A Report on Application of Box Model to Predict Reservoir Sedimentation

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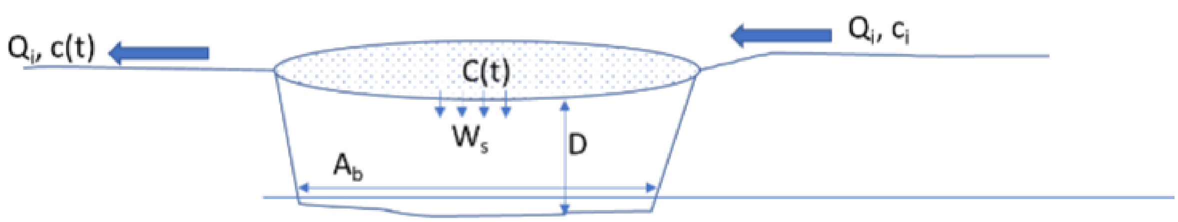
Introduction: Under different circumstances, the sediments fill the reservoir at varying speeds. This sedimentation rate is related to a reservoir's lifetime. The cumulative deposit of the incoming sediment load through a river is known as reservoir sedimentation. Both bedload and suspended loads move the sediments through a river. Sediments that roll along a river's bottom are known as bedloads, and sediments that dissolve in the water column are known as suspended loads. To calculate the reservoir's filling rate, a box model approach is used in this assignment. For that, we assume a square shaped box like area that has been taken as reservoir area. The sedimentation in this area can be carried by river (Bedload, Suspended Load) regularly and by flood in a time interval.

Theory and Numerical Methods: The Box Model concept is a model based on which reservoir trap efficiency can be calculated. This model also follows the mass balance theory. There are some assumptions in box model to analyze the trap efficiency of the reservoir sedimentation. These are:

- The water is well mixed with sediment
- Input and output water discharge is equal and is given as Qi
- The sediment concentration in the flood water is constant and given as Ci
- The volume of the reservoir is V and remain constant
- The mean water depth in the reservoir is D and remain constant
- Sediment settling velocity is Ws and remain constant
- Area of the bottom of the reservoir is Ab and remain constant



To fulfill the goal of the reservoir to calculate trap efficiency of a reservoir, we need to find the changes in the sediment concentration over time in the reservoir. For that the mass balance approach was taken into consideration with some assumptions. As the water is well mixed with sediments, the Input and output water discharge is equal and is given as Q_i . The sediment concentration(C_I) in the flood water, The volume of the reservoir(V), The mean water depth in the reservoir(D), Sediment settling velocity(W_S) and Area of the bottom of the reservoir(A_b) are constant. For mathematical simplification, parameters

such as Flashing time(T_e), Settling Time(T_s), equilibrium concentration(C_e) are also taken into account.
The equations that is used for the changes in concentration (C_t) over time is as follows:

C = Ce - (Ce - C0)*e^(-Bt)

For this test, the trap efficiency of a reservoir has been calculated assuming there is a 3 day flood.

To calculate the trap efficiency, we have to derive the following equation,

$$TE = \frac{\int_0^{t_{max}} W_s \, A_b \, C(t) \, dt}{\int_0^{t_{max}} Q_i(t) \, C_i(t) \, dt}$$

Here,

W_s = Sediment settling velocity = $\frac{D}{T_s}$

D = Mean depth of the reservoir

T_s = settling time

A_b = Area of the bottom of the reservoir

C_t $\cancel{C(t)}$ Concentration over time = $C_e - (C_e - C_0) e^{-Bt}$

Q_i = water discharge = $\frac{V}{T_f}$

$C_i(t)$ $\cancel{C(t)}$ = Sediment concentration in flood water = $C_e \times T_f \times B$

$$= C_e \times T_f \times \left(\frac{1}{T_f} + \frac{1}{T_s}\right)$$

$$= \frac{C_e (T_f + T_s)}{T_s}$$

C_e = equilibrium concentration

C_i = Initial Concentration

T_f = Flashing time

Now, we can write the equation as,

$$TE = \int_0^{t_{max}} \frac{\frac{D}{T_s} \times \frac{V}{D} \times \left[C_e - (C_e - C_o)\, e^{-Bt} \right] dt}{\left[\frac{V}{T_f} \times \frac{C_e (T_f + T_s)}{T_s} \right] dt}$$

