MATCHED MEDIAN FILTER FOR DETECTING QAM SIGNALS

Qin Liu, Jaakko Astola, and Yrjö Neuvo

Signal Processing Laboratory
Tampere University of Technology
P. O. Box 553, SF–33101 Tampere, Finland

Abstract – The complex Matched Median Filter (MMF) is developed in this paper for QAM signal detection under the maximum likelihood (ML) criterion. It is shown that the MMF has the so-called robustness property similar to median filters in case of impulsive noise. Based on the combination of the MMF and the Linear Matched Filter (LMF), extended matched filters are also introduced. These filters have properties of both MMF and LMF and behave well in a varying environment. Computer simulations demonstrate that the proposed detectors give a smaller detection error probability than the LMF when impulsive noise is present.

I. INTRODUCTION

When a QAM signal is transmitted, the whole system is mathematically equivalent to a baseband PAM system [4,5], but with complex impulse responses and information symbols. Suppose the simplified QAM transmission model is as follows (Fig. 1),

$$r_i = as_i + w_i, \qquad i = 1, ..., N. \tag{1.1}$$

where r_i, a, w_i, s_i are complex numbers [4]. r_i is the contaminated signal received, w_i is the additive white noise, s_i is the impulse response of the linear baseband channel, a is the transmitted signal symbol. Our goal is to estimate a from r_i, given s_i.

developed to overcome this problem, such as, the soft-limiter [3,6] for the ε-contaminated Gaussian noise. Unfortunately, we seldom know either the exact percentage or the parameters of the nominal distribution in practice. Sometimes the noise characteristics is time-varying. All of these make it difficult to obtain the best soft-limiter.

It is well known that median filters show particularly good performance when noise is of impulsive type. In [1], Astola *et al* defined a class of median type filters called MMFs for the estimation of the amplitude of a noisy signal. The well known robustness of median type filters guarantees that the MMFs are insensitive to noise characteristics so that they lead to a robust detection scheme performing well when strong impulsive noise corrupts the signal.

In this paper, we approach the QAM detection problem from the viewpoint of complex MMF. In the following, first we give a brief review of the matched median filter concept. Then we define the two dimensional MMF, discuss its robustness property, and propose two algorithms to calculate the output of the MMF. Also, we introduce a class of extended matched filters based on the MMF and the LMF. Finally, we present computer simulations

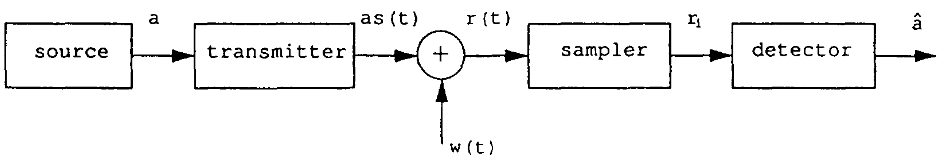

Fig. 1. *The baseband communication model.*

to demonstrate the effectiveness of the MMF and its extensions in the detection of QAM signals.

Usually stationary white Gaussian noise is assumed in the analysis of communication systems. For this kind of noise model, the optimum detector maximizing the detection probability (also the LMF maximizing the output SNR) is given by [4],

$$\hat{a}_l = \frac{\sum_{i=1}^{N} r_i s_i^*}{\sum_{i=1}^{N} s_i s_i^*} \tag{1.2}$$

where $*$ denotes complex conjugate.

However, this traditional LMF is very sensitive to impulsive noise, which occurs quite often in communication systems [5]. Various robust techniques have been then

II. MATCHED MEDIAN FILTERS

Consider the real baseband communication model of Fig. 1. Suppose that the real number a is to be communicated over the channel. Assume that the pulse $as(t)$ is then corrupted by additive biexponential white noise w with probability density function

$$f(w) = \frac{\beta}{2} e^{-\beta|w|} \tag{2.1}$$

 92CH3132-8/0000-0805 $3.00

After being sampled, the received pulse is

$$r_i = as_i + w_i, \qquad i = 1, ..., N. \tag{2.2}$$

Unlike that in (1.1), r, a, s and w here are all real numbers. Let us suppose that $s_i \neq 0$ only for $i = 1, ..., N$. Now, for a fixed i, r_i is a biexponentially distributed random variable with mean as_i. By the whiteness assumption, random variables $r_i - as_i$ are independent with identical biexponential distributions. Thus the likelihood function for a corresponding to the reception of this sequence is

$$L(a) = \alpha \prod_{i=1}^{N} e^{-\beta|r_i - as_i|} \tag{2.3}$$

where α and β are constants. This implies that the maximum likelihood estimate for a is the value $\hat{a}$ minimizing

