$$

$where \quad \alpha_0 = 20^{\circ}$

4.2 Shaft loading diagram

(see Fig.4.3)

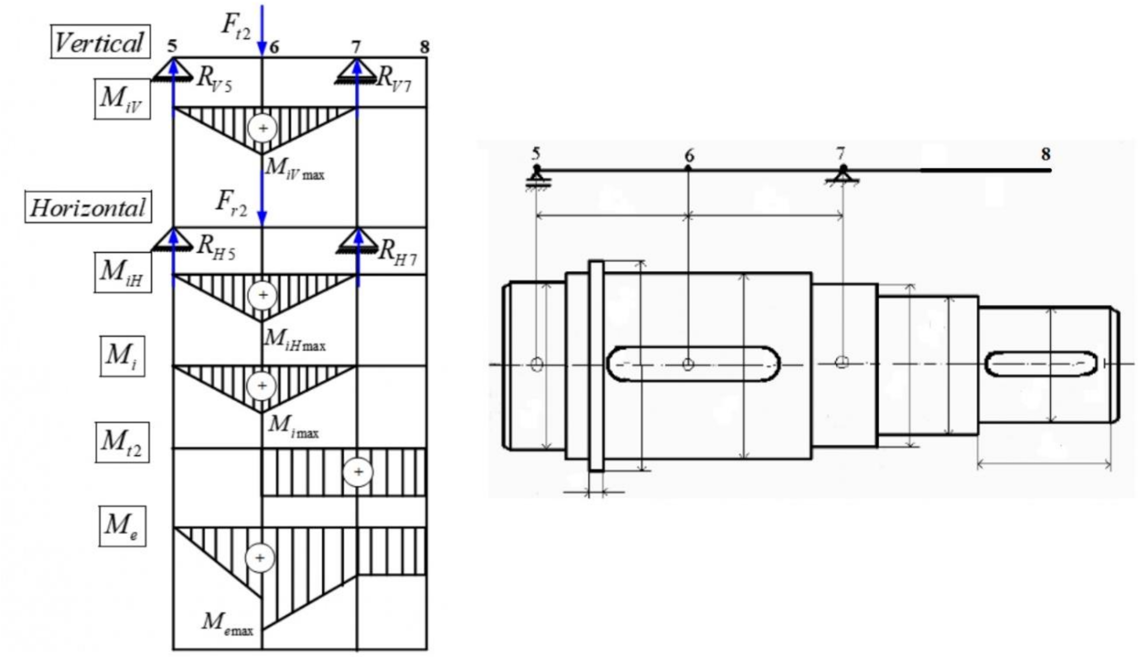


Fig. 4.3. *Shaft loading diagram [4]*

4.2.1 Reaction forces and bending moments in both planes

<u>Vertical plane</u>

$R_{V5} = R_{V7} = \frac{F_{t2}}{2}\ [daN] = \frac{220.71}{2} = 110.35\ [daN]$ –reaction forces on support points (5,7) (49)

$M_{iVmax} = \frac{1}{4} * F_{t2} * l\ [daN * cm] = 234.49[daN * cm]$ – maximum momentum produced by the tangential force on vertical plane. (50)

<u>Horizontal plane</u>

$R_{H5} = R_{H7} = \frac{F_{r2}}{2}\ [daN] = \frac{80.11}{2} = 40.05[daN]$ –reaction forces on support points (5,7) (51)

$M_{iHmax} = \frac{1}{4} * F_{r2} * l\ [daN * cm] = 170.23[daN * cm]$ –maximum momentum produced by the radial force on horizontal plane. (52)

Resulting reaction forces (algebraic sum of the forces acting on horizontal/vertical plane)

$R_5 = R_7 = \sqrt{R_{V5}^2 + R_{H5}^2}\ [daN] = \sqrt{13781.12} = 117.39[daN]$

<u>Resulting bending momentum</u> (algebraic sum of the momentums acting on horizontal/vertical plane)

$$M_{imax} = \sqrt{M_{iVmax}^2 + M_{iHmax}^2}\ [daN * cm] = 289.76\ [daN * cm] \quad (53)$$