$$= \int_0^{t_{max}} \frac{\frac{T_f}{C_e (T_s + T_f)} \times C_e \left[1 - \left(1 - \frac{C_o}{C_e}\right) e^{-Bt} \right] dt}{dt}$$

$$\cancel{= \frac{T_f}{T_f + T_s} \times \frac{\int_0^{t_{max}} dt - \int_0^{t_{max}} \left(1 - \frac{C_o}{C_e}\right) e^{-t}}{}}$$

$$= \frac{T_f}{T_f + T_s} \times \frac{\int_0^{t_{max}} dt - \int_0^{t_{max}} \left(1 - \frac{C_o}{C_e}\right) e^{-Bt}\, dt}{\int_0^{t_{max}} dt}$$

$$= \frac{T_f}{T_f + T_s} \times \frac{t_{max} - \left(1 - \frac{C_o}{C_e}\right) \int_0^{t_{max}} e^{-Bt}\, dt}{t_{max}}$$

$$= \frac{T_f}{T_f + T_s} \times \frac{t_{max} - \left(1 - \frac{C_o}{C_e}\right) \left[\frac{e^{-Bt}}{-B} \right]_0^{t_{max}}}{t_{max}}$$

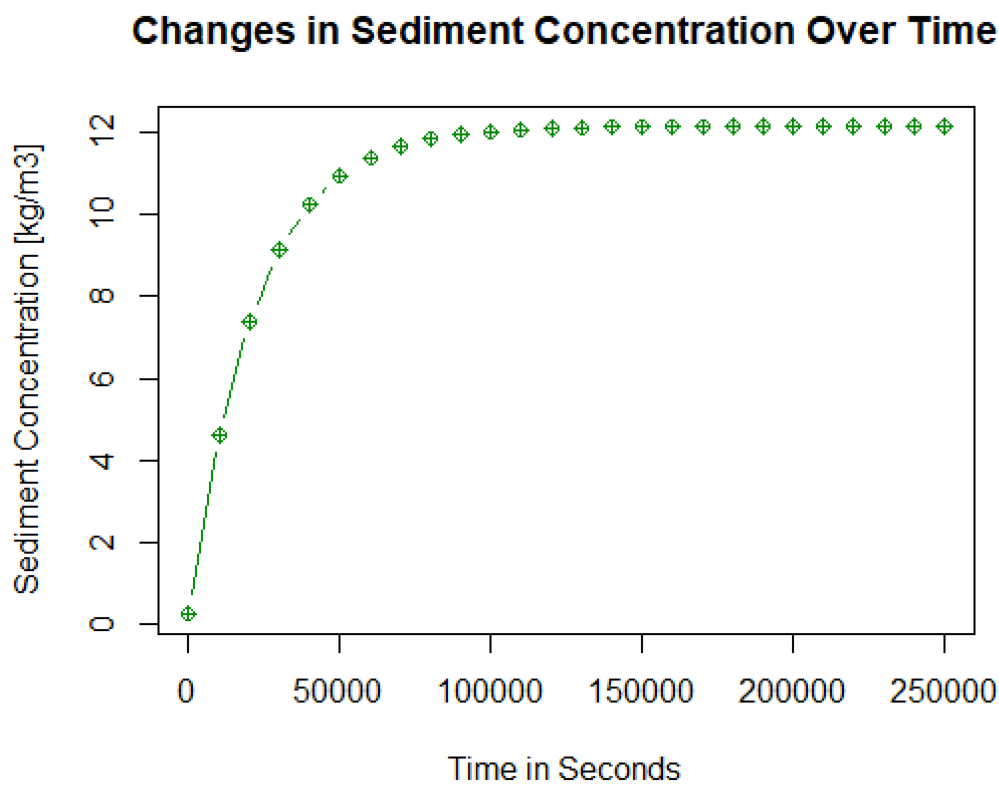
$$= \frac{T_f}{T_f + T_s} \times \frac{t_{max} - (1 - \frac{C_o}{C_e})\left(\frac{e^{-Bt_{max}} - 1}{-B}\right)}{t_{max}}$$

