Politecnico di Torino

Department of CONTROL AND COMPUTER ENGINEERING (DAUIN)

MSc in Mechatronic Engineering

2017/2018

Robotics- Prof. Rizzo

Report: Double pendulum, modeling, control and trajectory planning

Abdallah Bader(s251217)

May 7, 2018

Introduction

Objective of this report is to build a simulation model of a double pendulum and apply the following control techniques

1. Joint space control
    a. PID outer loop control of a centralized exact linearization architecture
    b. PD with gravity compensation controller design
2. Operational space control
    a. PD with gravity compensation controller
    b. Inverse dynamics method.

Another Objective is to provide convenient trajectories to demonstrate the simulated schemes working. Figure1,2 represents control schemes to be simulated using MATLAB Simulink including both joint space and operational space control techniques.

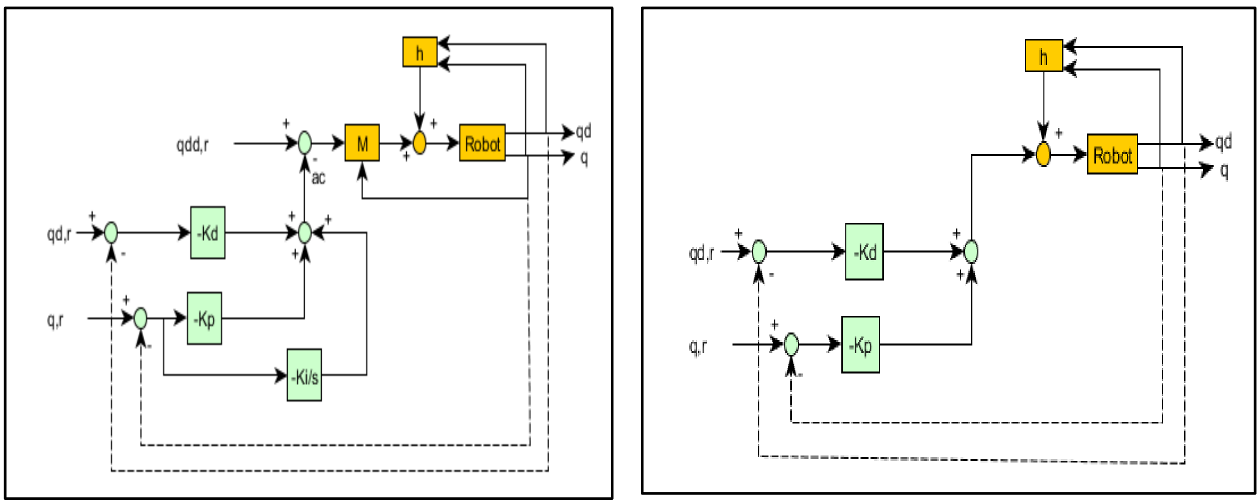


Fig. 1 Joint space control of both PID outerloop over exact linearized robot (left) and PD with gravity compensation(right)