4.2.2 Equivalent bending moment

$$M_{emax} = \sqrt{M_{imax}^2 + (\alpha T_2)^2}\ [daN * cm] = 1114.3[daN * cm] \quad (54)$$

$where\ \ T_2 = 1793.29[daN * cm] - output\ shaft\ torque$ (using relation (13))

$\alpha = \frac{\sigma_{aiIII}}{\sigma_{aiII}} = 0.6\ (OL50\ build\ material)$

→ Build material coefficient

$\sigma_{aiIII} = 450\ [daN/cm^2]\ \ \sigma_{aiII} = 750\ [daN/cm^2]$

4.2.3 Required diameter in critical section

$$d_{min} = \sqrt[3]{\frac{32 * M_{emax}}{\pi * \sigma_{aiIII}}}\ [cm] = 2.93[cm] \quad (55)$$

! Because a key will be used to assembly the gear with the shaft, the value of d_{min} must be increased by 4% (Ex.[4])

$$d_{\min} = d_{\min} + 4\% * d_{\min} = 30.47\ mm, round\ up\ the\ result\ to\ final\ d = 32\ \mathrm{mm} \quad (56)$$

$d \geq d_{min}$

4.3 Final geometry of the shaft

(see Fig.4.4)

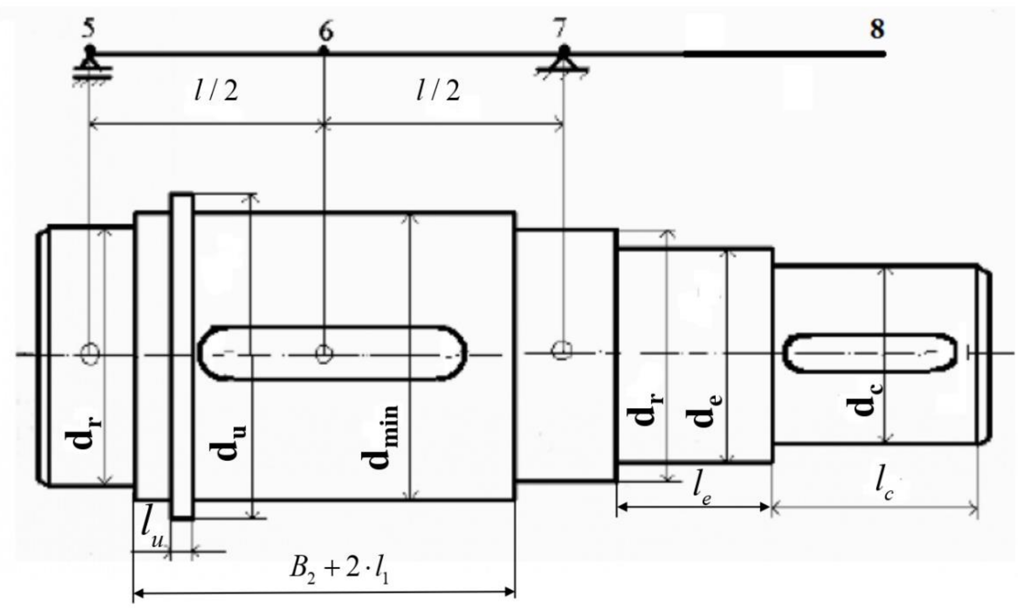


Fig. 4.4. *Final geometry of the shaft [4]*

Diameters in other points on the shaft

$\boldsymbol{d_u} > \boldsymbol{d} > \boldsymbol{d_r} \geq \boldsymbol{d_e} > \boldsymbol{d_c}$
$\boldsymbol{d_u} - is\ the\ shoulder\ diameter\ (to\ be\ chosen)$
$\boldsymbol{d} - is\ the\ diameter\ where\ the\ output\ gear\ will\ be\ mounted(calculated\ before)$
$\boldsymbol{d_r} - is\ the\ diameter\ of\ the\ shaft\ where\ the\ bearings\ will\ be\ mounted\ (to\ be\ chosen)$
$\boldsymbol{d_e} - is\ the\ diameter\ of\ the\ shaft\ where\ the\ seal\ will\ be\ mounted(to\ be\ chosen)$
$\boldsymbol{d_c}$ – is the diameter of the end of the shaft; end of the shaft will be connected to the process, in this case the industrial mixer through a coupling element (to be chosen). (Ex.[4])

