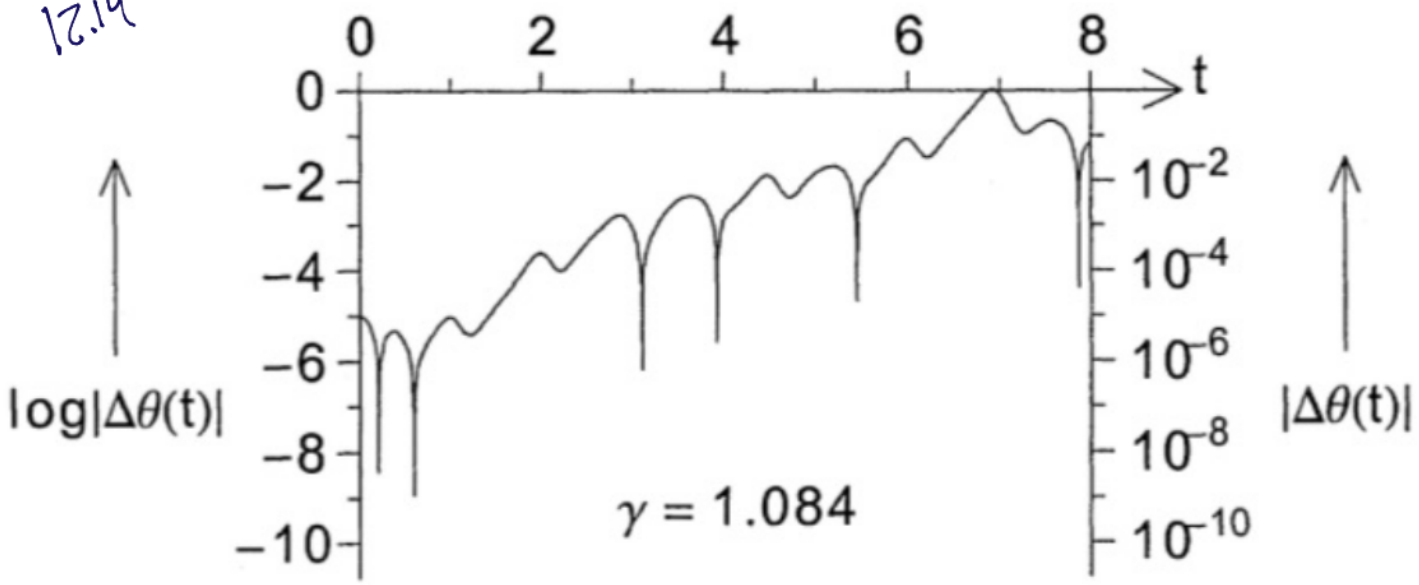
12.14



12.16 a) $t_{max} = \frac{1}{\lambda} \ln\left(\frac{\Delta\phi_{max}}{\Delta\phi(0)}\right) = 1 \ln(10^4) = 9.21$

b) $t_{max} = \frac{1}{\lambda} \ln[10^7] = 16.1$

12.23 fixed pts satisfy $x^* = f(x^*)$

$f(x) = r\sin(\pi x)$

always a fixed pt at $x = 0$

another appears when $r > \frac{1}{\pi}$ since $f'(0) > 1$ and $f(1) = 0$.

* $|f(x^*)| < 1$

$f'(0) = r\pi < 1$ if $r < \frac{1}{\pi}$ unstable at $r_0 = \frac{1}{\pi}$

the second fixed pt so long as * satisfied

or when $f'(x_2^*) = -1$

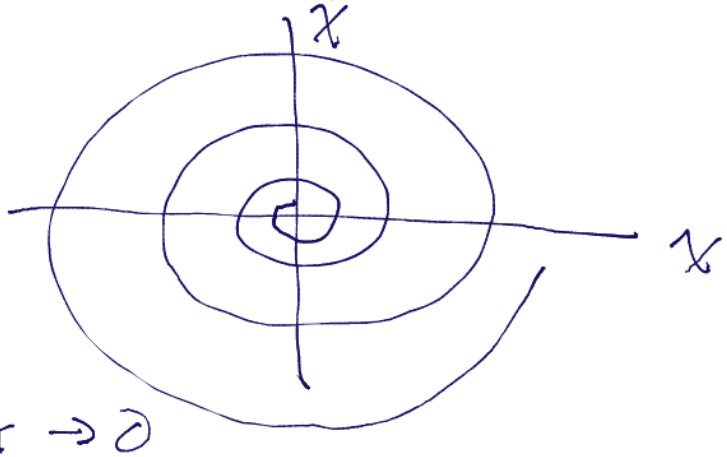
When $f(x_2^*) = x_2^*$ i.e. $r_1 \sin(\pi x_2^*) = x_2^*$

$f'(x_2^*) = -1$ i.e. $r_1 \pi \cos(\pi x_2^*) = -1$

↑ Combining and numerically solving gives $r_1 = 0.71996$.

12.20 general soln is $x(t) = Ae^{-\beta t}\cos(\omega t - \delta)$

(see eq 5.38) $\dot{x}(t) = -\omega A e^{-\beta t}\sin(\omega t - \delta)$



b) $t \to \infty$ $\dot{x}$ and $x \to 0$

$E \to 0$