Exam 2 - Classical Mechanics - PHYS-330

November 14, 2017

You must answer 2 of the following questions in class, the remaining 3 must be handed in at the start of class the next day. No electronic submissions. Show all of your work if you want full credit. You should work independently on your exam. each question is worth 20 points.

1. A wire rotates in a horizontal plane with an angular velocity ω about its centre of mass. The axis is fixed in an inertial frame. A bead is attached to the wire and is free to move along its length. The coefficient of friction between the bead and the wire is μ. If the bead is initially a distance x_0 from the axis of rotation and moving with speed v_0 along the wire.
   a. Determine the position, $x(t)$ of the bead on the wire in terms of ω, μ, x_0 and v_0.
   b. Determine the velocity, v_{0c}, such that the bead will come to a rest at the axis of rotation.
2. A particle slides on a frictionless incline. The angle, θ, of incline increases at a constant rate a.
   a. Write out a Lagrangian for the motion of the particle.
   b. If at $t = 0$ the particle is at rest and $\theta = 0$ find the subsequent motion of the particle (i.e. find the equation of motion and solve it).
3. a. Consider a central force of the form
$$f(r) = -\frac{1}{r^2} - \frac{\epsilon}{r^4}$$
   . What is the condition for stability for a circular orbit of radius a?
   b. Show that for a circular orbit of radius a, if $\partial^2 U/\partial r^2 > 0$ then the orbit is stable. Here U is the effective potential.
4. A mass, $m = 20$ kg, is attached to a spring with spring constant $k = 200$ N/m, is subjected to a linear damping force with constant $c = 60$ kg/s and a driving force given by $F = F_0 \cos \omega t$ where $F_0 = 50$ N.
   a. What value of ω results in a steady state oscillation with maximum amplitude?
   b. What is the maximum amplitude?
   c. What is the phase shift?
5. A disc is thrown up into the air in such a way that it wobbles. Friction due to air exerts a torque, $-c\boldsymbol{\omega}$, on the rotation of the disc.
   a. Show that the component of $\boldsymbol{\omega}$ in the direction of the symmetry axis decreases exponentially with time [Hint: the use of Euler's Equations will help]
   b. Show that the angle between the symmetry axis and angular velocity decreases with time if I_s (inertia in the direction of symmetry) is larger than I.