4.3.1 Diameter of the end shaft (point 8)

$$d_{c\ min} = \sqrt[3]{\frac{16\times T_2}{\pi\times\tau_{at}}}\ [cm] = 2.33[cm] = 23.3[mm] \quad (57)$$

Torsional shear stress for OL 50

$$\tau_{at} = (0.6 \div 0.65)\sigma_{all}\left[\frac{daN}{cm^2}\right] = 0.6 \times 1200 = 720\left[\frac{daN}{cm^2}\right] \quad (58)$$

$\sigma_{all} = 1200\left[\frac{daN}{cm^2}\right]$

$where\ \sigma_{all}\ is\ the\ allowable\ stress\ at\ traction\ for\ material\ OL\ 50, pulsating\ cycle, case\ II.$ (Ex.[4])

Choose a final value of diameter d_c and then the length l_c from the next slide(Ex.[4])

$$d_c \geq d_{c\ min} \rightarrow d_c = 24[mm] \quad (59)$$

4.3.2 Length of the end of the shaft

$$if\ d_c = 24[mm] \rightarrow\ l_c = 50[mm] \quad (60)$$

4.3.3 Seal diameter

$$d_e = d_c + (3..5)mm = 24 + 3 = 27[mm] \quad (61)$$

$$l_e = 0.5 \times d_p = 0.5 \times 40 = 20[mm] \quad (62)$$

$where\ d_p = 40[mm] \qquad (using\ relation\ (44))$

4.3.4 Bearings diameter

$$d_r = d_e + (3..5)mm = 27 + 3 = 30[mm] \quad (63)$$

4.3.5 Shoulder diameter and length

$$d_u = d + (3..5)mm = 32 + 3 = 35[mm] \quad (64)$$

$$l_u = 4[mm] \quad (65)$$

4.4 Choosing longitudinal parallel key

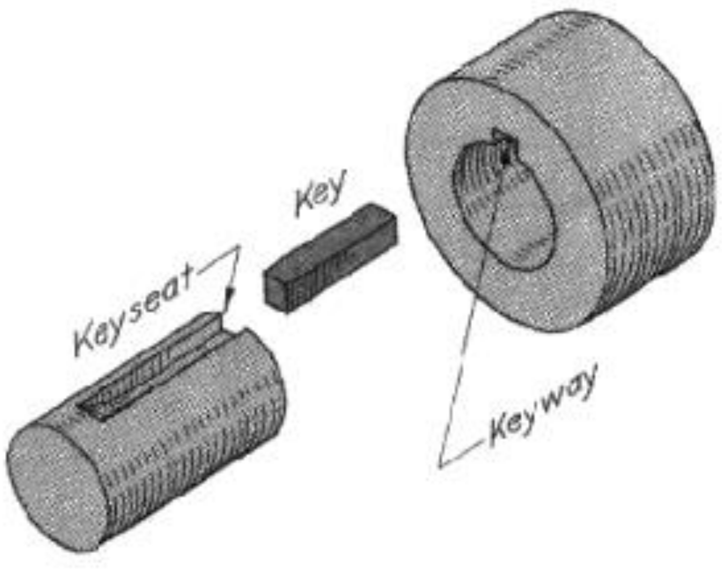


Fig. 4.5.*Key mechanism [4]*

Keys are used to transmit torque from a rotating machine element to the shaf. (see Fig.4.5) A key can fail by either being sheared off or can be crushed due to the compressive bearing forces (Ex.[4]). We can notice these in Fig.4.6.