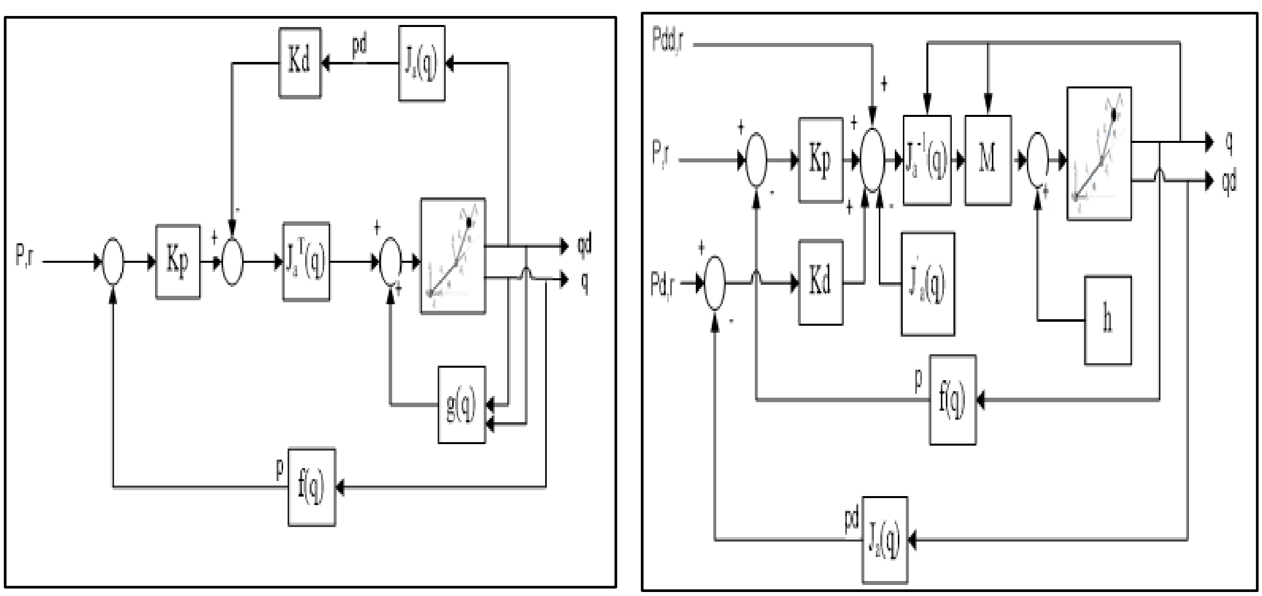


Fig. 2 Operational space control scheme of both PD with gravity compensation(left) and Inverse dynamics method (right)

Parameter	Abbrv.	Value	Unit
Length of 1st link	L_1	1	m
Length of 2nd link	L_2	1	m
Mass of 1st link	M_1	1	Kg
Mass of 2nd link	M_2	1	Kg
Moment of inertia of 1st link	I_1	3	$Kg.m^2$
Moment of inertia of 2nd link	I_2	2	$Kg.m^2$
Friction constant of joints J_1 , J_2	Beta	10	$N.s/m^2$

Table 1. Double pendulum parameters and their values

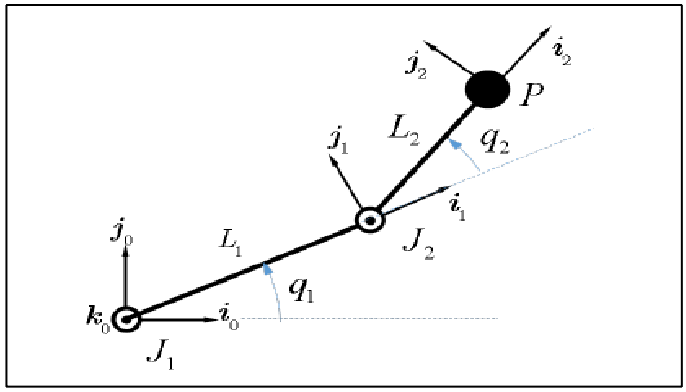


Fig. 3 Double pendulum scheme with relevant symbols (picture from the provided document)

Joint space control

In this section, two control techniques were applied, a PID outerloop controller while having an exact linearized model. Fig4. represents a Simulink block diagram of the exact linearized double pendulum, that is controlled using a PID outer loop controller. Other Simulink models are shown in Appendix A. Simulink models include mainly the double inverted pendulum block, along with other relevant blocks including inertia matrix M, velocity relevant terms matrix h, reference trajectory, controller blocks, and finally input/output blocks to work with data between MATLAB workspace and Simulink.

