Inverse equation of state construction from mass-radius-relations of compact stars

Bachelor thesis

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2.1 The Tolman-Oppenheimer-Volkoff equation

Before we get to the main topics of this thesis, it is important to set up a basis that our work emerges from.
Firstly, this involves a derivation of the equation used for determining the structure of compact stars; here, we will use the Tolman-Oppenheimer-Volkoff [TOV] equation. We later choose our unit system to be $c = G = 1$, so that every unit is a power of a length (also, that choice makes numerical evaluation a lot easier).
The derivation is based off assuming the star matter as a perfect/ideal fluid. The system shall further not evolve in time, therefore staying spherically symmetric. In terms of the stress-energy tensor we are left with ($c = 1$):

$$T_{\mu\nu} = (\epsilon + P)\, u_\nu u_\mu - P g_{\mu\nu} \tag{1}$$

Spherical symmetry leads to a certain form of the metric, which then gives us the stress-energy tensor omponents:

$$T_{\mu\nu} = diag(\epsilon e^{\nu(r)},\ Pe^{\lambda(r)},\ Pr^2,\ Pr^2 sin^2(\theta)) \tag{2}$$

In conclusion we get the full TOV equation:

$$\frac{dP}{dr} = \frac{(\epsilon + P)(m + 4\pi r^3 P)}{2mr - r^2} \tag{3}$$

Together with a second equation for the mass:

$$\frac{dm}{dr} = 4\pi r^2 \epsilon \tag{4}$$

2.2 Numerical solution

In order to generate an initial mass-radius relation to test our reverse algorithm with, we use a fourth order Runge-Kutta algorithm to solve the TOV equation along with the mass differential equation numerically. By looking at both equations, our ODE system is in the form

$$\dot{y}(t) = f(y(t), t) \tag{5}$$

where $\dot{y}(t)$ is a two component "vector":

$$\dot{y}(t) = \begin{pmatrix} dP/dr \\ dm/dr \end{pmatrix} \tag{6}$$

$f(y(t), t)$ then contains the right hand side of equations 3 and 4. As stated before, to create an initial mass radius relation to check the inverted algorithm with, we use a fourth order Runge-Kutta [RK4] code. The RK4 method works as follows:

$$y(t + \tau) = y(t) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{7}$$

with k_i:

$$\begin{aligned}
k_1 &= f(y(t), t) \cdot \tau \\
k_2 &= f(y(t) + k_1/2, t + \tau/2) \cdot \tau \\
k_3 &= f(y(t) + k_2/2, t + \tau/2) \cdot \tau \\
k_4 &= f(y(t) + k_3, t + \tau) \cdot \tau
\end{aligned}$$

All k_i are of course also two component "vectors".

Messung der Fadenlänge	l (m)
1	
2	
3	
4	
5	
Mittelwert $\bar{l}$	
Standardabweichung σ_l	

References

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[2] H. Kuchling, *Taschenbuch der Physik, 21. Auflage*, Fachbuchverlag Leipzig, 2014.