AGA0503 - Métodos Numéricos

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Primeiro Exercício Programa (EP1)

2a) $x = .1101011011 \times 2$

No sistema decimal:

$$x = 2\left(1\cdot10^{-1} + 1\cdot10^{-2} + 0 + 1\cdot10^{-4} + 0 + 1\cdot10^{-6} + 1\cdot10^{-7} + 0 + 1\cdot10^{-9} + 1\cdot10^{-10}\right)$$

Logo: $x = 0,2202022022$ //

b) O valor de ε_m corresponde àquele menor que ainda pode ser representado no sistema: $\varepsilon_m = 0,0009727626$ //

3) $\underbrace{\log_{10}(x^2 - 1) = 0}_{10^0 = x^2 - 1 \Rightarrow x^2 = 2}$; $\varepsilon = 0,0001$

$\tilde{x}_i = \frac{b_i + a_i}{2}$; $\varepsilon_i = \left|\frac{b_i + a_i}{2}\right|$

Utiliza-se a

determinação numérica da raiz:

$x^2 - 2 = 0$

i	a_i	b_i	$\tilde{x}_i$	ε_i	sinal
0	0	2	1	1	−
1	1	2	1,5	0,5	+
2	1	1,5	1,25	0,25	−
3	1,25	1,5	1,375	0,125	−
4	1,375	1,5	1,4375	0,0625	+
5	1,375	1,4375	1,406	0,03...	−
⋮					
14	1,4141845...	1,4143066...	1,4142456...	6,1e-5	

(*) Anexo a tabela completa.

4) $f(x) = x^2 - 2$

$f'(x) = 2x$

$f''(x) = 2$

P/ convergência:

$$\left| \varepsilon_0 \frac{f''(x)}{f'(x)} \right| < 2$$

$$\left| 0{,}0001 \cdot \frac{2x}{2} \right| < 2 \quad \forall |x| < 2e+4$$

Aplica-se $x_0 = 2$:

i	x_i	$f(x)$	$f'(x)$	
0	2	2	4	
1	1,5	0,25	3	0,25
2	1,4166667	0,0069444	2,83333	0,0555
3	1,414286	0,000204	2,82857	0,0017 ✓
4	1,4141411	-0,000205	2,8282822	
5				

EP1-tabela-problema3

0, 0, 2, 1.0, 1.0
1, 1.0, 2, 1.5, 0.5
2, 1.0, 1.5, 1.25, 0.25
3, 1.25, 1.5, 1.375, 0.125
4, 1.375, 1.5, 1.4375, 0.0625
5, 1.375, 1.4375, 1.40625, 0.03125
6, 1.40625, 1.4375, 1.421875, 0.015625
7, 1.40625, 1.421875, 1.4140625, 0.0078125
8, 1.4140625, 1.421875, 1.41796875, 0.00390625
9, 1.4140625, 1.41796875, 1.416015625, 0.001953125
10, 1.4140625, 1.416015625, 1.4150390625, 0.0009765625
11, 1.4140625, 1.4150390625, 1.41455078125, 0.00048828125
12, 1.4140625, 1.41455078125, 1.414306640625, 0.000244140625
13, 1.4140625, 1.414306640625, 1.4141845703125, 0.0001220703125
14, 1.4141845703125, 1.414306640625, 1.41424560546875, 6.103515625e-05

5) $f(x) = x^2 - 2$, $\varepsilon_0 = 0{,}0001$

$x_1 = 1$, $x_2 = 2$

i	x_i	$f(x)$	ε_i
3	1,333...	-0,2222	0,010453
4	1,400...	-0,03999	0,0003
5	1,41463...	0,001189	1,5e-6 ✓
6	1,41422114...	-6,0e-6	

$$x_{i+1} = x_i - \frac{(x_i - x_{i-1})\, f(x_i)}{f(x_i) - f(x_{i-1})}$$