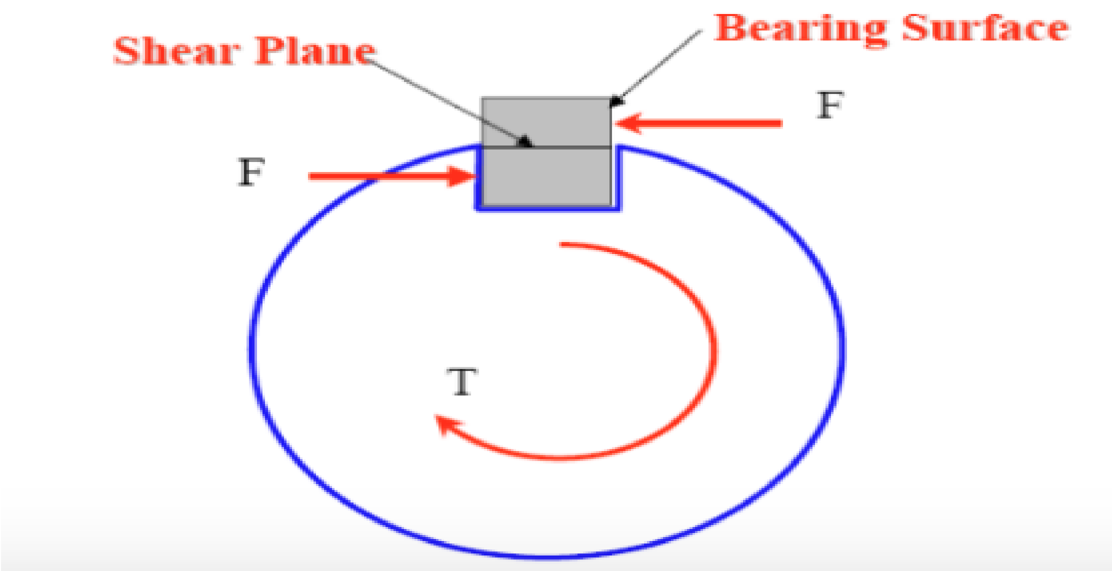


Fig. 4.6.*. Forces acting on keys [4]*

4.4.1 Calculate the required key length at contact stress

$b = 10\ [mm] \rightarrow$ key section (66)

$h = 8\ [mm] \rightarrow$ key section (67)

$t_1 = 5\ [mm] \rightarrow$ keyway depth (68)

$t_2 = 3.3\ [mm] \rightarrow$ keyway depth (69)

We can see $b, h,\ t_1, t_2$ in Fig.4.7.

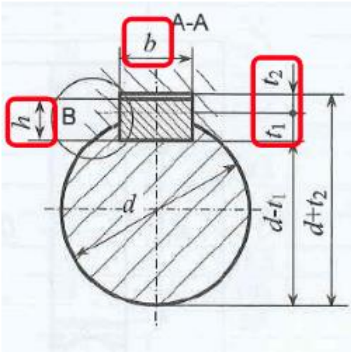


Fig.4.7. *Key dimensions [4]*

$l_{min} = \frac{4T_2}{h \times d \times p_a}\ [cm] = 3.5\ [cm] = 35\ mm -$ key minimum length (70)

where $p_a = 800\ \left[\frac{daN}{cm^2}\right]$ −contact pressure

! The final length of the key must be correlated with the width B2 of the output gear. (Ex.[4])
$l_{\boldsymbol{p}} = 0.8 \times B_2 = 0.8 \times 45 = 36[mm] \geq l_{min} = 35[mm]\ and\ is\ between\ (22{,}110)$
(accepted lengths for $30 < d \leq 38$). I will use the standardized value $l_{pa} = 40[mm]$ (71)
$where\ l_{pa} \geq\ l_{\boldsymbol{p}}$

4.4.2 Verification for shear stress (already dimensioned for contact pressure)

$\tau_f = \frac{2T_2}{b \times d \times l_{pa}} < \tau_{af}\ \left[\frac{daN}{cm^2}\right] \rightarrow \tau_f = \frac{2 \times 1793.29}{1 \times 3.2 \times 4} = 280.2[daN/cm^2] < \tau_{af}$ (72)
$\tau_{af} = 960[daN/cm^2] - OL50$