$$= \frac{T_f}{T_f + T_s} \times \frac{t_{max} - (1 - \frac{C_o}{C_e})\left(\frac{1 - e^{-Bt_{max}}}{B}\right)}{t_{max}}$$

$$\Rightarrow TE = \frac{T_f}{T_f + T_s} \times \frac{t_{max}\left[1 - \frac{(1 - \frac{C_o}{C_e})(1 - e^{-Bt_{max}})}{Bt_{max}}\right]}{t_{max}}$$

$$\therefore TE = \frac{T_f}{T_f + T_s} \left\{1 - \left(\frac{1 - \frac{C_o}{C_e}}{Bt_{max}}\right)\left(1 - e^{-Bt_{max}}\right)\right\}$$

Result: The changes in sediments concentration over 3 days is shown in the graph below:

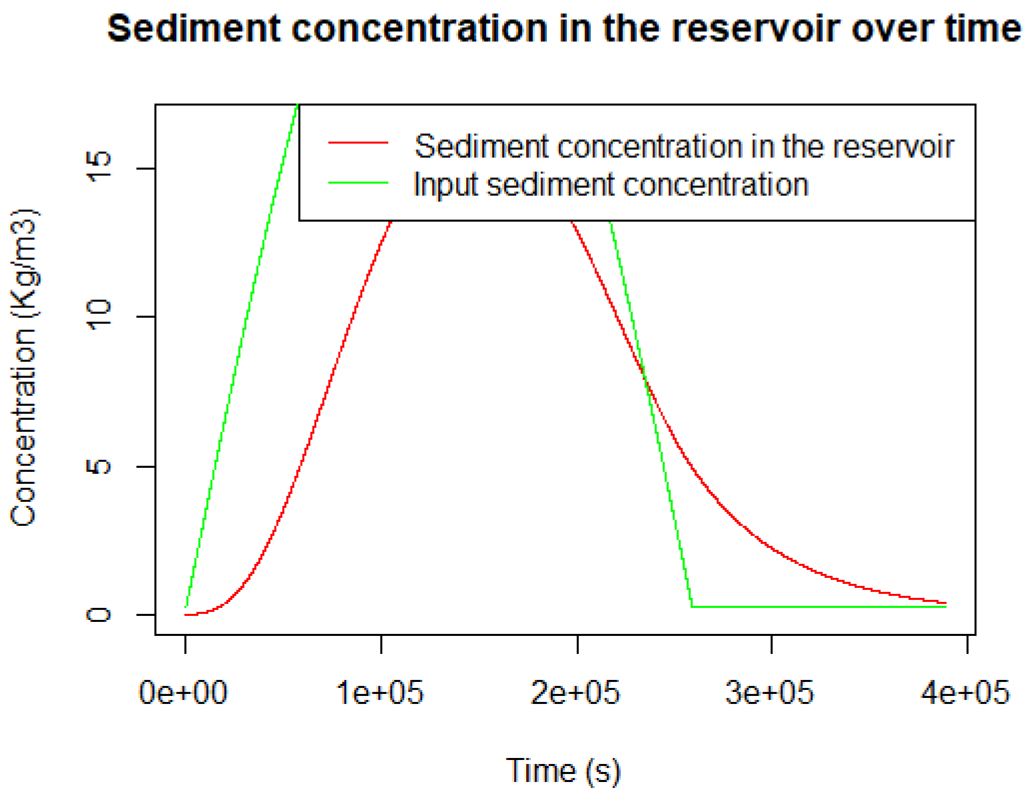


Here in the graph we can see that the concentration of sediments increases gradually till 100000 seconds or 27 hours approximately. Then the trend slowed down and the concentration of sediments was at a constant rate.