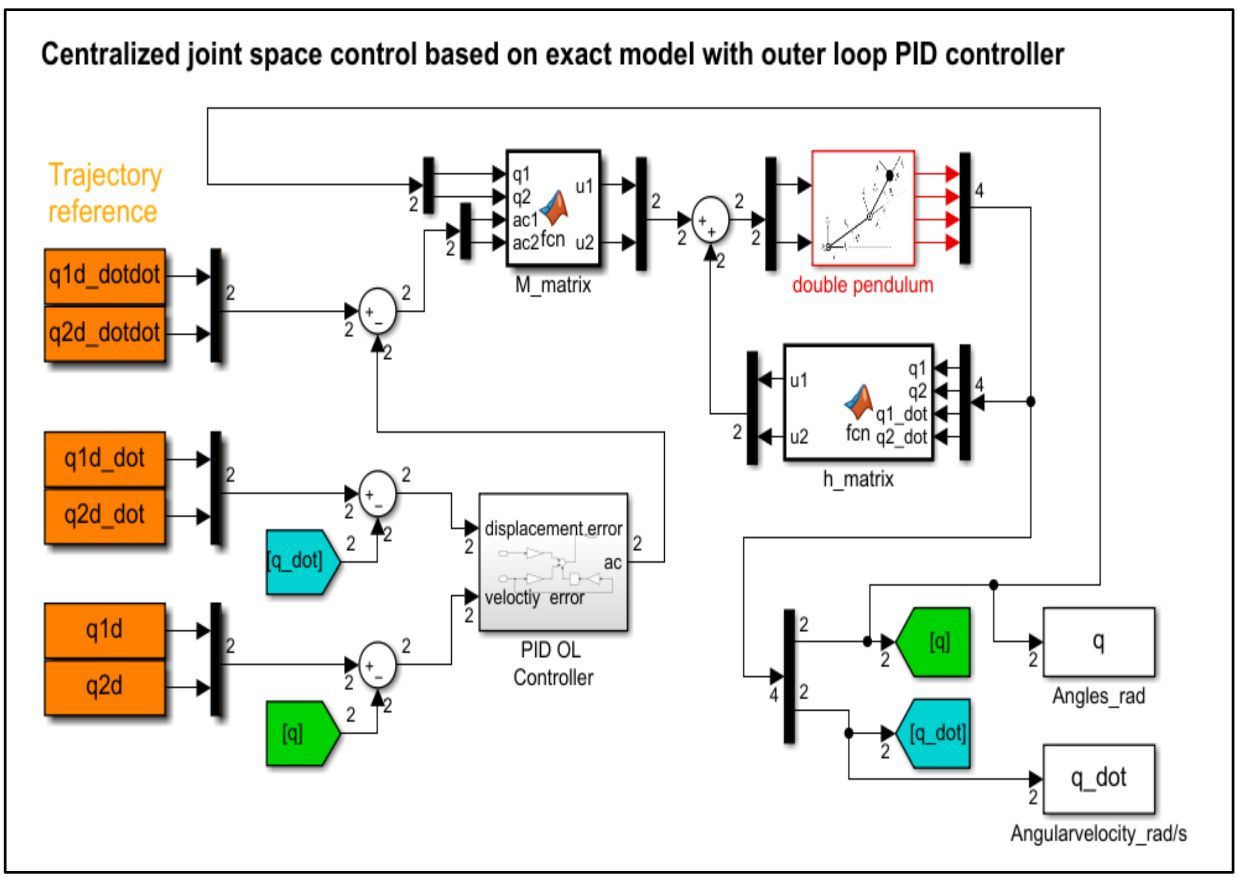


Fig. 4 Centralized joint space control based on double inverted pendulum's exact model with outer loop PID controller

Controller design

Table 2 shows controller gains used. These values were decided over a set of trials. It can be seen that Kp gain using PD, is 4 times larger than one using PID controller. Which then can introduce a large stiffness in performance.

Gain	PID w/ exact linearization	PD /w gravity compensation
Kp	500	2000
Kd	80	80
Ki	200	n/a

Table 2 controller gains for joint space control

Reference trajectory

A convex trajectory is planned over the joint space, this was done using "jtraj" command from robotics toolbox. Figure 5 shows reference angle, angular velocity, and angular acceleration.