$where\ b = 1[cm](13), d = 3.2[cm](66), l_{pa} = 4[cm](56), T_2 = 1793.29[daN * cm](64)$

4.5 Verification of shaft deflection and critical speed

4.5.1 Deflection in vertical plane

$f_V = \frac{F_{t2} * l^3}{48 * E * I}\ [cm] = 261.60 \times 10^{-6}\ [cm]$ (73)

where:
$E = 2.1 * 10^6\ [daN/cm^2]$
$I = \pi * \frac{d^4}{64}\ [cm] = 5.14\ [cm]$ (74)
l=8.5 $[cm] -$ the shaft length (using relation (42))

4.5.2 Deflection in horizontal plane

$f_H = \frac{F_{r2} * l^3}{48 * E * I}\ [cm] = 94.95 \times 10^{-6}\ [cm]$ (75)

where:
$E = 2.1 * 10^6\ [daN/cm^2]$
$I = \pi * \frac{d^4}{64}\ [cm] = 5.14\ [cm]$ (using relation (67))
l=8.5 $[cm]$ – the shaft length (using relation (42))

4.5.3 Final deflection and admitted deflection

$f = \sqrt{f_V^2 + f_H^2} < f_{adm}\ [cm] \rightarrow f = 278.29 \times 10^{-6}[cm] = 0.00027829[cm] < f_{adm}\ [cm]$ (76)
$f_{adm} = 5 \times 10^{-3} \times m = 3 \times 10^{-4} \times l\ [cm] = 0.00255[cm]$ (77)

4.5.4 Weight of the output gear + static deflection (check vibrations)

$$\begin{cases} G = \gamma \times V = \gamma \times \frac{\pi \times D_d^2}{4} \times B_2\ [daN] = 7.2[daN] - \text{weight of the output gear} \\ \gamma = 7.8 \times 10^{-3} \left[\frac{daN}{cm^3}\right] \end{cases} \tag{78}$$

4.5.5 Static deflection

$$\begin{cases} f_{st} = \frac{G * l^3}{48 * E * I}\ [cm] = \frac{7.2 \times (8.5)^3}{48 \times 2.1 \times 10^6 \times 5.14} = 8.53 \times 10^{-3}[cm] \\ E = 2.1 \times 10^6 \left[\frac{daN}{cm^2}\right] \\ I = \pi \times \frac{d^4}{64}[cm] = 5.14[cm] \end{cases} \tag{79}$$

4.5.6 Critical speed

$$n_{cr} = \frac{30}{\pi} * \sqrt{\frac{g}{f_{st}}}\ [RPM] = 102\ 376\ [RPM] \tag{80}$$

$g = 981 \left[\frac{cm}{s^2}\right]$ – gravitational acceleration

$n_2 = 553.84[RPM] < n_{cr}$ (from relation (7))

Chapter 5. Rolling Bearing Selection

5.1 Calculate dynamic basic bearing load rating C

5.1.1 Introduction – General facts

The ball bearings place in our design (see Fig.5.1):

