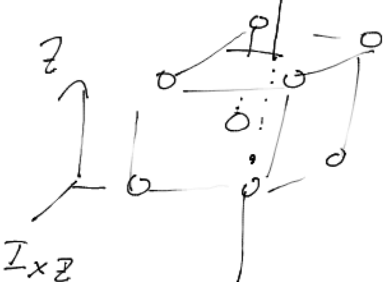
10.22

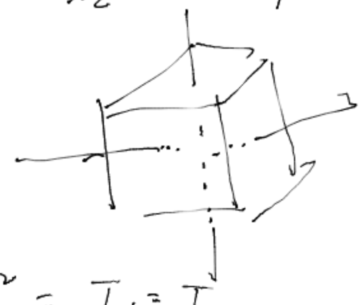
$$I_{xx} = \sum m_\alpha (y_\alpha^2 + z_\alpha^2) = m\left(\sum y^2 + \sum z_\alpha^2\right) = 4a^2 + 4a^2$$

$$I_{xx} = 8a^2$$



$$I_{yy} = 8a^2 = I_{zz}$$

$$I_{xy} = -\sum x_\alpha y_\alpha = -2ma^2 = I_{yz} = I_{xz}$$



$$I = ma^2 \begin{bmatrix} 8 & -2 & -2 \\ -2 & 8 & -2 \\ -2 & -2 & 8 \end{bmatrix}$$

(b) $I_{xx} = \sum m_\alpha (y_\alpha^2 + z_\alpha^2) = 4ma^2 = I_{yy} = I_{zz}$

$$I = ma^2 \begin{bmatrix} 4 & & 0 \\ & 4 & \\ 0 & & 4 \end{bmatrix}$$

$y = \frac{a}{2}$ $z = \frac{a}{2}$

$x = \frac{a}{2}$

#2 10.12 $A = \sqrt{3}a^2$ - Triangular ends

$$I_{zz} = \frac{M}{V}\int x^2 + y^2 \, dV = \frac{M}{A}\int x^2 dx\, dy + \frac{M}{A}\int dx \int y^2 dy$$

$$= \frac{M}{A}\left(\frac{\sqrt{3}a^4}{6}\right) + \frac{M}{A}\left(\frac{\sqrt{3}a^4}{6}\right)$$

$I_{xz}, I_{yz} = 0$

reflection sym

$$= \frac{M}{A}\frac{1}{\sqrt{3}}a^4 = \frac{1}{3}Ma^2$$

$$I_{xx} = \rho\int (y^2 + z^2)\, dV = \frac{M}{V}\int y^2 dV + \frac{M}{V}\int z^2 dV$$

$$= \frac{1}{2} I_{zz} + \frac{Ah^3}{12}$$

$$= \frac{1}{6}Ma^2 + \frac{Ah^3}{12}$$

$$I = \frac{1}{12}M \begin{bmatrix} 2a^2 + h^2 & & 0 \\ & 2a^2 + h^2 & \\ 0 & & 4a^2 \end{bmatrix}$$

#3 10.44

$\lambda_1 = \lambda_2 \quad \therefore \lambda_3 \dot{\omega}_3 = \Gamma$

$\omega_3 = \omega_{30}(1 + 2\beta t)$ ω_{30} initial value

$\beta = \dfrac{\Gamma}{2\lambda_3 \omega_{30}}$

$$\dot{\omega}_1 = -\frac{\lambda_3 - \lambda_1}{\lambda_1}\omega_3\omega_2 = -\Omega(1+2\beta t)\omega_2$$

$$\dot{\omega}_2 = \frac{\lambda_3 - \lambda_1}{\lambda_1}\omega_3\omega_1 = \Omega(1+2\beta t)\omega_1$$

$\Omega = \omega_{30}(\lambda_3 - \lambda_1)/\lambda_1 \qquad \eta = \omega_1 + i\omega_2$

$\dot{\eta} = i\Omega(1+2\beta t)\eta$

has soln $\eta = \omega_{10}\, e^{i\Omega(t+\beta t^2)}$

$\omega_1 = \omega_{10}\cos(\Omega t - \beta\Omega t^2) \quad \omega_2 = \omega_{10}\sin\Omega(t+\beta t^2)$

#4 $\theta = 90^\circ = \text{constant} \quad \dot{\theta} = \ddot{\theta} = 0$

$mgl = I_s S\dot{\phi}$

$$\dot{\phi} = \frac{mgl}{I_s S} = \frac{m(9.8)(0.02)}{\frac{1}{2}m(0.02)^2(20)(2\pi\,\text{rad/s})}$$

$= 7.8\,s^{-1}$

$$\frac{2\pi}{\dot{\phi}} = \frac{2\times 3.14159}{7.8\,s^{-1}} = 0.81$$

(note $mgl\sin\theta = I\ddot{\theta} + I_s S\dot{\phi}\sin\theta - I\dot{\phi}^2\cos\theta\sin\theta$

11.13. eqn of motion

$$① \quad m\ddot{x}_1 = -b\dot{x}_1 - (k+k_2)x_1 + k_2 x_2$$

$$② \quad m\ddot{x}_2 = -b\dot{x}_2 + k_2 x_1 - (k+k_2)x_2$$

add to get

$$m\ddot{\xi}_1 = -b\dot{\xi}_1 - k\xi_1 \qquad \boxed{\ddot{\xi}_1 + 2\beta\dot{\xi}_1 + \omega_0^2\xi_1 = 0} \quad ③$$

where $\beta = \frac{b}{2m}$

$$① - ② \quad \boxed{\ddot{\xi}_2 + 2\beta\dot{\xi}_2 + \omega_0'^2\xi_2 = 0} \quad ④ \qquad \omega_0' = (k+2k_2)/m$$

ξ_1 & ξ_2 are uncoupled.

Soln to ③ & ④

$$\xi_1 = e^{-\beta t}(B_1\cos(\omega_1 t) + C_1\sin(\omega_1 t))$$

$$\xi_2 = e^{-\beta t}(B_2\cos(\omega_2 t) + C_2\sin(\omega_2 t))$$

where $\omega_1 = \sqrt{\omega_0^2 - \beta^2} \qquad \omega_2 = \sqrt{\omega_0'^2 - \beta^2}$

$\xi_1(0) = \xi_2(0) = \frac{A}{2}$ and $B_1 = B_2 = \frac{A}{2}$

for $\beta \ll \omega_0 \qquad C_1 = C_2 = 0$

$$x_1 = \xi_1 + \xi_2 = (A/2)\,e^{-\beta t}(\cos(\omega_1 t) + \cos(\omega_2 t))$$

$$x_2 = \xi_1 - \xi_2 = (A/2)\,e^{-\beta t}(\cos(\omega_1 t) - \cos(\omega_2 t))$$