

a) $Y = \sum_{i=1}^n Y_i$, Poisson(λ)

$$P(Y) = \frac{e^{-\lambda} \lambda^Y}{\sum Y_i!}$$

$$\ell[P(Y)] = \frac{e^{-\lambda} \lambda^{\sum Y_i}}{\sum Y_i!}$$

$$\log[\ell(P(Y))] = L(Y) = -\lambda + \sum_{i=1}^n Y_i \ln(\lambda) - \ln(\sum Y_i!)$$

b) $\hat{\lambda} = \bar{Y} = \bar{Y}/n$

$$\log[\ell(P(Y))] = -1 + \frac{\sum_{i=1}^n Y_i}{\lambda} \rightarrow \frac{\sum_{i=1}^n Y_i}{\lambda} = 1$$
$$\Rightarrow \sum_{i=1}^n Y_i = \lambda$$

$$\Rightarrow \bar{Y} = \frac{\lambda}{n}$$

c) $E(X) = \text{Var}(X) = \lambda$

$$\text{Var}(\hat{\lambda}) = \text{Var}(\bar{Y}/n) = \frac{\lambda}{n^2}$$

$$d) I(\lambda) = E \lambda (-\frac{\partial^2}{\partial \lambda^2} \ln f(Y|\lambda)) = E \lambda (\frac{1}{\lambda^2}) = \frac{\lambda}{\lambda^2} = \frac{1}{\lambda}$$

$$nI(\lambda) = \frac{n}{\lambda}$$

No, they are not the same.

e) Mean = $(3+6+3+3+5+4+6+9+3+3)/10 = 4.5$

$$Z_w = \frac{4.5 - 3}{0.671} = \frac{1.5}{0.671}$$

$$SE = \frac{\sqrt{4.5}}{\sqrt{10}} = 0.671$$



$Z \text{ score} = 0.9875$

Answer: 0.025