Assignment on 1-D Diffusion Groundwater Flow

Course Name: Computational Geology Lab

GML 505

Submitted By,

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Session: 2022-23

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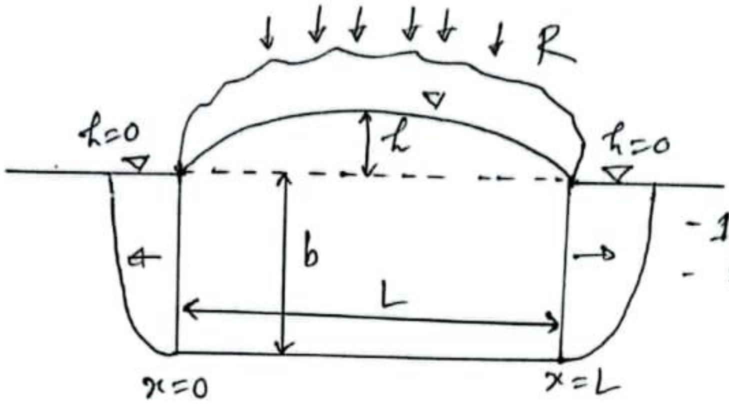
1-D Diffusion Groundwater Flow



- 1 Directional
- steady state flow

fig: Cross Section view of an island

Here, R is the Recharge
h is the hydraulic head
b is the thickness of the aquifer
L is the length of the island

we know,

$$T\frac{d^2h}{dx^2}+R=0 \quad \text{(steady state condition)}$$

$$\Rightarrow \frac{d^2h}{dx^2}=-\frac{R}{T}$$

$$\Rightarrow d^2h=-\frac{R}{T}dx^2$$

$$\Rightarrow \iint d^2h=-\frac{R}{T}\int dx\int dx$$

$$\Rightarrow \int dh=-\frac{R}{T}\int dx\,(x+c_1)$$

$\Rightarrow \int dh = -\frac{\rho}{T}\left(\int x\,dx + c_1\int dx\right)$

$\Rightarrow h = -\frac{\rho}{T}\left(\frac{x^2}{2} + xc_1 + c_2\right)$

NOW, the boundary condition is

$x=0,\ h=0$

$x=L,\ h=0$

When, $x=0, h=0$

$\therefore 0 = -\frac{\rho}{T}\cdot\frac{0^2}{2} + (c_1\times 0) + c_2$

$\Rightarrow c_2 = 0$

When, $x=L,\ h=0$

$0 = -\frac{\rho}{T}\left(\frac{L^2}{2} + Lc_1 + c_2\right)$

$\Rightarrow Lc_1 = -\frac{L^2}{2}$

$\Rightarrow c_1 = \frac{L}{2}$

$\therefore h = -\frac{\rho}{T}\left(\frac{x^2}{2} - \frac{L}{2}x + 0\right)$

$\Rightarrow h = -\frac{\rho x^2}{2T} + \frac{\rho L x}{2T}$

$\Rightarrow h = \frac{\rho}{2T}(Lx - x^2)$ A.

Numerical Solution:

1D- steady state condition.

