ExOSAM equations ~~and~~ and parameters for modeling Keplerian orbital motion, ~~flu~~ position, ~~flux~~ flux, and R-M effect

Parameters used in ExOSAM model:

I: inclination angle of planet,
b: impact parameter
$\frac{R_p}{R_*}$: radius ratio of planet to star for flux ~~determin~~ determination
M_*: mass of the star
M_p: mass of the planet
R_p: radius of the planet
R_*: radius of the star
a_p: semi-major axis of planetary orbit around center-of-mass,
a_*: semi-major axis of stellar ~~around~~ orbit around center-of-mass,
e: eccentricity of planetary orbit.
K_{amp}: ~~semi ar~~ RV semi-amplitude, constrain RV slope during transit
RV_{offset}: RV offset between RV datasets,
V_{macro}: stellar macroturbulance parameter,
q_1, q_2: Quadratic limb darkening coefficients,
ω: argument of periastron
P_{orb}: orbital period of the planet,
T_0: mid-transit time
$v\sin(i)$: Rotational velocity of the star,
λ: spin-orbit angle,
V_{RM}: The radial velocity anomaly of the R-M effect
$F_{blocked}$: fraction flux blocked by transiting planet.
RV: radial velocity of the star,

Describing the orbital motion and position

- $a_p = \left(\frac{P_{orb}^2 \cdot G \cdot (M_* + m_p)}{4\pi^2} \right)^{1/3}$

  uncertainty in M_* should be taken into account and used as a prior in MCMC.

- $a_* = \left[\left(\frac{P_{orb}^2 \cdot G \cdot M_p^3}{(M_* + M_p)^2} \right) \cdot \frac{1}{4\pi^2} \right]^{1/3}$

  a_* is small thus M_p can be constant in model

  $a_* << a_p$

  P_{orb} is a prior in MCMC

- $R_* = \frac{a_p \cdot \cos(I)}{b} \cdot \frac{1 - e^2}{1 + e \cdot \sin(\omega)}$

  I, b, e, and ω should be used as priors in MCMC. If orbit is circular, then e and ω are fixed and not used in MCMC.

- $R_p = \frac{R_p}{R_*} \cdot R_*$ $\frac{R_p}{R_*}$ have priors in MCMC

- true anomaly start

  $\nu_{start} = \pi - \omega$ ω is a prior in MCMC if $e > 0$

- If $\nu_{start} \geq \pi$: eccentric anomaly start

  $E_{start} = 2 \cdot \tan^{-1}\left(\tan\left(\frac{\nu_{start}}{2}\right) \cdot \sqrt{\frac{1-e}{1+e}} \right) + 2\pi$

  elif $\nu_{start} \leq -\pi$:

  $E_{start} = \ldots\ldots - 2\pi$

  else:

  $E_{start} = 2 \cdot \tan^{-1}\left(\tan\left(\frac{\nu_{start}}{2} \cdot \sqrt{\frac{1-e}{1+e}}\right)\right)$

- Mean anomaly transit
  if $\omega > 180$:
  $$M_{transit} = 2\pi - \omega + \pi$$
  else:
  $$M_{transit} = \pi - \omega$$

- Time of Periastron:
  $$T_\omega = T_0 - \frac{M_{transit} \cdot P_{orb}}{2\pi}$$
  T_0 is a prior in MCMC

- Time of periapsis passage:
  $$T_{pass} = T_0 - T_\omega$$

- True anomaly at transit mid time:
  $$\nu_{transit} = \frac{3\pi}{2} - \omega$$

- eccentric anomaly at transit mid time:
  if $\nu_{transit} \geq \pi$:
  $$E_{transit} = 2.0 \cdot \tan^{-1}\left[\tan\left(\frac{\nu_{transit}}{2}\right) \cdot \sqrt{\frac{1-e}{1+e}}\right] + 2\pi$$
  elif $\nu_{transit} \leq -\pi$:
  $$E_{transit} = \ldots\ldots - 2\pi$$
  else:
  $$E_{transit} = 2 \cdot \tan^{-1}\left[\tan\left(\frac{\nu_{transit}}{2}\right) \cdot \sqrt{\frac{1-e}{1+e}}\right]$$