$$\sum_{i=1}^{N} |r_i - \hat{a}s_i| \tag{2.4}$$

Writing this sum as

$$\sum_{i=1}^{N} |s_i||\hat{a} - \frac{r_i}{s_i}| \tag{2.5}$$

Thus we get the matched median filter which is matched to the pulse shape s_i, $i = 1, ..., N$ for the input signal r_i by specifying its output as the value $\hat{a}$ minimizing (2.5). The fact that the minimizing value is always one of the input samples gives us a simple method to compute the output of the matched median filter using comparisons. In fact, the output of the matched median filter is the weighted median of normalized received signal values and that the weights come from the pulse shape. The well known robustness property of median type filter makes it possible that the matched median filter is insensitive to impulsive noise. In the following section, we extend the concept of matched median filter to the baseband QAM detection, which is a complex signal detection problem indeed.

III. MATCHED MEDIAN FILTER AS DETECTOR FOR QAM SIGNALS

3.1. Complex matched median filters and their properties

Rewrite the QAM transmission model,

$$r_i = as_i + w_i, \qquad i = 1, ..., N. \tag{3.1}$$

Suppose noise w_i comes from the circularly symmetric distribution,

$$f(u, v) = \rho e^{-\beta|(u,v)-(u_0,v_0)|} \tag{3.2}$$

u, v are the real and the imaginary part of the noise ($u + jv = w$), respectively, ρ, β are constants and the center (u_0, v_0) of the distribution is at the origin $(0, 0)$. $|.|$ denotes the Euclidean distance. For convenience, the complex number $x + jy$ is also denoted by a vector (x, y).

By the whiteness assumption, the log likelihood function of a is

$$L(a) = \sum_{i=1}^{N} |r_i - as_i| = \sum_{i=1}^{N} |s_i||a - g_i| = L(x, y) \tag{3.3}$$

where $r_i/s_i = g_i = (x_i, y_i)$ $(i = 1, ..., N)$, $a = (x, y)$. Thus the ML estimate of a, the output of the two dimensional MMF, is the complex value minimizing $L(x, y)$.

Apparently, the output of complex MMF is not necessarily one of the input samples compared with the scalar MMF. In subsection *3.2*, we will discuss the calculation of it. First, let us consider some of the properties of the minimization function $L(x, y)$. It can be proven that $L(x, y)$ has the following two properties:

Property 1. $L(x, y)$ is a linear combination with positive coefficients of convex functions $|a - g_i|$ and thus convex.

Property 2. Let $O : (x_o, y_o)$ be a minimum point for $L(x, y)$. If point $A : (x_N, y_N)$ slides along the line $\overline{OA}$, O will still maintain its minimal property.

The second property demonstrates the robustness of MMF while the first property shows the uniqueness of the optimal solution of MMF. However, the minimum solution of (3.3) cannot be found in a closed form. In the following subsection, two algorithms are proposed for finding the solution.

3.2. The Calculation of Matched Median Filter Output

A. Vector Median Method

This algorithm is adopted directly from vector median (VM) [2], i.e., we estimate a by choosing one of (x_i, y_i) $(i = 1, ..., N)$ which minimizes (3.3). Although this algorithm does not give us the exact ML estimate, it is easy to implement (Fig.2).

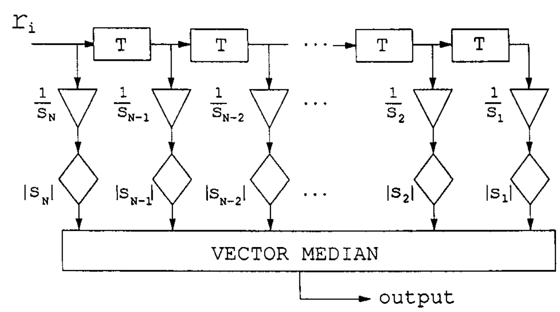

Fig. 2. *Vector Median Method.*

B. Iterative Algorithm

Since $L(x, y)$ is not a continuously differentiable function at the point (x_i, y_i) $(i = 1, .., N)$, the conventional gradient methods can not be used to get the optimal solution. On the other hand, $L(x, y)$ is a convex function as *Property 1* shows. Thus various search methods in optimization [8,9] can be adopted to get the exact output of MMF.

If the actual minimum point of Eq. (3.3) is chosen as the output, the filtering operation defines the optimal matched median filter (OMMF). It is optimal in the sense that it gives the ML estimate for a of the transmission model (3.1), (3.2).