$\therefore \frac{d^2h}{dx^2} = 0$

$h=0$ R = Recharge $h=0$

h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}	h_{11}

For, h_1

$$h_2 - 2h_1 + h_0 = -dx^2 \frac{R}{T}$$

For, $h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}$ below

$$h_1 - 2h_2 + h_3 = -dx^2 \frac{R}{T}$$

$$h_2 - 2h_3 + h_4 = -dx^2 \frac{R}{T}$$

$$h_3 - 2h_4 + h_5 = -dx^2 \frac{R}{T}$$

$$h_4 - 2h_5 + h_6 = -dx^2 \frac{R}{T}$$

$$h_5 - 2h_6 + h_7 = -dx^2 \frac{R}{T}$$

$$h_6 - 2h_7 + h_8 = -dx^2 \frac{R}{T}$$

$$h_7 - 2h_8 + h_9 = -dx^2 \frac{R}{T}$$

$$h_8 - 2h_9 + h_{10} = -dx^2 \frac{R}{T}$$

$$h_9 - 2h_{10} + h_{11} = -dx^2 \frac{R}{T}$$

Now organize them for matrix visualization

$$-2h_1 + h_2 = -dx^2 \cdot \frac{P}{T} - h_0$$

$$h_1 - 2h_2 + h_3 = -dx^2 \frac{P}{T}$$

$$h_2 + 2h_3 + h_4 = -dx^2 \frac{P}{T}$$

$$h_3 - 2h_4 + h_5 = -dx^2 \frac{P}{T}$$

$$h_4 - 2h_5 + h_6 = -dx^2 \frac{P}{T}$$

$$h_5 - 2h_6 + h_7 = -dx^2 \frac{P}{T}$$

$$h_6 - 2h_7 + h_8 = -dx^2 \frac{P}{T}$$

$$h_7 - 2h_8 + h_9 = -dx^2 \frac{P}{T}$$

$$h_8 - 2h_9 + h_{10} = -dx^2 \frac{P}{T}$$

$$h_9 - 2h_{10} \cancel{+h_{11}} = dx^2 \frac{P}{T} - h_{11}$$

$$\therefore \begin{bmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 \end{bmatrix} \times \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \\ h_{10} \end{bmatrix} = \begin{bmatrix} -dx^2 \frac{P}{T} - h_0 \\ -dx^2 \frac{P}{T} \\ -dx^2 \frac{P}{T} \\ -dx^2 \frac{P}{T} \\ -dx^2 \frac{P}{T} \\ -dx^2 \frac{P}{T} \\ -dx^2 \frac{P}{T} \\ -dx^2 \frac{P}{T} \\ -dx^2 \frac{P}{T} \\ -dx^2 \frac{P}{T} - h_{11} \end{bmatrix}$$

$$\therefore AX = B \text{, where, } X = A^{-1}B$$

Here,
Recharge, $R = 0.002$ m/d
Transmissivity, $T = 1500$ m²/d
Length, $L = 5000$ m

$$\therefore dx = \frac{L}{10} = \frac{5000\text{ m}}{10} = 500\text{ m}$$

$$\therefore dx^2 = (500)^2 = 250000$$

$$\therefore -dx^2 \frac{R}{T} = \frac{-250000 \times 0.002}{5000} = -0.3333$$

From the Matrix solution in excel we found the $h_1, h_2 \ldots h_{10}$ is given below

$h_1 = 1.66667$ m
$h_2 = 3$ m
$h_3 = 4$ m
$h_4 = 4.6667$ m
$h_5 = 5$ m
$h_6 = 5$ m
$h_7 = 4.6667$ m
$h_8 = 4$ m
$h_9 = 3$ m
$h_{10} = 1.6667$ m.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	**Parameters**						dx^2(R/T)-h0	-0.33333							