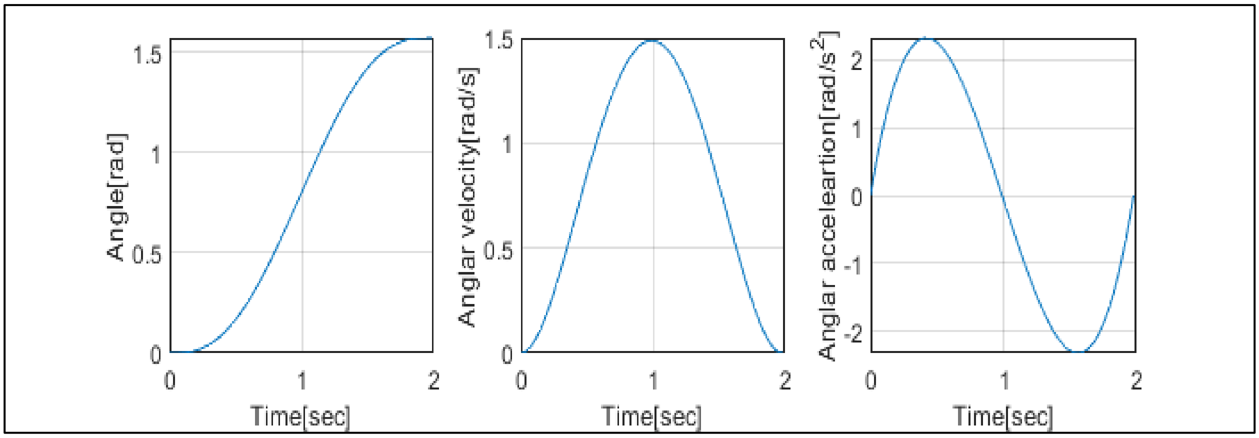


Fig. 5 Reference trajectory used for joint space control scheme

Operational space control

For operational space control, two different techniques are used, PD with gravity compensator control, and Inverse dynamics method. Simulation block diagrams of the proposed methods can be seen in Appendix A.

Controller design

Table 3 shows controller gains tuned for both PD with gravity controller and Inverse dynamics method. Those gains were decided over a set of trials. It can be seen that Kp gain for PD with gravity controller is 5 times larger than inverse dynamics one.

Gain	PD w/ gravity compensator	Inverse Dynamics Method
Kp	2500	500
Kd	50	50

Table 3 Controller gains for operational space control

Reference trajectory

For PD w/ gravity compensator, only desired reference trajectory of displacement is required, therefore, a trajectory is planned to pass through set of points in the operational space using "ctraj" MATLAB command. While for inverse dynamics method, reference displacement, velocity, and acceleration are required, therefore "mtraj" MATLAB command is used for multi axis trajectory generation. Which produces compatible convex trajectories in task space to achieve passing through a set of desired points.

Results

Joint space results

Figure 6. shows Trajectory tracking results in joint space, where a desired convex trajectory is performed with a final amplitude of $\pi/2$ for $q1$, and 0 reference for $q2$. This is eventually the upright position of the double pendulum. For angle q1, a matching signal can be seen while performing centralized joint space control, while a slight offset is noticed when applying PD with control gravity compensation controller. For angle q2, a very small deviation around zero is noticed using the exact linearized PID controller, while a clear offset of 0.001 can be seen while applying PD control with gravity.