3.3. Extended Matched Filters

Combining the VM output with the LMF output in the following way,

$$\hat{a}_{E_0} \in \{\hat{a}_{VM}, \hat{a}_l\} \tag{3.4}$$

which minimizes (3.3). Here $\hat{a}_{VM}$ is the VM output. Actually, because the point is restricted to the minimal function (3.3), this extended algorithm can only lead to a solution much closer to the minimum point of (3.3) than the VM method.

As we know, the LMF is based on the following minimization function

$$\sum_{i=1}^{N} |s_i|^2 |\hat{a}_l - g_i|^2 \tag{3.5}$$

while the MMF is based on

$$\sum_{i=1}^{N} |s_i||\hat{a} - g_i| \tag{3.6}$$

It is easy to verify

$$\sqrt{\sum_{i=1}^{N} |s_i|^2 |\hat{a} - g_i|^2} \le \sum_{i=1}^{N} |s_i||\hat{a} - g_i| \le \sqrt{N}\sqrt{\sum_{i=1}^{N} |s_i|^2 |\hat{a} - g_i|^2} \tag{3.7}$$

Now consider another kind of extension (Fig.3)

$$\hat{a}_{E_\xi} \in \{\hat{a}_{VM}, \hat{a}_l\} \tag{3.8}$$

which minimizes function

$$F(\hat{a}) = \sum_{i=1}^{N} |s_i||\hat{a} - g_i| + \sqrt{\xi}\sqrt{\sum_{i=1}^{N} |s_i|^2 |\hat{a} - g_i|^2} \tag{3.9}$$

The square root of (3.9) makes the squared part and the absolute part comparable. ξ is usually chosen between 1 and N as inequality (3.7) indicates with its corresponding extension called as E_ξ (e.g., E_1 for $\xi = 1$). In this way, the output of the corresponding LMF is included in the set of possible outputs so that we can expect to obtain a better estimate if the corrupting noise contains both Gaussian and impulsive components. This conjecture is confirmed by our computer simulations.

Although the detection scheme we discussed is based on the transmission model (3.1) and (3.2), simulations in the following section show that it is a robust detection scheme performing well also for ε-Gaussian noise when contaminated Gaussian noise corrupts the signal.

IV. DETECTION SIMULATION

The theoretical detection error probability of the proposed methods is difficult to analyze. Instead, computer simulations are conducted to evaluate the performance of them in this section.

Without losing generality we consider the following 16-point QAM signal detection in our simulations. Here, signal $1 + j1$ is transmitted, and the correct detection is that the output $\hat{a}_1 + j\hat{a}_2$ of the filter should satisfy $0 \le \hat{a}_1 < 2$, $0 \le \hat{a}_2 < 2$ (Fig.4).

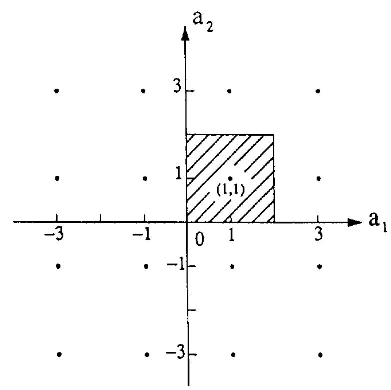

Fig. 4. *16-point QAM constellation. Points fall in the dark area are correct detection if (1,1) was transmitted.*

The simulation results are obtained from 1000000 experiments of various percentage of impulsive noise. The pulse carrying the symbol was assumed raised cosine. For each realization, we have 11 samples (i.e. N=11). The noise is white and modeled as a mixture of Gaussian and truncated Cauchy distribution of maximum value equal to 2000. Signal to noise ratio is defined as $10log(\Delta/\sigma)$ (dB), Δ is the energy of signal (as_i), σ is the noise variance.