- time of transit:
  if $e == 0$:
  $$T_{transit} = \frac{E_{transit} \cdot P_{orb}}{2\pi} + T_{pass}$$
  else:
  $$T_{transit} = \frac{E_{transit} - e \cdot \sin(E_{transit}) \cdot P_{orb}}{2\pi} + T_{pass}$$

- update data time:

$$Time\_array(l) = time\_array(l) + T_{transit}$$

- update RV values based RV offset:

$$RV(l) = RV(l) + RV_{offset}$$

RV_{offset} is a prior in MCMC

Steping through Each RV datum:

- if $e == 0$:

$$E = \frac{2\pi}{P_{orb}} \cdot (Time\_array(l) - T_\omega)$$

else:

Bessel function for determining eccentric anomaly

for eccentric orbits

$$\begin{cases} J = J_n(order, order \cdot e) \\ E_{sum} = E_{sum} + \frac{2}{order} \cdot J \cdot \sin\left(order \cdot \frac{2\pi}{P_{orb}} \cdot (time - T_\omega)\right) \\ E = \frac{2\pi}{P_{orb}} \cdot (Time\_array(l) - T_\omega) + E_{sum} \end{cases}$$

time_array(l)

- $$\nu = 2 \cdot \tan^{-1}\left[\tan\left(\frac{E}{2}\right) \cdot \sqrt{\frac{1+e}{1-e}}\right]$$

true anomaly

- Calculate planet star separation at each datum

if $e == 0$:

$$D = a_p + a_*$$

else:

$$D = \left(\frac{(a_p + a_*) \cdot (1 - e^2)}{1 + e \cdot \cos(\nu)}\right)$$

- reference time to mid transit time:

$$Time_{ref}(l) = time\_array(l) - T_{transit}$$

- position of planet in 3D space relative to star:

$$X = D \cdot \left(-\sin(\nu) \cdot \sin(\omega) + \cos(\nu) \cdot \cos(\omega)\right)$$

$$Y = D \cdot \left(\cos(\nu + \pi) \cdot \cos(I) \cdot \sin(\omega) + \sin(\nu + \pi) \cdot \cos(I) \cdot \cos(\omega)\right.$$

$$Z = D \cdot \left(-\cos(\omega) \cdot \sin(I) \cdot \sin(\nu) - \cos(\nu) \sin(I) \cdot \sin(\omega)\right)$$

- planet - star apparent distance:

$$D_{sep} = \sqrt{X^2 + Y^2}$$

- Phase of the planet:

$$Phase = \cos^{-1}\left[\sin(\nu + \omega) \cdot \sin(I)\right]$$

- normalized flux from star with ~~no~~ transit:

$$A_{planet} = \pi \cdot R_p^2$$

bond albedo

$$flux = 1.0 + \left(\frac{A_{planet}}{4 \pi D^2}\right) \cdot A_{bond} \cdot 0.5 \cdot (1.0 + \cos(phase))$$

not important for R-m work

- Location of the center of the planet along the x-axis of the rotational axis of the star:

$$X' = X \cdot \cos(\lambda) + Y \cdot \sin(\lambda)$$ spin-orbit angle

- sub planet velocity: $V_{sub} = v\sin(i) \cdot \frac{X'}{R_*}$

- If $D_{sep} \leq R_* + R_p$ and $D_{sep} \geq R_* - R_p$:

$$D_{sep_1} = \frac{D_{sep}^2 + R_*^2 - R_p^2}{2 \cdot D_{sep}}$$

$$X_1 = \frac{X \cdot D_{sep_1}}{D_{sep}} + \frac{Y}{D_{sep}} \cdot \sqrt{R_*^2 - D_{sep_1}^2}$$

$$Y_1 = \frac{Y \cdot D_{sep_1}}{D_{sep}} - \frac{X}{D_{sep}} \cdot \sqrt{R_*^2 - D_{sep_1}^2}$$

$$X_2 = \frac{X \cdot D_{sep_1}}{D_{sep}} - \frac{Y}{D_{sep}} \cdot \sqrt{R_*^2 - D_{sep_1}^2}$$

$$Y_2 = \frac{Y \cdot D_{sep_1}}{D_{sep}} + \frac{X}{D_{sep}} \cdot \sqrt{R_*^2 - D_{sep_1}^2}$$