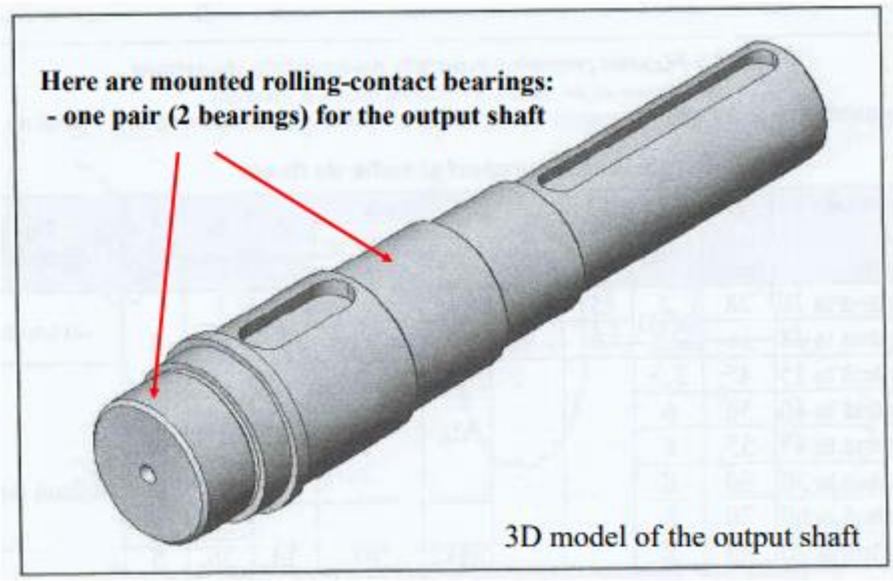


Fig. 5.1. *Bearings location [5]*

Important features:

Bearings are not designed, they are chosen accordingly to the project that you are working on from catalogues.

Another important feature of bearings is that they are consumables and are replaced periodically.

They need to be changed in pairs, both at the same time, in order to have the same characteristics and the same performances at a certain moment of time.

Considering our project, we will choose bearings which are built to support purely radial load (type A Fig. 5.2) because all our forces are acting perpendicular on the shaft.

Type of rolling-contact bearings:

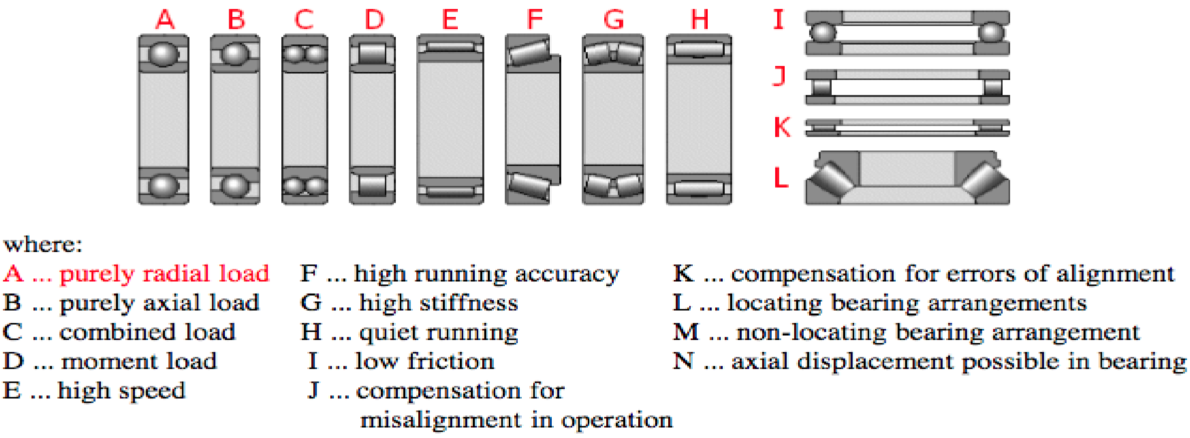


Fig. 5.2. *Bearings types [6]*

Cause of early bearings failure (see Fig. 5.3):

Fig. 5.3. *Bearings location [6]*

5.1.2 Fundamental calculation

We will use: Life's equation (Lundberg & Palmgren) where:

C - dynamic basic bearing load rating [daN], calculated so that the bearings can support the load C for 1 million rotations.