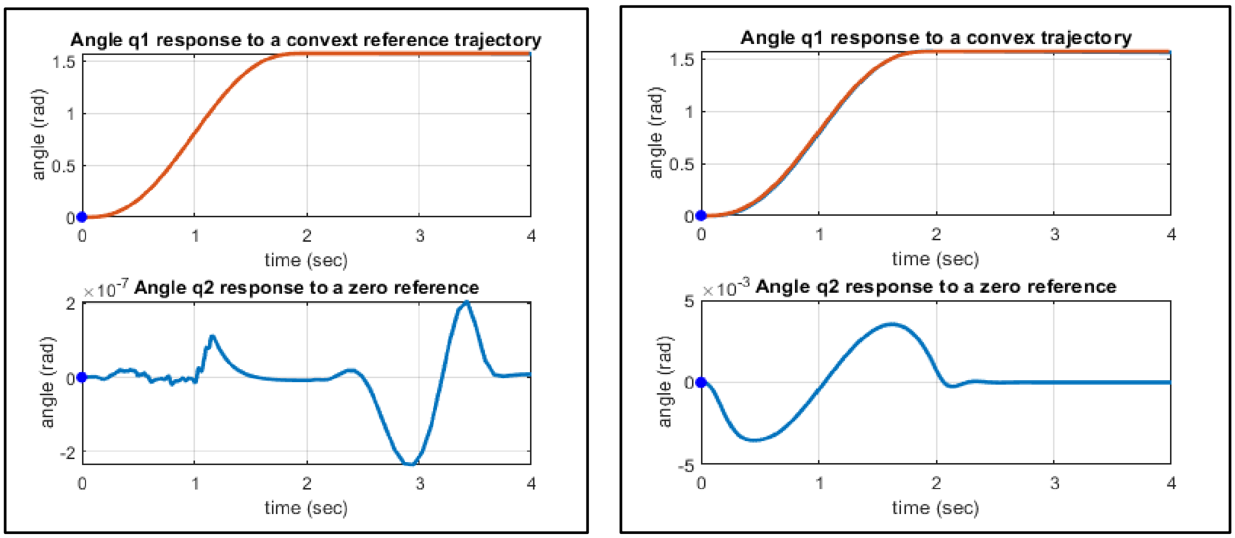


Figure 6 angle q1, and q1, response for a trajectory to an upright pendulum position, PID outerloop(left), PD with gravity (right)

Operational space results

Figure 7. shows a demonstration of trajectory tracking in the operational space using both, PD controller with gravy compensation, and the inverse dynamics method. Using first technique, a poor tracking of the trajectory can be clearly seen. While using the second technique, a matching trajectory is performed.

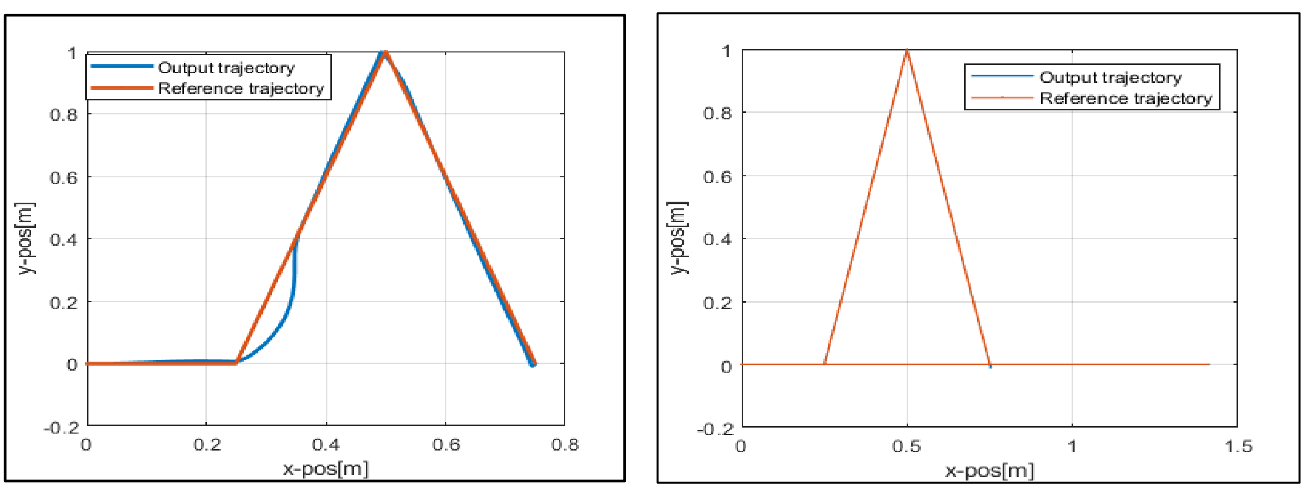


Figure 7 Demonstration drawing of a trajectory following in operational space (x,y), PD with gravity(left), inverse dynamics method(right)

Discussion

Joint space control

Two different control techniques were applied, it can be seen from results that PID with outer loop control, a matching output signal was acquired with very small errors. While results when using PD control with gravity compensator, shows a less matching output signal. With slightly larger errors. It can be also noticed that PD technique requires large control gains which imposes a less compliant system.