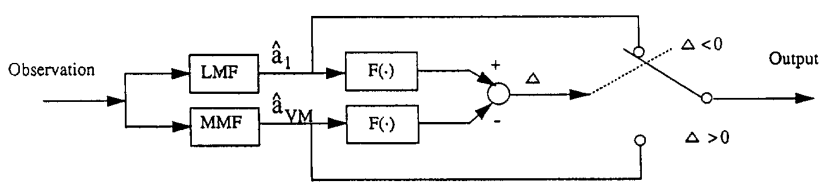

Fig. 3. *Extended Filters.*

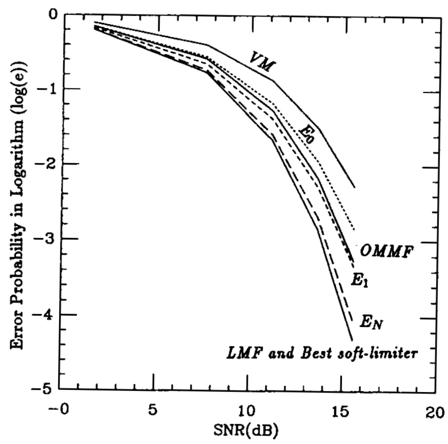

Fig. 5. *Gaussian Noise.*

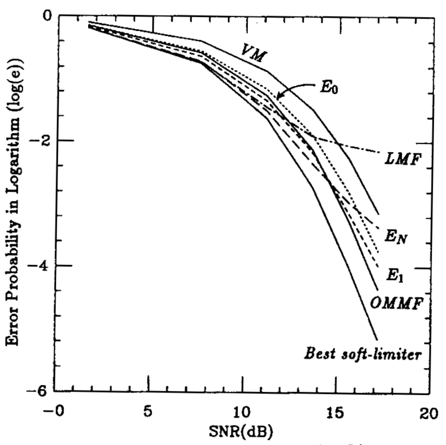

Fig. 6. *Contam. Gaussian with* 1% *truncated Cauchy.*

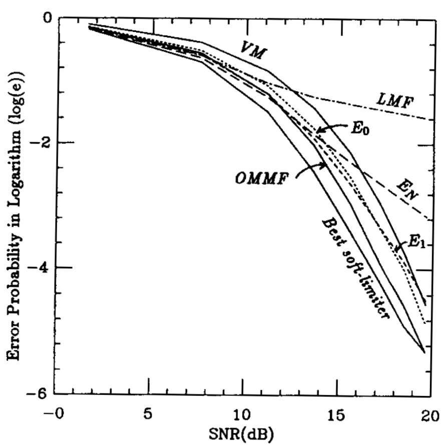

Fig. 7. *Contam. Gaussian with* 5% *truncated Cauchy.*

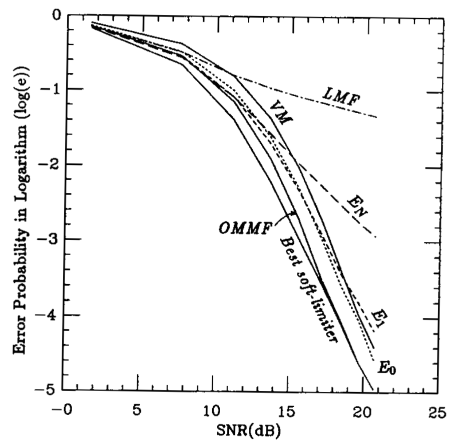

Fig. 8. *Contam. Gaussian with* 10% *truncated Cauchy.*

The results show clearly that without considering the best soft-limiter filter, the MMF performs best when impulsive noise is relatively high, and the LMF is the best when the noise is nearly Gaussian, while extended filters give compromise performance between these two.

V. CONCLUSION

In this paper, we have proposed a robust detector for QAM signal detection. Analogous to standard median filter, this MMF has the similar robustness property in case of impulsive noise. By combining the LMF and the MMF, extended matched median filters have also been introduced. Simulations illustrate the MMF and its various extended filters give a smaller detection error probability than the LMF when impulsive noise is present.

REFERENCES

[1] J. Astola and Y. Neuvo, "A Median Type Matched Filter," *Proc. ICASSP-90*, New Mexico, April 1990.

[2] J. Astola, P. Haavisto, and Y. Neuvo, "Vector Median Filters,"*Proc. IEEE*, vol.78, no.4, April 1990.

[3] S. A. Kassam and H. V. Poor, "Robust Techniques for Signal Processing: A Survey," *Proc. IEEE*, vol.73, no.3, March 1985.

[4] A. P. Clark, *Equalizers for Digital Modems*, John Wiley & Sons, New York, 1985.

[5] A. P. Clark, *Adaptive Detectors for Digital Modems*, Pentech Press, London(1989).

[6] P. J. Huber, "Robust estimation of a location parameter," *Ann. Math. Stat.*, vol.35, pp.73-104, 1964.

[7] A. H. El-Sawy, V. D. Vandelinde, "Robust Detection of Known Signals," *IEEE Trans. Information Theory*, vol.23, no.6, pp.722-727, Nov.1977.

[8] E. Polak, *Computational Methods in Optimization: A Unified Approach*, Academic Press, New York, 1971.

[9] P. R. Adby and M. A. Dempster, *Introduction to Optimization Methods*, Chapman and Hall, London, 1974.