L - the basic life measurable in millions of rotations

$$L = \frac{60 \times n_2 \times L_h}{10^6}\ [milions\ of\ rotations] = 664.6\ [milions\ of\ rotations] \tag{81}$$

$$L = \left(\frac{C}{F}\right)^p \tag{82}$$

where:	$L_h = 20000$ [hours] – number of running hours (check page 4) $n_2 = 553.84[RPM]$ –rotational speed [RPM] (check relation (10)) F= bearing load [daN]

5.1.3 Equivalent bearing load F_e

Considering the reaction forces which occur on the support points 5 and 7 (check figure 5.4), we can compute the resulting total radial force $\mathbf{F_r}$ with the following formula:

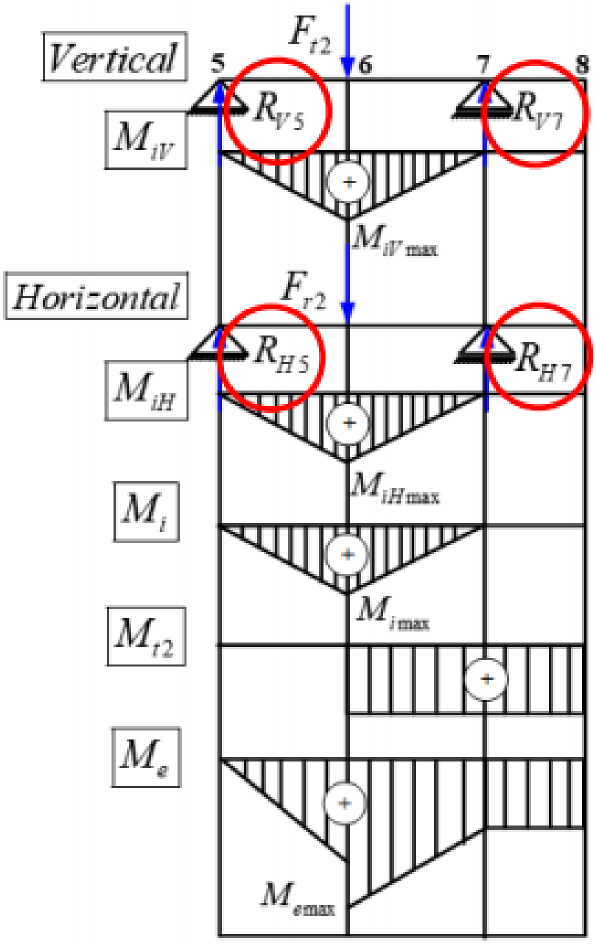


Fig.5.4. *Shaft loading diagram [5]*

$$F_e = F_r = R_5 = R_7 = \sqrt{R_{V5}^2 + R_{H5}^2}\ [daN] = 117.39\ [daN] \quad (83)$$

$R_{V5} = 110.35[daN](49)$, $R_{H5} = 40.05[daN](51)$

where F_e – *equivalent bearing load*

5.1.4 Dynamic basic bearing load rating ($\mathbf{C_{min}}$)

$$\left(\frac{C_{min}}{F_e}\right) = L^{0.33},\ using\ (64)\ and\ p = 0.33 - exponent\ factor$$

$$C_{min} = F_e * L^{0.33} = 1001.33\ [daN] = 10.01\ [kN] \quad (84)$$

5.1.5 Bearing selection from the Table 5.1

The selection will be done taking into consideration $C_{min} = 10.01$[kN] (84),
$d_r = 30\ [mm]$ (63) (check 4.3 Final geometry of the shaft) and the following table:

d_r [mm]	D_r [mm]	B [mm]	Sarcina dinamică de bază C [kN]	Simbol
15	32	9	5,6	6002
	35	11	7,8	6202
	42	13	11,4	6302
17	35	10	6	6003
	40	12	9,5	6203
	47	14	13,4	6303
	62	17	23,6	6403
20	42	12	9,3	6004
	47	14	12,7	6204
	52	15	17,3	6304
	72	19	30,5	6404
25	47	12	10	6005
	52	15	14,3	6205
	62	17	22,4	6305
	80	21	36	6405
30	55	13	12,7	6006
	62	16	19,3	6206
	72	19	29	6306
	90	23	42,5	6406
35	62	14	16,3	6007
	72	17	25,5	6207
	80	21	33,5	6307
	100	25	55	6407
40	68	15	17	6008
	80	18	29	6208
	90	23	42,5	6308
	110	27	63	6408

Table 5.1. *Choose bearing type [5]*

In conclusion the symbol of our bearings is: 6006 with C=12.7[kN] ≥ $\mathbf{C_{min} = 10.01}$[kN] (from the Table 5.1).