2	**Recharge**	0.002	m/d				dx^2(R/T)	-0.33333							
3	**Transmissivity**	1500	m^2/d				dx^2(R/T)	-0.33333							
4	**Length**	5000	m	**dx^2**			dx^2(R/T)	-0.33333							
5	**dx**	5000/10	500	250000			dx^2(R/T)	-0.33333							
6							dx^2(R/T)	-0.33333							
7							dx^2(R/T)	-0.33333							
8							dx^2(R/T)	-0.33333							
9							dx^2(R/T)	-0.33333							
10							dx^2(R/T)-h11	-0.33333							
11															
12															
13	**Coefficient Matrix**														
14	-2	1	0	0	0	0	0	0	0	0		h1		-0.33333	
15	1	-2	1	0	0	0	0	0	0	0		h2		-0.33333	
16	0	1	-2	1	0	0	0	0	0	0		h3		-0.33333	
17	0	0	1	-2	1	0	0	0	0	0		h4		-0.33333	
18	0	0	0	1	-2	1	0	0	0	0	X	h5		-0.33333	
19	0	0	0	0	1	-2	1	0	0	0		h6		-0.33333	
20	0	0	0	0	0	1	-2	1	0	0		h7		-0.33333	
21	0	0	0	0	0	0	1	-2	1	0		h8		-0.33333	
22	0	0	0	0	0	0	0	1	-2	1		h9		-0.33333	
23	0	0	0	0	0	0	0	0	1	-2		h10		-0.33333	
24															
25															
26		**Inverse Coefficient Matrix**												**Solution**	
27	-0.909090909	-0.81818	-0.72727	-0.63636	-0.54545	-0.45455	-0.363636364	-0.27273	-0.18182	-0.09091		-0.33333	**h1**	**1.666667**	
28	-0.818181818	-1.63636	-1.45455	-1.27273	-1.09091	-0.90909	-0.727272727	-0.54545	-0.36364	-0.18182		-0.33333	**h2**	**3**	
29	-0.727272727	-1.45455	-2.18182	-1.90909	-1.63636	-1.36364	-1.090909091	-0.81818	-0.54545	-0.27273		-0.33333	**h3**	**4**	
30	-0.636363636	-1.27273	-1.90909	-2.54545	-2.18182	-1.81818	-1.454545455	-1.09091	-0.72727	-0.36364		-0.33333	**h4**	**4.666667**	
31	-0.545454545	-1.09091	-1.63636	-2.18182	-2.72727	-2.27273	-1.818181818	-1.36364	-0.90909	-0.45455	X	-0.33333	**h5**	**5**	
32	-0.454545455	-0.90909	-1.36364	-1.81818	-2.27273	-2.72727	-2.181818182	-1.63636	-1.09091	-0.54545		-0.33333	**h6**	**5**	
33	-0.363636364	-0.72727	-1.09091	-1.45455	-1.81818	-2.18182	-2.545454545	-1.90909	-1.27273	-0.63636		-0.33333	**h7**	**4.666667**	
34	-0.272727273	-0.54545	-0.81818	-1.09091	-1.36364	-1.63636	-1.909090909	-2.18182	-1.45455	-0.72727		-0.33333	**h8**	**4**	
35	-0.181818182	-0.36364	-0.54545	-0.72727	-0.90909	-1.09091	-1.272727273	-1.45455	-1.63636	-0.81818		-0.33333	**h9**	**3**	
36	-0.090909091	-0.18182	-0.27273	-0.36364	-0.45455	-0.54545	-0.636363636	-0.72727	-0.81818	-0.90909		-0.33333	**h10**	**1.666667**	
37															

