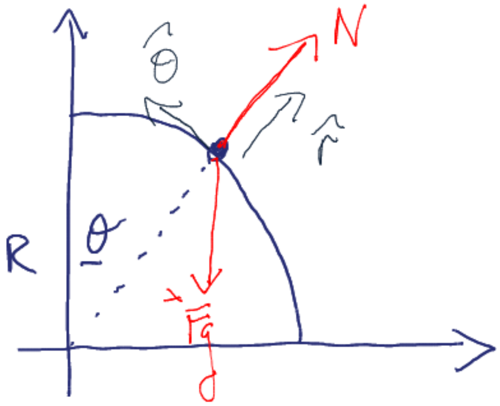



$$m\dot{\vec{v}} = m\vec{g} + \vec{N}$$

$$m\frac{v^2}{R} = mg\cos\theta - N$$

$$\Delta K = \pm mgy$$

$$\frac{1}{2}\cancel{m}v^2 - \frac{1}{2}\cancel{m}v_0^2 = \cancel{m}gR(1-\cos\theta)$$

$$v^2 = v_0^2 + 2gR(1-\cos\theta)$$

$$m\left(v_0^2 + 2gR(1-\cos\theta)\right) = mg\cos\theta - N$$

$$N = mg\left(3\cos\theta - 2 - \frac{v_0^2}{Rg}\right)$$

#2 a) $$\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}m\vec{v}\cdot\vec{v}\right) = m\frac{d\vec{v}}{dt}\cdot\vec{v}$$

$$= \vec{F}\cdot\frac{dr}{dt}$$

Hence $dK = F\cdot dr$

b) $$dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy + \frac{\partial U}{\partial z}dz = \nabla U\cdot dr$$

$$dU = -\vec{F}\cdot d\vec{r} = dK$$

Thus $d(U+K) = 0$

c) $$dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy + \frac{\partial U}{\partial z}dz + \frac{\partial U}{\partial t}dt$$

$$= (\vec{\nabla}U \cdot d\vec{r}) + \frac{\partial U}{\partial t}dt$$

$$= -\vec{F}\cdot d\vec{r} + \frac{\partial U}{\partial t}dt$$

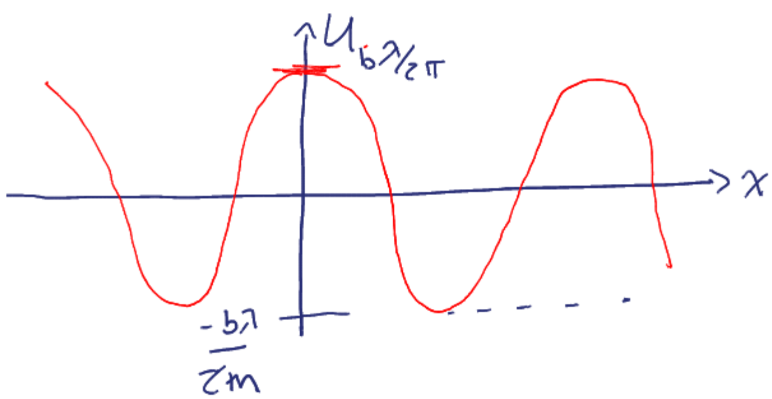
$$= -dK + \frac{\partial U}{\partial t}dt$$

$$\frac{d}{dt}(U+K) = \frac{\partial U}{\partial t}$$

---

#3 $$F = b\sin\left(\frac{2\pi x}{\lambda}\right)\hat{x}$$

$$U = -b\int \sin\frac{2\pi x}{\lambda}\,dx = \frac{b\lambda}{2\pi}\cos\left(\frac{2\pi x}{\lambda}\right)$$




if $E > b\lambda/2m$ motion unbounded

max velocity when

$x = (n+\frac{1}{2})\lambda$

for $E < \frac{b\lambda}{2m}$ Turning points

at $\frac{b\lambda}{2\pi}\cos\left(\frac{2\pi x}{\lambda}\right) = E$

stable equilibrium $x = \left(n+\frac{1}{2}\right)\lambda$

unstable $x = n\lambda$

---

#4 Since the mass moves in a circle $a_r = -\frac{v^2}{r}$

$$-m\frac{v^2}{r} = F_r = -\frac{dU}{dr} = nkr^{n-1}$$

$$mv^2 = nkr^n = nU$$

Thus $T = \frac{1}{2}mv^2 = \frac{1}{2}nU$

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#5 $W = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

angular momentum conserved
$mvr = mv_0 r_0$

Thus $W = \left(\frac{r_0^2}{r^2} - 1\right)\frac{1}{2}mv_0^2$

$W < 0$ if $r > r_0$

thus work done by particle if radius increases.