Operational space control

Inverse dynamics-based method shows a better tracking performance, with much smaller controller gain over other PD with gravity control method. Also, inverse dynamics method suggests a more deterministic approach since trajectories of displacement, velocity, and acceleration are set as reference. A similar observation compared to joint space control is the much larger controller gains of PD with gravity controller, which can introduce a less compliant system than other method.

Conclusion

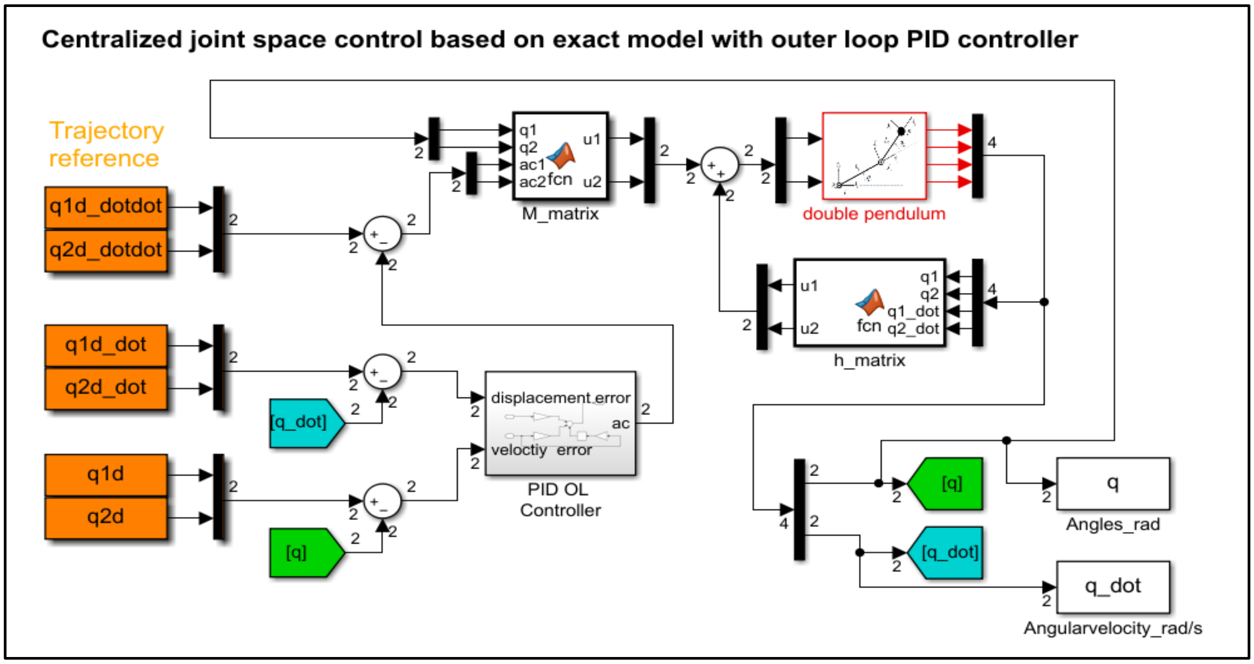
A simulation model of double pendulum was simulated. First, joint space control was applied using both PID outerloop over exact linearized system, and PD with gravity compensation. PID technique showed more effective with smaller controller gains. Trajectory tracking were demonstrated over a convex trajectory obtained from Robotics toolbox. Second, Operational space control was applied, using both PD with gravity compensation and the inverse dynamics method. Inverse dynamics method showed more effective to follow reference trajectory, which is obtained from multi-axis trajectory generation using Robotics toolbox.

Further possible work is to investigate torques required for each control technique, also a more structured criteria for choosing controller gains can be adopted. Investigating frequency behavior can be done to conclude the bandwidth of each controller. Also, some interesting topics to consider is how to generate paths where required torque is more efficient.

While only cartesian positions were planned, and controlled for, including orientation would add more dexterity to the end effector, which can possibly add a plus of more efficient controlled forces.