As per the given parameters, we found 0.36036 or 36.036% of trapping efficiency of the reservoir. It means that after 3 days 36.036% of the sediments were transported into the reservoir during a flood.

The table here shows the values used to calculate the changes in sedimentation over 3 days and the trapping efficiency of the reservoir for a given time period in R language.

Values	
B	4.57444444444445e-05
C	num [1:26] 0.265 4.626 7.386 9.133 10.239 …
C0	0.265
Ce	12.1447656060238
Ci	20
d	1e-05
D	5
g	9.8
i	26L
n	0.001
Pp	2650
Pw	1000
Qi	400
r	5e-06
r2	2.5e-11
t	num [1:26] 0 10000 20000 30000 40000 50000…
TE	0.360359746836682
Tf	36000
tmax	259200
Ts	55658.6270871985
V	14400000
Ws	8.98333333333333e-05



This graph shows the sediment concentration in reservoir and input of sediment due to a 3 day flood. The computation time has been taken 4.5 days because of the sediment settling time. The graph also shows that, there is a difference between input sediment concentration and reservoir sediment concentration. The equilibrium concentration is approximately 2.5e+05 seconds after the beginning of the sedimentation due to flood.

Conclusion: As we've shown, the box model technique, given certain presumptions, is a useful model for determining the sedimentological possibilities. The box model method will not meet our needs if these presumptions are ignored. In this instance, we calculated the reservoir's trapping effectiveness for a certain amount of time as well as the variations in sedimentation over the course of three days flooding event.

Appendix 1:

```
#Given Values are
D = 5 #Mean Water Depth in m
V = 1.44*10^7 #Reservoir Volume in m3
d = 0.00001 #Diameter of Sediments in m
r = d/2 #Radius of Sediments in m
r2= r*r
Pw= 1000 #Density of Water in kg/m3
Pp= 2650 #Density of Sediments in kg/m3
n = 0.001 #Dynamic Viscosity in kg/m-s
g = 9.8 #Gravitational Acceleration in m/s2
tmax = 3*24*60*60 #Duration of flood in seconds
Qi= 400 #River Discharge in m3/s
Ci= 20 #Average Sediment Concentration in flood water in kg/m3
C0= 0.265 #Initial Concentration of Sediments in kg/m3
#Now we Need do determine some parameters using the given values to calculate the
changes in
Ws=(2*r2*(Pp-Pw)*g)/(9*n) #sediment concentration over time
Tf= V/Qi #Flashing time
Ts= D/Ws #Settling time
B = {(1/Tf)+(1/Ts)}
Ce= (Ci/(Tf*B)) #Equilibrium Concentration
t=seq(0,3*24*60*60,by=10000)
C=rep(NA,length(t))
for (i in 1:length(t))
  C[i]=Ce-{(Ce-C0)*exp(-B*t[i])}
plot(t,C,type = "b",pch=10,col="green4",main="Changes in Sediment Concentration
Over Time",
     xlab ="Time in Seconds",ylab = "Sediment Concentration [kg/m3]")
#Now we need to Determine the trap efficiency
TE=(Tf/(Tf+Ts))*(1-((1-C0/Ce)/(B*tmax))*(1-exp(-B*tmax)))
```

