

Inverse equation of state construction from mass-radius-relations of compact stars

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1 Introduction

2 Preparations

2.1 The Tolman-Oppenheimer-Volkoff equation

Before we get to the main topics of this thesis, it is important to set up a basis that our work emerges from.

Firstly, this involves a derivation of the equation used for determining the structure of compact stars; here, we will use the Tolman-Oppenheimer-Volkoff [TOV] equation. We choose our unit system as $c = G = 1$, so that every unit is a power of a length.

The derivation is based off assuming the star matter as a perfect/ideal fluid. The system shall further not evolve in time, therefore staying spherically symmetric. In terms of the metric components, we are left with;

In conclusion we get the full TOV equation(s):

$$\frac{dP}{dr} = \frac{(\epsilon + P)(m + 4\pi r^3 P)}{2mr - r^2} \quad (1)$$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon \quad (2)$$

2.2 Numerical solution

In order to generate an initial mass-radius relation to test our reverse algorithm with, we use a fourth order Runge-Kutta algorithm to solve the TOV equation along with the mass differential equation numerically. By looking at both equations, our ODE system is in the form

$$\dot{y}(t) = f(y(t), t) \quad (3)$$

where $\dot{y}(t)$ is a two component "vector":

$$\dot{y}(t) = \begin{pmatrix} dP/dr \\ dm/dr \end{pmatrix} \quad (4)$$

$f(y(t), t)$ then contains the right hand side of equations 1 and 2. As stated before, to create an initial mass radius relation to check the inverted algorithm with, we use a fourth order Runge-Kutta [RK4] code. The RK4 method works as follows:

$$y(t + \tau) = y(t) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (5)$$

with k_i :

$$\begin{aligned} k_1 &= f(y(t), t) \cdot \tau \\ k_2 &= f(y(t) + k_1/2, t + \tau/2) \cdot \tau \\ k_3 &= f(y(t) + k_2/2, t + \tau/2) \cdot \tau \\ k_4 &= f(y(t) + k_3, t + \tau) \cdot \tau \end{aligned}$$

All k_i are of course also two component "vectors".

Messung der Fadenlänge	l (m)
1	
2	
3	
4	
5	
Mittelwert \bar{l}	
Standardabweichung σ_l	

References

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- [2] H. Kuchling, *Taschenbuch der Physik, 21. Auflage*, Fachbuchverlag Leipzig, 2014.