EE 599 Midterm - Neema Badihian

1.

$$
\begin{array}{r}
XXXXXX \\
ZZZZZZ \\
-\ 111XYZ \\
-\ XYZ111 \\
X11X11 \\
1Y11Y1
\end{array}
$$

$$XXXXXX \atop ZZZZZZ \Big) = -YYYYYY$$

$$XXXXXX \atop ZZZZZZ \Big) = -YYYYYY$$

$$\boxed{-1 \text{ w/ prob } 1}$$

3. a) $d = 3$

$$
\begin{array}{cccccccc}
1 & 1 & 1 & 1 & 1 & X & Z & Y \\
1 & 1 & 1 & 1 & X & 1 & Y & Z \\
1 & 1 & 1 & 1 & Y & Z & X & 1 \\
1 & 1 & 1 & 1 & Z & Y & 1 & X \\
1 & X & Z & Y & 1 & 1 & 1 & 1 \\
X & 1 & Y & Z & 1 & 1 & 1 & 1 \\
Y & Z & X & 1 & 1 & 1 & 1 & 1 \\
Z & Y & 1 & X & 1 & 1 & 1 & 1
\end{array}
$$

```
In[ ]:= X = {{0, 1}, {1, 0}}
        Y = {{0, -I}, {I, 0}}
        Z = {{1, 0}, {0, -1}}
        Id = IdentityMatrix[2]
Out[ ]=
        {{0, 1}, {1, 0}}
Out[ ]=
        {{0, -i}, {i, 0}}
Out[ ]=
        {{1, 0}, {0, -1}}
Out[ ]=
        {{1, 0}, {0, 1}}

In[ ]:= G = {}
Out[ ]=
        {}

In[ ]:= AppendTo[G, KroneckerProduct[Z, Z, Z, Z, Id, Id, Id, Id]]
        AppendTo[G, KroneckerProduct[X, X, X, X, Id, Id, Id, Id]]
        AppendTo[G, KroneckerProduct[Id, Id, Id, Id, Z, Z, Z, Z]]
        AppendTo[G, KroneckerProduct[Id, Id, Id, Id, X, X, X, X]]
        AppendTo[G, KroneckerProduct[X, X, Id, Id, X, X, Id, Id]]
        AppendTo[G, KroneckerProduct[Id, X, Y, Z, Id, X, Y, Z]]
        AppendTo[G, KroneckerProduct[Id, Z, X, Y, Id, Z, X, Y]]

In[ ]:= Dimensions[G]
Out[ ]=
        {7, 256, 256}

In[ ]:= weight3 = Select[
                    Tuples[{X, Y, Z, Id}, 8],
                    Count[#, Id] == 5 &]

In[ ]:= Dimensions[weight3]
Out[ ]=
        {1512, 8, 2, 2}

In[ ]:= xlogical = {}
Out[ ]=
        {}
```

In[ ]:=
```
For[i = 1, i < 1513, i++,
 If[(KroneckerProduct @@ weight3[[i]]).G[[1]] == G[[1]].(KroneckerProduct @@ weight3[[i]]) &&
   (KroneckerProduct @@ weight3[[i]]).G[[2]] == G[[2]].(KroneckerProduct @@ weight3[[i]]) &&
   (KroneckerProduct @@ weight3[[i]]).G[[3]] == G[[3]].(KroneckerProduct @@ weight3[[i]]) &&
   (KroneckerProduct @@ weight3[[i]]).G[[4]] == G[[4]].(KroneckerProduct @@ weight3[[i]]) &&
   (KroneckerProduct @@ weight3[[i]]).G[[5]] == G[[5]].(KroneckerProduct @@ weight3[[i]]) &&
   (KroneckerProduct @@ weight3[[i]]).G[[6]] == G[[6]].(KroneckerProduct @@ weight3[[i]]) &&
   (KroneckerProduct @@ weight3[[i]]).G[[7]] == G[[7]].(KroneckerProduct @@ weight3[[i]]),
  AppendTo[xlogical, weight3[[i]]]]]
```