- Distance from center of star to center area of planet inside stellar disk

$$D_{in} = \frac{|D_{sep}| - R_p + R_*}{2}$$

$$Len = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2}$$

$$\theta_{in} = \tan^{-1}\left(\frac{Y}{X}\right)$$

$$X_{in} = D_{in} \cdot \cos(\theta + \pi)$$

$$Y_{in} = |D_{in} \cdot \sin(\theta)|$$

$$X_{in}' = X_{in} \cdot \cos(\lambda) + Y_{in} \cdot \sin(\lambda)$$

- $V_{sub} = \dfrac{V\sin(i) \cdot X'_{in}}{R_*}$

- following calculates area of planet within the stellar disk:

$$\beta_1 = 2 \cdot \sin^{-1}\left(\frac{0.5 \cdot Len}{R_p}\right)$$

$$\alpha_1 = 2 \cdot \cos^{-1}\left(\frac{0.5 \cdot Len}{R_*}\right)$$

- if $D_{sep} \geq \sqrt{R_*^2 - R_p^2}$:

$$A_p = \frac{1}{2} \cdot R_*^2 \cdot (\alpha_1 - \sin(\alpha_1)) + \frac{1}{2} \cdot R_p^2 \cdot (\beta_1 - \sin(\beta_1))$$

normalized Area of planet inside stellar disk,

$$I_{o_p} = I_o * A_p$$

limb darkening coefficients

$$\text{where } I_o = \frac{6}{\pi R_*^2 (6 - (2 \cdot q_1) - q_2)}$$

- if $D_{sep} < \sqrt{R_*^2 - R_p^2}$

$$A_p = \pi R_p^2 \left(\frac{1}{2} \cdot R_*^2 \cdot (\alpha_1 - \sin(\alpha_1))\right) - \frac{1}{2} \cdot R_p^2 \cdot (\beta_1 - \sin(\beta_1))$$

$$I_{o_p} = I_o * A_p$$

- if $D_{sep} < R_* - R_p$

$$A_p = \pi * R_p^2 \qquad I_{o_p} = I_o A_p$$

- Determining the flux blocked by the planet.
  - $F_{blocked} = I_{o_p} \cdot \left[1 - q_1 \cdot \left(1 - \sqrt{\left|1 - \frac{D_{in}^2}{R_*}\right|}\right) - q_2 \cdot \left(1 - \sqrt{\left|1 - \frac{D_{in}}{R_*^2}\right|}\right)^2\right]$

- Rossister-McLaughlin effect:

$$V_{RM} = -\left(F_{blocked} \cdot V_{sub}\right) \cdot \left(\frac{2 \cdot V_{macro}^2 + 2 v\sin(i)^2}{(2 \cdot V_{macro}^2) + v\sin(i)^2}\right)^{3/2} \cdot \left(1 - \frac{V_{sub}^2}{(2 \cdot V_{macro}^2) + v\sin(i)^2}\right)$$

$$F_{*} = 1 - F_{blocked}$$

- Radial velocity of the star:

$$RV = K_{amp} \cdot \left(\cos(M + \nu + \omega) + e \cdot \cos(\omega + M)\right) + V_{RM}$$

prior on parameter in MCMC

Proposed Prior Parameters in MCMC:

$$\boxed{b,\ I,\ \frac{R_p}{R_*},\ M_*,\ e,\ V_{macro},\ q_1,\ q_2,\ \omega,\ P_{orb},\ T_0,\ K_{amp}}\ \boxed{v\sin(i)}$$

length of transit set by: $b, I, \frac{R_p}{R_*}, M_*, P_{orb}$

from its effect on a_p

transit Duration:

$$T_D = \frac{P_{orb}}{\pi} \cdot \sin^{-1}\left[\frac{R_*}{a_p} \cdot \frac{\sqrt{\left(1 + \frac{R_p}{R_*}\right)^2 - b^2}}{\sin(I)}\right] \cdot \frac{\sqrt{1 - e^2}}{1 + e \cdot \sin(\omega)}$$

Transit duration is not directly used in model

Free parameters in MCMC:

Fixed parameters in MCMC: $\underbrace{R_*,\ R_p}_{\text{both vary from } \frac{R_p}{R_*} \text{ prior}},\ M_p$