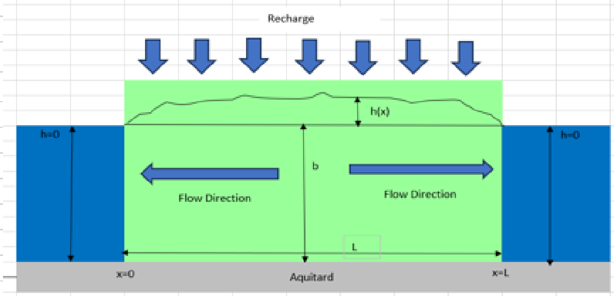


Figure 1 Matrix Solution by Excel

1-D Diffusion solution by R

Analytical Solution of the 1-D diffusion groundwater flow:

```
R=0.002 # Recharge is m/d
T=1500 #Transmissivity is m^2/d
L=5000 #Length is m
dx=500 #small distance divided by the total cell number
x=seq(from=0, to=L, by=dx)
h=R*(L*x-x^2)/(2*T)
plot(x,h, type="l",col="red",lwd=3, main = "Analytical Solution for island problem", xlab =
    "Horizontal distance", ylab = "Water Head")
```

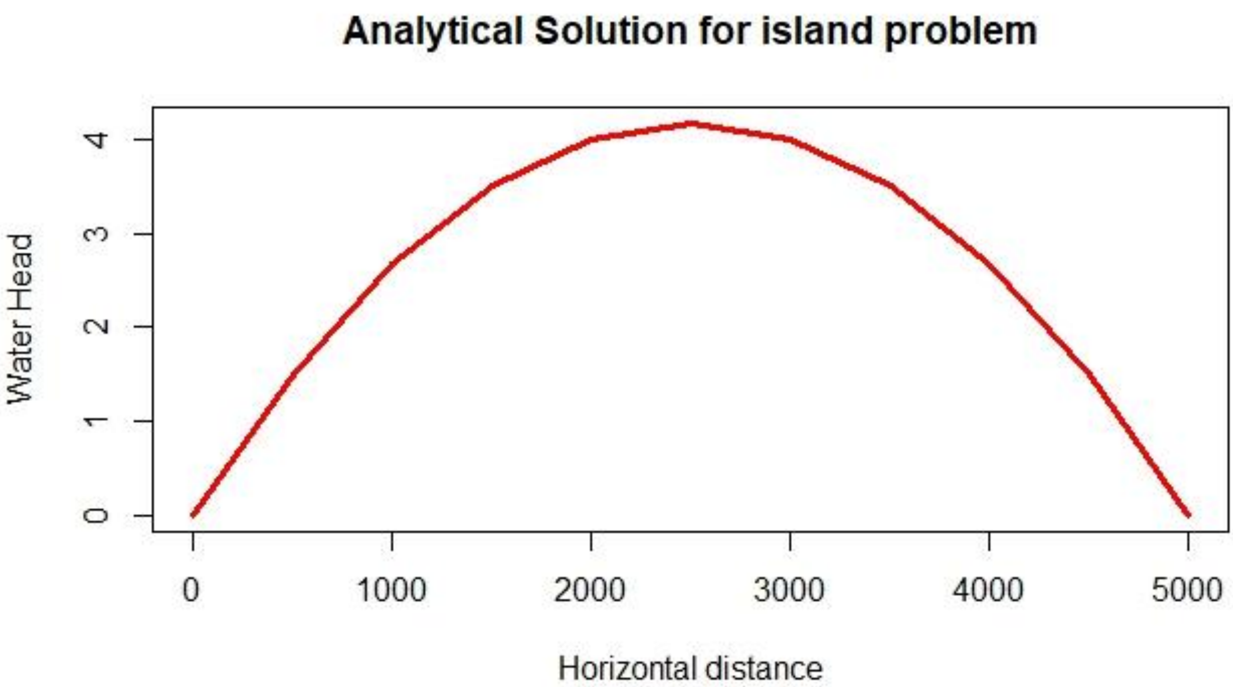


Figure 2 Analytical Solution Graph

Numerical Solution of the 1-D diffusion groundwater flow:

```
z=10 #number of equations
A = matrix(c(rep(c(-2,1,rep(0, times=z-2),1), times=z-1),-2),
        nrow = z, ncol = z,byrow = TRUE)
a=inv(A) #used by matlib package
R=0.002
T=1500
L=5000
dx=L/(z+1)
x=seq(from=0, to=L, by=dx)
p=-dx^2*(R/T)
a1=a*p
h1=rowSums(a1)
h=c(0,h1,0)
plot(x,h,type="l",col="red",lwd=3, main = "Numerical Solution for island problem", xlab =
    "Horizontal distance", ylab = "Water Head")
```