5.2 Bearing selection from SKF catalogue

From SKF catalogue we have selected a bearing of W 6006 (from Table 5.2)

Designation	Principal dimensions			Basic load ratings		Speed ratings	
				dynamic	static	Reference speed	Limiting speed
	d [mm]	D [mm]	B [mm]	C [kN]	C_0 [kN]	[r/min]	[r/min]
62206-2RS1	30	62	20	19.5	11.2		7 500
62306-2RS1	30	72	27	28.1	16		6 300
63006-2RS1	30	55	19	13.3	8.3		8 000
☆ ■ 6306	30	72	19	29.6	16	20 000	13 000
■ 6306 N	30	72	19	29.6	16	20 000	13 000
■ 6306 NR	30	72	19	29.6	16	20 000	13 000
☆ ■ 6306-2RS1	30	72	19	29.6	16		6 300
■ 6306-2RZ	30	72	19	29.6	16	20 000	11 000
☆ ■ 6306-2Z	30	72	19	29.6	16	20 000	11 000
■ 6306-2ZNR	30	72	19	29.6	16	20 000	11 000
■ 6306-RS1	30	72	19	29.6	16		6 300
■ 6306-Z	30	72	19	29.6	16	20 000	13 000
■ 6306-ZNR	30	72	19	29.6	16	20 000	13 000
6406	30	90	23	43.6	23.6	18 000	11 000
■ ICOS-D1B06 TN9	30	62	16	20.3	11.2		6 500
W 6006	30	55	13	11.4	8.15	28 000	17 000
☆ W 6006-2RS1	30	55	13	11.4	8.15		8 000
☆ W 6006-2RS1/VP311	30	55	13	11.4	8.15		8 000
☆ W 6006-2Z	30	55	13	11.4	8.15	28 000	14 000

Table 5.2. *Choose bearing type from SKF catalogue [7]*

5.3 Technical specification of the selected bearing

Its basic load rating (dynamic) C is 11.4 kN.
Dimensions of the bearings are as follow (see Fig.5.5 and Fig.5.6):

Generated from www.skf.com on 2021-05-07

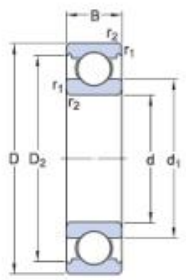
W 6006 Deep groove ball bearings

Deep groove ball bearings

Bearing data
Tolerances,
Normal (metric), P6, P5, Normal (inch),
Radial internal clearance,
Matched bearing pairs, Stainless steel
d < 10 mm, Other bearings

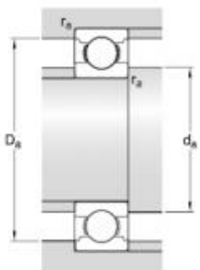
Bearing interfaces
Seat tolerances for standard conditions,
Tolerances and resultant fits

Technical specification



DIMENSIONS

d	30 mm
D	55 mm
B	13 mm
d_1	≈ 38 mm
D_2	≈ 49.95 mm
$r_{1,2}$	min. 1 mm



ABUTMENT DIMENSIONS

d_a	min. 35 mm
D_a	max. 50.5 mm
r_a	max. 1 mm

Fig. 5.5. *Bearings W 6006 specifications (1) [7]*

SKF

Generated from www.skf.com on 2021-05-07

CALCULATION DATA

Basic dynamic load rating	C	11.4 kN
Basic static load rating	C_0	8.15 kN
Fatigue load limit	P_u	0.355 kN
Reference speed		28 000 r/min
Limiting speed		17 000 r/min
Calculation factor	k_r	0.03
Calculation factor	f_0	14.7

MASS

Mass bearing	0.1 kg

Fig. 5.6. *Bearings W 6006 specifications (2) [7]*

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ANNEXES

A1 - Technical drawing of the output shaft