Annotated Literature on Multi-Scale Spatial Point Processes and Potential for Chi-Squared Cox Processes (CSCPs)

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Epidemiology

- **Iftimi, A., et al (2018). "A multi-scale area-interaction model for spatio-temporal point patterns."** Applied a multi-scale Gibbs process to varicella case locations in Spain. Found small-scale clustering due to contagion and larger-scale regularity. *Why CSCP:* Suggests two independent processes at different scales; a CSCP could model these as additive stochastic components with clear diagnostics. Not sure the inhibitive aspect would be compatible though.
- **Iftimi, A., et al. (2017). "Measuring spatial inhomogeneity at different spatial scales using hybrids of Gibbs point process models."** Proposed hybrid Gibbs point processes to capture multiple clustering/inhibition scales in disease data. *Why CSCP:* A CSCP would offer a simpler, random-field-based alternative with a direct diagnostic via $g(h) - 1$, if we focus only on clustering.

Ecology

- **Wiegand, T., Gunatilleke, C.V.S., Gunatilleke, I.A.U.N., Okuda, T. (2007). "Analyzing the spatial structure of a Sri Lankan tree species with multiple scales of clustering." Ecology 88:3088–3102.** Showed that *Shorea congestiflora* trees display nested clusters at two distinct radii (25 m, 8 m), implying different recruitment/habitat mechanisms. *Why CSCP:* A textbook (???) case for additive superposition: separate squared-GRF components can represent large- and small-scale clustering.
- **Levin, S.A. (1992). "The problem of pattern and scale in ecology." Ecology 73:1943–1967.** This seems to be a seminal paper on how multiple processes at distinct scales shape ecological patterns. *Why CSCP:* Supports the philosophy of explicitly modeling independent multi-scale clustering. Could be good to point to this.
- **Yau, C.Y. & Loh, J.M. (2012), "A Generalization of the Neyman–Scott Process." Statistica Sinica 22: 1717–1736** Generalizes the Neyman–Scott cluster process to allow regular (non-Poisson) parent processes (e.g., Strauss) to mix clustering and inhibition, develops inference for the generalized model. *Why CSCP:* May not be too relevant, but again could show the need for multi-scale Cox processes.

Astronomy

- **Coil, A. (2012). "Large Scale Structure of the Universe *in* Planets, Stars and Stellar Systems vol 6** Described the halo model: galaxy clustering is explained as the sum of "one-halo" (within halos) and "two-halo" (between halos) terms. *Why CSCP:*

This is directly analogous to additive multi-scale clustering; CSCP could possibly model halo-scale and cosmic-web-scale fields as independent components ??

- **Martínez, V.J., Saar, E. (2002). *Statistics of the Galaxy Distribution.*** Apparently surveyed point process models (including Cox and segment Cox) for cosmic structures, showing multi-scale clustering signatures. I can't get immediate access to this book to inspect more closely. *Why CSCP:* Provides motivation for additive Cox models in cosmology, which could capture both fine and coarse clustering hierarchies. Could be quite handy.

Seismology

- **Marianna Siino, Giada Adelfio, Jorge Mateu, Marcello Chiodi & Antonino D'Alessandro. (2017). "Spatial pattern analysis using hybrid models: an application to the Hellenic seismicity." Stoch. Env. Res. Risk Assess. 31:1633–1648.** Modeled Greek earthquake locations as a hybrid: background seismicity plus clustered aftershocks. *Why CSCP:* Earthquakes are a natural additive process (background + aftershocks); a CSCP could formalize this as independent intensity fields.

- **Pei, T., et al. (2012). "Multi-scale decomposition of point process data." GeoInformatica 16:625–652.** Developed methods to decompose point patterns into scale-specific components, illustrated with seismic data. *Why CSCP:* Their decomposition in spirit should align with a CSCP's generative additive structure.

Theory and Methodology

- **McCullagh, P., Møller, J. (2006). "The permanental process." Advances in Applied Probability 38(4):873–888.** Defined and studied permanental (boson) Cox processes, with intensities given by sums of squared Gaussian fields (chi-square distributed). Provided existence proofs, joint densities, and links to permanents. *Why CSCP:* This is the theoretical foundation; we can make our work differ by 1) allowing different kernels per component (important given the aforementioned applications) and 2) emphasizing applied diagnostics/inference/fitting tools.

- **Shirai, T., Takahashi, Y. (2003). "Random point fields associated with fermion, Poisson and boson processes." J. Funct. Anal. 205(2):414–463; ASPM 39:345–354.** Formalized boson/permanental processes in the probability literature. *Why CSCP:* Establishes the connection between squared-GRF intensities and chi-square Cox processes. Need to look at this in more detail.

- **Walder, C.J., Bishop, A.N. (2017). "Fast Bayesian Intensity Estimation for the Permanental Process." ICML 2017.** Proposed efficient Bayesian inference for permanental processes using kernel methods. *Why CSCP:* Shows computational feasibility of additive chi-square intensities... not too keen on worrying about the Bayesian angle just yet, but should keep this in mind.

- **Nicolis, O., et al (2022). "Temporal Cox Process with Folded Normal Intensity." Axioms 11(10):513.** Considered alternative positive transforms of Gaussian processes (absolute value – I mentioned this earlier today in a chat with Adrian!!), and reviewed related Gamma/chi-square intensity models. *Why CSCP:* Confirms interest in non-log link functions; folded-normal is a sibling to chi-square. Definitely warrants at least a skim-read...