

Annotated Literature on Multi-Scale Spatial Point Processes and Potential for Chi-Squared Cox Processes (CSCPs)

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Epidemiology

- Iftimi, A., et al (2018). “**A multi-scale area-interaction model for spatio-temporal point patterns.**” Applied a multi-scale Gibbs process to varicella case locations in Spain. Found small-scale clustering due to contagion and larger-scale regularity. *Why CSCP*: Suggests two independent processes at different scales; a CSCP could model these as additive stochastic components with clear diagnostics. Not sure the inhibitive aspect would be compatible though.
- Iftimi, A., et al. (2017). “**Measuring spatial inhomogeneity at different spatial scales using hybrids of Gibbs point process models.**” Proposed hybrid Gibbs point processes to capture multiple clustering/inhibition scales in disease data. *Why CSCP*: A CSCP would offer a simpler, random-field-based alternative with a direct diagnostic via $g(h) - 1$, if we focus only on clustering.

Ecology

- Wiegand, T., Gunatilleke, C.V.S., Gunatilleke, I.A.U.N., Okuda, T. (2007). “**Analyzing the spatial structure of a Sri Lankan tree species with multiple scales of clustering.**” *Ecology* **88**:3088–3102. Showed that *Shorea congestiflora* trees display nested clusters at two distinct radii (25 m, 8 m), implying different recruitment/habitat mechanisms. *Why CSCP*: A textbook (???) case for additive superposition: separate squared-GRF components can represent large- and small-scale clustering.
- Levin, S.A. (1992). “**The problem of pattern and scale in ecology.**” *Ecology* **73**:1943–1967. This seems to be a seminal paper on how multiple processes at distinct scales shape ecological patterns. *Why CSCP*: Supports the philosophy of explicitly modeling independent multi-scale clustering. Could be good to point to this.
- Yau, C.Y., Loh, J.M. (2012). “**A Neyman–Scott cluster process with two cluster scales.**” *Environmental and Ecological Statistics* **19**:411–430. Extended the Neyman–Scott model to allow two independent cluster scales, applied to ecological data. *Why CSCP*: As above. Shows the need for multi-scale Cox processes; CSCP offers a different random-field based route.

Astronomy

- Coil, A. (2012). “**Large Scale Structure of the Universe.**” *NED Review*. Described the halo model: galaxy clustering is explained as the sum of “one-halo” (within halos) and “two-halo” (between halos) terms. *Why CSCP*: This is directly analogous to

additive multi-scale clustering; CSCP could possibly model halo-scale and cosmic-web-scale fields as independent components ??

- **Martínez, V.J., Saar, E. (2002).** *Statistics of the Galaxy Distribution*. Surveyed point process models (including Cox and segment Cox) for cosmic structures, showing multi-scale clustering signatures. *Why CSCP*: Provides motivation for additive Cox models in cosmology, which could capture both fine and coarse clustering hierarchies. Could be quite handy.

Seismology

- **Siino, M., Adelfio, G., D’Alessandro, A. (2017).** “Spatial pattern analysis using hybrid models: an application to the Hellenic seismicity.” *Stoch. Env. Res. Risk Assess.* **31:1633–1648**. Modeled Greek earthquake locations as a hybrid: background seismicity plus clustered aftershocks. *Why CSCP*: Earthquakes are a natural additive process (background + aftershocks); a CSCP could formalize this as independent intensity fields.
- **Pei, T., et al. (2012).** “Multi-scale decomposition of point process data.” *GeoInformatica* **16:625–652**. Developed methods to decompose point patterns into scale-specific components, illustrated with seismic data. *Why CSCP*: Their decomposition in spirit should align with a CSCP’s generative additive structure.
- **Adelfio, G., Chiodi, M. (2020).** “Overview of methods for multi-structure point processes.” *Stoch. Env. Res. Risk Assess.* **34:17–36**. Reviewed methods for modeling multi-structured point processes across domains. Highlighted superposition as a key tool for multi-scale clustering. *Why CSCP*: Reinforces the idea that additive models like CSCPs are often more natural than multiplicative LGCPs. This could be quite an important paper to cite.

Theory and Methodology

- **McCullagh, P., Møller, J. (2006).** “The permanental process.” *Advances in Applied Probability* **38(4):873–888**. Defined and studied permanental (boson) Cox processes, with intensities given by sums of squared Gaussian fields (chi-square distributed). Provided existence proofs, joint densities, and links to permanents. *Why CSCP*: This is the theoretical foundation; we can make our work differ by 1) allowing different kernels per component (important given the aforementioned applications) and 2) emphasizing applied diagnostics/inference/fitting tools.
- **Shirai, T., Takahashi, Y. (2003, 2004).** “Random point fields associated with fermion, Poisson and boson processes.” *J. Funct. Anal.* **205(2):414–463**; *ASPM* **39:345–354**. Formalized boson/permanental processes in the probability literature. *Why CSCP*: Establishes the connection between squared-GRF intensities and chi-square Cox processes. Need to look at this in more detail.
- **Walder, C.J., Bishop, A.N. (2017).** “Fast Bayesian Intensity Estimation for the Permanental Process.” *ICML 2017*. Proposed efficient Bayesian inference for permanental processes using kernel methods. *Why CSCP*: Shows computational feasibility of additive chi-square intensities... not too keen on worrying about the Bayesian angle just yet, but should keep this in mind.

- **Walder, C.J., Bishop, A.N. (2017).** “Gamma Gaussian Cox Processes.” **Methodology and Computing in Applied Probability** 19(2):447–465. Studied Cox processes with intensity given by a squared GP, linking to Gamma/chi-square distributions. *Why CSCP*: Again– need to look at this more carefully. Closest modern methodological contribution to what we seem to be doing; as above, our proposed CSCP extends to multi-scale and applied contexts.
- **Nicolis, O., Fernández, L., Muñoz, J.E. (2022).** “Temporal Cox Process with Folded Normal Intensity.” **Axioms** 11(10):513. Considered alternative positive transforms of Gaussian processes (absolute value – I mentioned this earlier today in a chat with Adrian!!), and reviewed related Gamma/chi-square intensity models. *Why CSCP*: Confirms interest in non-log link functions; folded-normal is a sibling to chi-square. Definitely warrants at least a skim-read...