Written Assignment #4

CPEN 432: Real-time System Design

Due: April 1, 2018, by 11:59 p.m.

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   (a) Three soft real-time tasks have to be scheduled on the same processor. Two tasks have fixed execution times but their periods/frequencies are flexible. The third task has a fixed periodicity but variable execution times because of supporting imprecise execution. The parameters of the tasks are as in the following table (Table 1).

Task	$e_{\min}$	$e_{\max}$	e_0	$P_{\min}$	$P_{\max}$	P_0	k
T_1	2	2	2	8	12	10	1
T_2	4	4	4	5	10	6	2
T_3	4	10	5	12	12	12	1.5

   Table 1: Task set for question 1a

   k is the elasticity of each task and e_0 and P_0 are the *nominal* execution times and nominal periods for the tasks. Assume that elastic scheduling is used. What then are the execution parameters of these three tasks? Recall that the compression formula for elastic scheduling is as follows:

   $$\forall T_i \in V : U_i = U_{i,0} - (U_0 - U_b + U_M)\frac{k_i}{\sum_{T_j \in V} k_j}.$$

   Solve the problem for both EDF scheduling and RM scheduling using the Liu and Layland bound.

   (b) In the elastic task model, we wish to adopt a different strategy for compressing tasks. Given a desired utilization $U_d < U_0$, where $U_0 = \sum_{i=1}^{n} e_i / P_{i,0}$, we want to compress tasks so that the system utilization becomes U_d. Derive the equation(s) that perform utilization compression. Justify your answer.

   (c) (**Bonus**) Show that, under the assumption that task execution times are integers (or can be represented by integers), the greedy procedure we used to solve the LP of the imprecise computation model is optimal. That is, the greedy procedure, upon termination, produces execution times $e_1, \ldots, e_n$ that satisfy the constraints of the LP and minimize the total QoS loss $\sum_{i=1}^{n}(e_{i,\max} - e_i)$.

2. Packet processors are used in most commercial networking equipment to support a variety of traffic management and monitoring functionality. Examples of such functionality include packet classification, spam detection, virus and malware blockers and encryption support.

Consider a simple packet processing system with three stages: a packet classification stage, a spam detector, a virus signature analyzer, and a cryptography module. All traffic enters the packet classifier and packets are identified as email, general application and VoIP traffic (in 2 time units). Email traffic is routed through the remaining three stages and each email packet requires 4 time units, 5 time units and 3 time units respectively at the three stages. General application traffic does not require spam detection but does require virus scanning and encryption; these packets require 6 time units and 7 time units respectively at these two stages. VoIP traffic requires encryption only: 3 time units.

Suppose that email traffic arrives at the rate of 1 packet every 20 time units (traffic shapers can ensure this regular arrival rate), general application traffic arrives at the rate of 1 packet every 12 time units, and VoIP traffic arrives at the rate of 1 packet every 15 time units. The priority levels for the different classes of traffic are such that VoIP > general application > email.

(a) What is the maximum delay experienced by a packet of email traffic? (You may make reasonable assumptions to determine this delay.)

(b) What is the [approximate] maximum email traffic rate that this packet processing system can support assuming that all non-email traffic arrives at the fixed rate specified?

3. Consider n sporadic, implicit-deadline tasks $T_1, \ldots, T_n$. Task T_i has WCET $e_i > 0$, period $P_i \in \mathbb{N}$, and worst case (peak) memory requirement $M_i \in \mathbb{N}$ (in MB). The tasks are *nonpreemptible* and migrations are not allowed. Moreover, the tasks cannot be rearranged: In any allocation onto processors, the tasks must appear in the same order as they are given in the input. You are given a utilization bound $U_b \in (0, 1]$ and a pool of identical processors, each with a schedulable utilization U_b. Given an allocation of processors to the input tasks, the peak memory demanded by processor π_j under the given allocation is defined as $\max\{M_i : T_i \text{ is assigned to processor } \pi_j\}$. Our goal is to assign tasks to as many processors as needed, respecting the given processor schedulable utilization and the no rearrangement constraint, so that the overall peak memory usage is minimized; that is, the sum of the peak memory on all processors is the minimum possible. Develop an algorithm to solve this problem optimally and prove its correctness. Derive the asymptotic running time of your algorithm in terms of $n, e_i, P_i, U_i, M_i, U_b$ (or any subset thereof). You may assume that all task parameters are rational numbers except where explicitly stated otherwise.

   HINT: Use Dynamic Programming.

   Bonus: If your algorithm runs in time that is *polynomial* in the size of the input (in binary encoding), I will give you the entire credits allotted to *all the written assignments **and** the final exam*, regardless of your performance on the written assignments.

4. We need to execute three periodic tasks on a partitioned multiprocessor system using EDF at each processor. The utilizations of the three tasks are 0.4, 0.3 and 0.5, respectively. Assume that a processor has a reliability of 0.95. In other words, at any given instant the probability that a processor may fail is 5%. We would like the reliability of each task to be at least 0.999. This implies that the probability that a task would fail at any given instant is 0.1%. How many processors do we need to ensure this level of reliability? How should tasks be partitioned to obtain this reliability level? Here we are allowed to replicate tasks, but jobs belonging to one replica execute on the same processor.