In[ ]:=
```
MatrixForm[xlogical /. {X → x, Y → y, Z → z, Id → id}]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} x & id & y & z & id & id & id & id \\ y & z & x & id & id & id & id & id \\ z & y & id & x & id & id & id & id \\ id & x & z & y & id & id & id & id \\ id & id & id & id & x & id & y & z \\ id & id & id & id & y & z & x & id \\ id & id & id & id & z & y & id & x \\ id & id & id & id & id & x & z & y \end{pmatrix}$$

In[ ]:=
```
weight4 = Select[
              Tuples[{X, Y, Z, Id}, 8],
              Count[#, Id] == 4 &]
Dimensions[weight4]
```

Out[ ]=
```
{5670, 8, 2, 2}
```

In[ ]:=
```
weight4Gcomm = {}
For[i = 1, i < 5671, i++,
 If[(KroneckerProduct @@ weight4[[i]]).G[[1]] == G[[1]].(KroneckerProduct @@ weight4[[i]]) &&
   (KroneckerProduct @@ weight4[[i]]).G[[2]] == G[[2]].(KroneckerProduct @@ weight4[[i]]) &&
   (KroneckerProduct @@ weight4[[i]]).G[[3]] == G[[3]].(KroneckerProduct @@ weight4[[i]]) &&
   (KroneckerProduct @@ weight4[[i]]).G[[4]] == G[[4]].(KroneckerProduct @@ weight4[[i]]) &&
   (KroneckerProduct @@ weight4[[i]]).G[[5]] == G[[5]].(KroneckerProduct @@ weight4[[i]]) &&
   (KroneckerProduct @@ weight4[[i]]).G[[6]] == G[[6]].(KroneckerProduct @@ weight4[[i]]) &&
   (KroneckerProduct @@ weight4[[i]]).G[[7]] == G[[7]].(KroneckerProduct @@ weight4[[i]]),
  AppendTo[weight4Gcomm, weight4[[i]]]]]
```

Out[ ]=
```
{}
```

In[ ]:=
```
Dimensions[weight4Gcomm]
```

Out[ ]=
```
{66, 8, 2, 2}
```

```
In[ ]:= xlogicalkron = {}
        For[i = 1, i < 9, i++, AppendTo[xlogicalkron, KroneckerProduct @@ xlogical[[i]]]]
Out[ ]=
        {}

In[ ]:= weight4Gcommkron = {}
        For[i = 1, i < 67, i++, AppendTo[weight4Gcommkron, KroneckerProduct @@ weight4Gcomm[[i]]]]
Out[ ]=
        {}

In[ ]:= zlogical = {}
        For[i = 1, i < 67, i++,
         If[weight4Gcommkron[[i]].xlogicalkron[[1]] == -xlogicalkron[[1]].weight4Gcommkron[[i]] &&
           weight4Gcommkron[[i]].xlogicalkron[[2]] == -xlogicalkron[[2]].weight4Gcommkron[[i]] &&
           weight4Gcommkron[[i]].xlogicalkron[[3]] == -xlogicalkron[[3]].weight4Gcommkron[[i]] &&
           weight4Gcommkron[[i]].xlogicalkron[[4]] == -xlogicalkron[[4]].weight4Gcommkron[[i]] &&
           weight4Gcommkron[[i]].xlogicalkron[[5]] == -xlogicalkron[[5]].weight4Gcommkron[[i]] &&
           weight4Gcommkron[[i]].xlogicalkron[[6]] == -xlogicalkron[[6]].weight4Gcommkron[[i]] &&
           weight4Gcommkron[[i]].xlogicalkron[[7]] == -xlogicalkron[[7]].weight4Gcommkron[[i]] &&
           weight4Gcommkron[[i]].xlogicalkron[[8]] == -xlogicalkron[[8]].weight4Gcommkron[[i]],
          AppendTo[zlogical, weight4Gcomm[[i]]]]]
Out[ ]=
        {}
```

In[ ]:= **MatrixForm[zlogical /. {X → x, Y → y, Z → z, Id → id}]**

Out[ ]//MatrixForm=

x	id	x	id	x	id	x	id
x	id	x	id	z	z	id	id
x	id	x	id	id	x	id	x
x	id	x	id	id	id	z	z
x	id	id	x	x	id	id	x
x	id	id	x	y	y	id	id
x	id	id	x	id	x	x	id
x	id	id	x	id	id	y	y
y	y	id	id	x	id	id	x
y	y	id	id	y	y	id	id
y	y	id	id	id	x	x	id
y	y	id	id	id	id	y	y
y	id	y	id	y	id	y	id
y	id	y	id	z	id	id	z
y	id	y	id	id	y	id	y
y	id	y	id	id	z	z	id
z	z	id	id	x	id	x	id
z	z	id	id	z	z	id	id
z	z	id	id	id	x	id	x
z	z	id	id	id	id	z	z
z	id	id	z	y	id	y	id
z	id	id	z	z	id	id	z
z	id	id	z	id	y	id	y
z	id	id	z	id	z	z	id
id	x	x	id	x	id	id	x
id	x	x	id	y	y	id	id
id	x	x	id	id	x	x	id
id	x	x	id	id	id	y	y
id	x	id	x	x	id	x	id
id	x	id	x	z	z	id	id
id	x	id	x	id	x	id	x
id	x	id	x	id	id	z	z
id	y	id	y	y	id	y	id
id	y	id	y	z	id	id	z
id	y	id	y	id	y	id	y
id	y	id	y	id	z	z	id
id	z	z	id	y	id	y	id
id	z	z	id	z	id	id	z
id	z	z	id	id	y	id	y
id	z	z	id	id	z	z	id
id	id	y	y	x	id	id	x
id	id	y	y	y	y	id	id
id	id	y	y	id	x	x	id
id	id	y	y	id	id	y	y
id	id	z	z	x	id	x	id
id	id	z	z	z	z	id	id
id	id	z	z	id	x	id	x
id	id	z	z	id	id	z	z