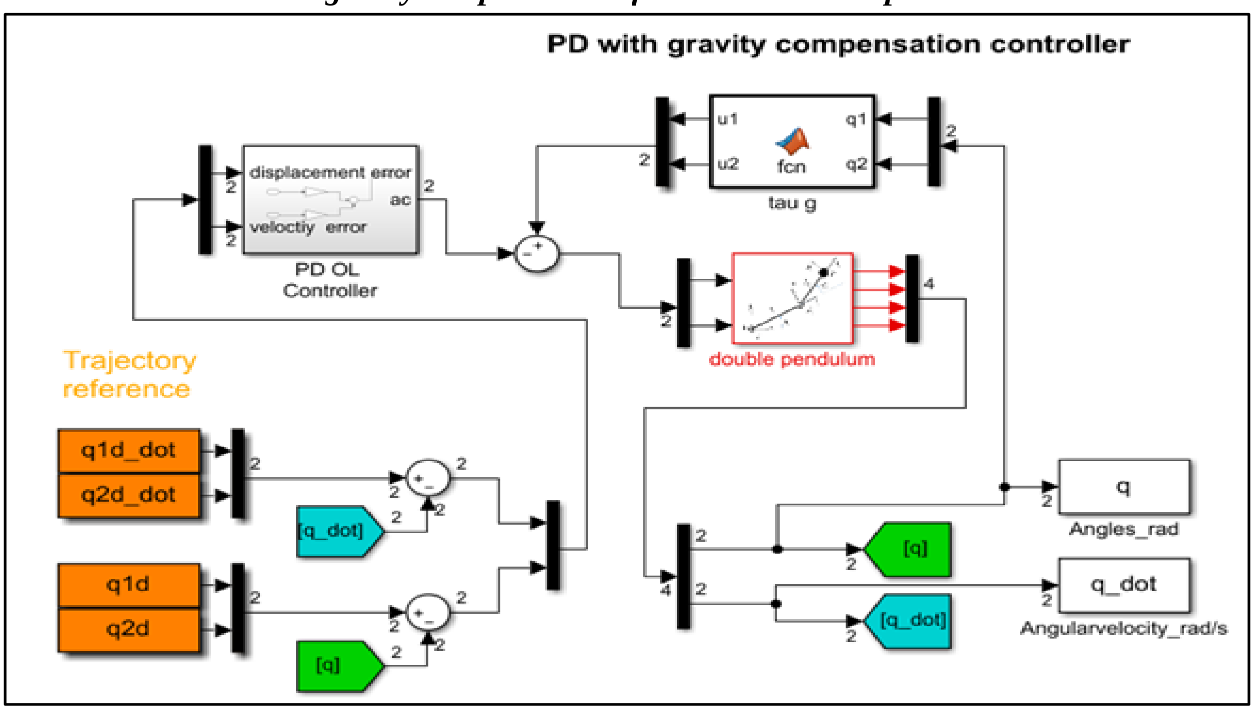
Appendix A

Centralized joint space control based on double inverted pendulum's exact model with outer loop PID controller