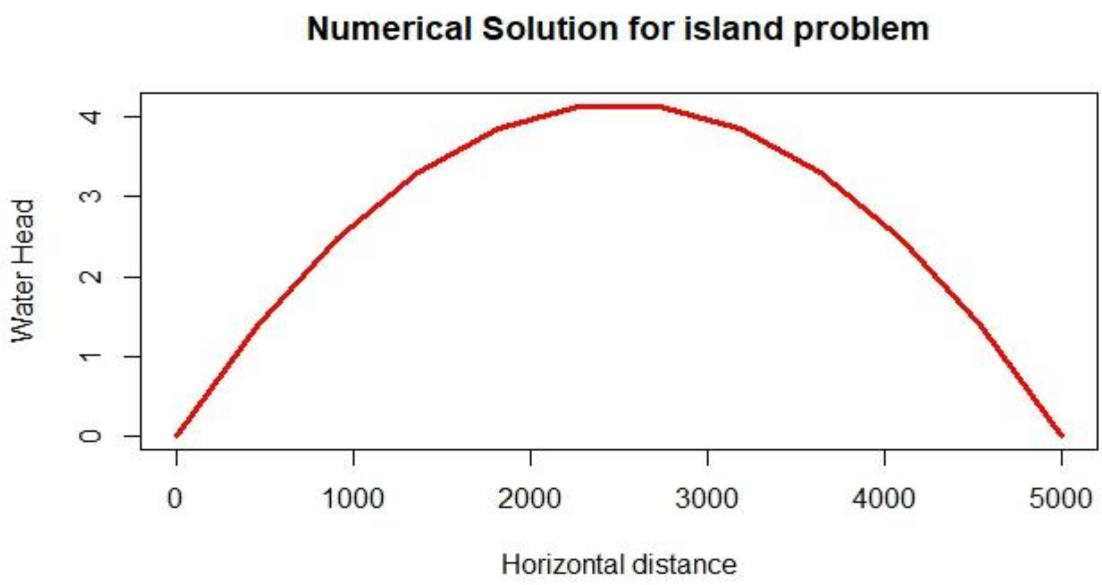


Figure 3 Numerical Solution Graph

Numerical Solution for 5th equation of the 1-D diffusion groundwater flow:

```
z=10 #number of equations
s=5 #discharge equation no
A = matrix(c(rep(c(-2,1,rep(0, times=z-2),1), times=z-1),-2),
      nrow = z, ncol = z,byrow = TRUE)
a=inv(A) #requires matlib package
R=0.002 #recharge
R1=-0.001 #discharge
T=1500 #
L=5000
dx=L/(z+1)
x=seq(from=0, to=L, by=dx)
p=-dx^2*(R/T)
p1=-dx^2*(R1/T)
q=c(rep(p, times=z))
Q=matrix(q, nrow = z, ncol = 1,byrow = TRUE)
Q[s,1]=p1
h1=a%*%Q
h=c(0,h1,0)
plot(x,h,type="l",col="red",lwd=3, main = "Numerical Solution for island problem", xlab =
    "Horizontal distance", ylab = "Water Head")
```

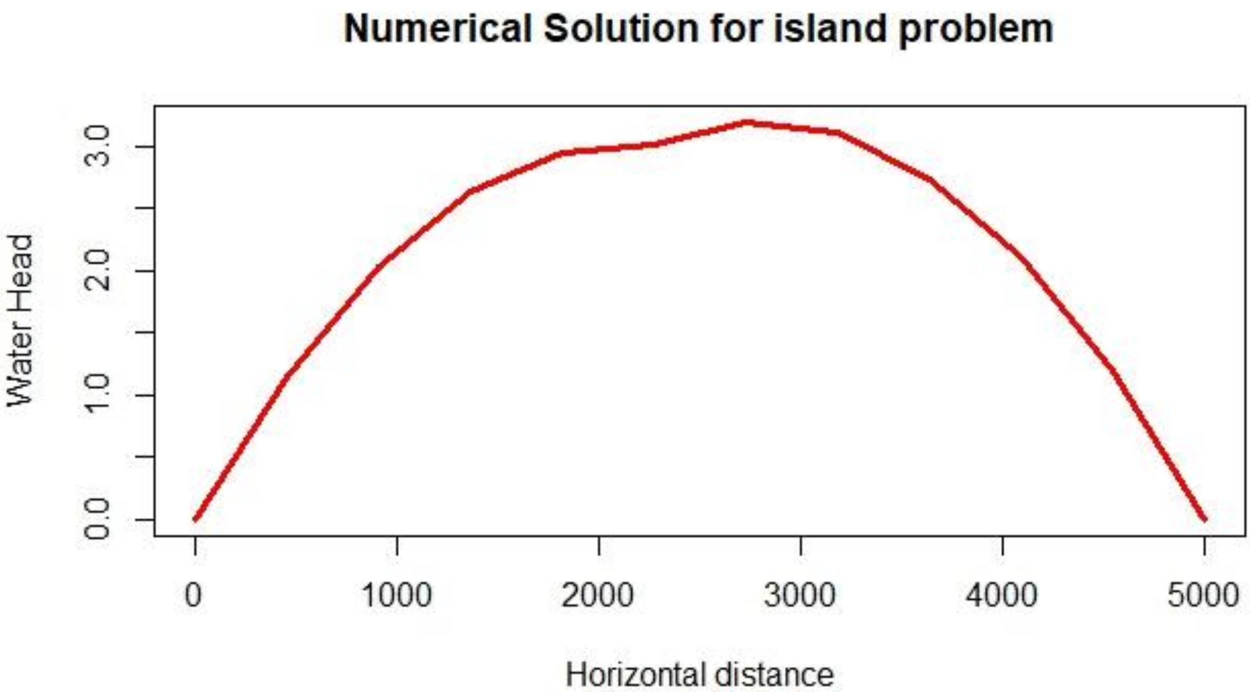


Figure 4 Numerical Solution graph of 5th Equation