Appendix 2:

```
# Purpose of Program & Origin
# This script solves the box model equation for the sediment concentration
# created by a sinusoidal flood entering a shallow lake or reservoir.

# Define Model Parameters
gravity <- 9.8  # Acceleration of gravity (m/s)
V <- 1.44e7  # Volume of the reservoir (cubic meters)
D <- 5  # Depth of the reservoir (meters)
Ds <- 0.00001  # Sediment diameter (meters)
waterdensity <- 1000  # Water density (kg/cubic meter)
seddensity <- 2650  # Sediment density (kg/cubic meter)
```

```
viscosity <- 0.001  # Viscosity of water (kg/m/s)
flooddurationdays <- 3  # Duration of the flood (days)
Qmaxoverqi <- 20  # Ratio of maximum water discharge to initial water discharge during
sinusoidal flood (cubic meters per second)
initialdischarge <- 22  # Water discharge at the beginning of the flood (cubic meters per
second)
cmaxoverci <- 100  # Ratio of maximum sediment input concentration to initial input
sediment concentration during sinusoidal flood (kg/cubic meter)
initialci <- 0.0001 * seddensity  # Initial input sediment concentration (kg/cubic meter)

# Convert Units
flooddursec <- flooddurationdays * 24 * 3600

# Parameters that Determine Accuracy
lengthofsimulationdays <- 4.5  # Duration of computations, in days

lengthofsimsec <- lengthofsimulationdays * 24 * 3600  # Convert to seconds
numberoftimesteps <- 5200
dt <- lengthofsimsec / (numberoftimesteps - 1)  # Time step in seconds

# Perform Preliminary Computations
settlingvelocity <- (gravity * (seddensity - waterdensity) / (18 * viscosity)) * (Ds^2)
Ab <- V / D  # Surface area of reservoir bottom (square meters)
Ts <- D / settlingvelocity  # Settling time scale (seconds)

# Preallocate arrays to store Qi, ci, c, and t
Qi <- rep(0, numberoftimesteps)  # Input water discharge (cubic m/s)
ci <- rep(0, numberoftimesteps)  # Input sediment concentration
c <- rep(0, numberoftimesteps)  # Sediment concentration in reservoir
t <- rep(0, numberoftimesteps)  # Time

# Define initial values of specified parameters
Qi[1] <- initialdischarge
ci[1] <- initialci
Tf <- V / Qi[1]
B <- (1 / Tf) + (1 / Ts)
ce <- ci[1] / (Tf * B)
c[1] <- ce
t[1] <- 0

# Loop over time computing c at each time step
for (i in 2:numberoftimesteps) {
  t[i] <- dt * i  # Determine the "time"

  # Compute flood discharge and concentration
  if (t[i] <= flooddursec) {  # Sinusoidal flood
```

```
    Qi[i] <- Qi[1] * (1 + Qmaxoverqi * sin(2 * pi * t[i] / (2 * flooddursec)))
    ci[i] <- ci[1] * (1 + cmaxoverci * sin(2 * pi * t[i] / (2 * flooddursec)))
  } else {  # Constant discharge and concentration after the flood
    Qi[i] <- initialdischarge
    ci[i] <- initialci
  }

  # Compute time-varying parameters Tf, ce
  Tf[i] <- V / Qi[i]
  B[i] <- (1 / Tf[i]) + (1 / Ts)
  ce[i] <- ci[i] / (Tf[i] * B[i])

  # Compute c(i) using the implicit finite difference approximation
  c[i] <- (((t[i] - t[i - 1]) * B[i] * ce[i]) + c[i - 1]) / (1 + (t[i] - t[i - 1]) * B[i])
}

# Plot Sediment Concentration and Input Concentration Over Time
plot(t, c, type = "l", col = "red", xlab = "Time (s)", ylab = "Concentration (Kg/m3)", main =
"Sediment concentration in the reservoir over time")
lines(t, ci, col = "green")
legend("topright", legend = c("Sediment concentration in the reservoir", "Input sediment
concentration"), col = c("red", "green"), lty = 1)

# Loop over time to compute sediment deposited and supplied during each time step
totalsedimentmasssupplied <- 0  # in kg
totalsedimentmassdeposited <- 0  # in kg
for (i in 2:numberoftimesteps) {
  # Compute sediment supplied during the current time step
  sediment_supplied_during_current_time_step <- Qi[i] * ci[i] * (t[i] - t[i - 1])

  # Compute sediment deposited during the current time step
  sediment_deposited_during_current_time_step <- settlingvelocity * Ab * c[i] * (t[i] - t[i -
1])

  # Update total sediment masses
  totalsedimentmasssupplied <- sediment_supplied_during_current_time_step +
totalsedimentmasssupplied
  totalsedimentmassdeposited <- sediment_deposited_during_current_time_step +
totalsedimentmassdeposited
}

# Compute Trap Efficiency
trapefficiency <- totalsedimentmassdeposited / totalsedimentmasssupplied
```