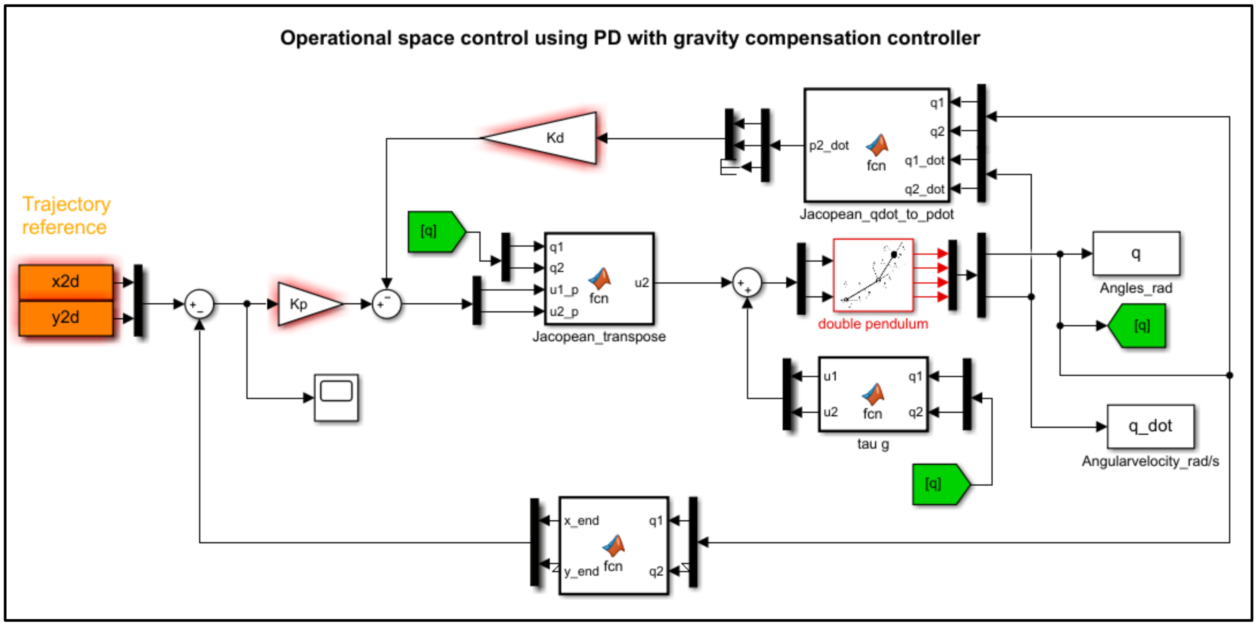
FigureA1. Centralized joint space control based on double inverted pendulum's exact model with outer loop PID controller.

PD control scheme with gravity compensation of a double inverted pendulum



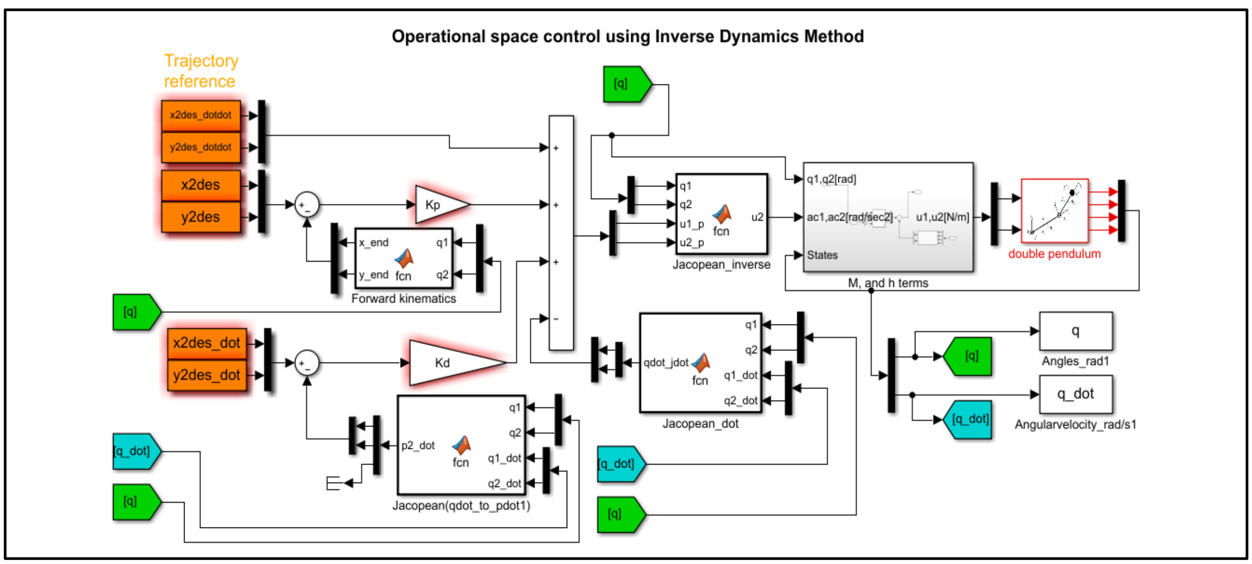
FigureA2. PD control scheme with gravity compensation of a double inverted pendulum.

Operational space control using PD with gravity compensation controller



FigureA3. Operational space control using PD with gravity compensation controller.

Operational space control using Inverse dynamics method



FigureA4. Operational space control using Inverse dynamics method.