3.14

from 3.29 $m\dot{v} = -\dot{m} v_{ex} + F^{ext}$ ⊛

note that $\dot{m} = -k \Rightarrow dt = -\frac{dm}{k}$

$$m\frac{dv}{dt} = -\frac{dm}{dt} v_{ex} - bv$$

$$-m\frac{dv}{dm}k = k v_{ex} - bv$$

$$\frac{k\,dv}{k v_{ex} - bv} = -\frac{dm}{m}$$

$$\therefore \frac{k}{b}\ln\left(\frac{k v_{ex} - bv}{k v_{ex}}\right) = \ln\left(\frac{m}{m_0}\right) \Rightarrow v = \frac{k v_{ex}}{b}\left(1 - \left(\frac{m}{m_0}\right)^{b/k}\right)$$

3.23

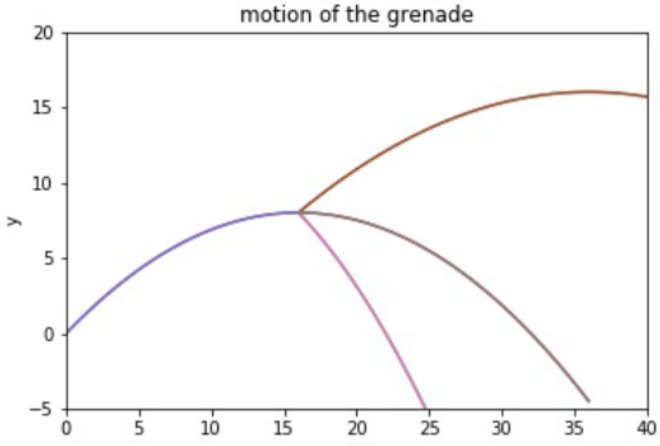
$$\vec{r} = v_0 t + \frac{1}{2} g t^2$$

$$x = v_{0x} t \qquad y(t) = v_{0y} t + \frac{1}{2} g t^2$$

pieces have equal masses $m_1 = m_2 = M/2$

$$\vec{v}_1 + \vec{v}_2 = 2v \qquad v_2 = 2v - v_1 = v - \Delta v$$

motion of the grenade



3.22 M - mass of hemisphere

$$\rho = \frac{M}{V} = \frac{M}{2\pi R^3/3} \quad \text{- density of hemisphere}$$

$$\vec{R} = \int \frac{\rho \vec{r}\, dV}{M} = \int \vec{r}\, \frac{dV}{V} \qquad \begin{array}{l} X = 0 \\ Y = 0 \end{array}$$

$$Z = \frac{1}{V}\int z\, dV = \frac{3}{2\pi R^3}\int_0^R r^2 dr \int_0^{\pi/2} \sin\theta\, d\theta \int_0^{2\pi} d\phi\, r\cos\theta$$

$$Z = \frac{3}{2\pi R^3}\, \frac{R^4}{4} \cdot \frac{1}{2}\, 2\pi = \frac{3}{8} R$$

3.26 $F = f(r)\,\hat{r}$ $\qquad \Gamma = \vec{r} \times \vec{F} = 0$

$\Downarrow$

angular momentum l is constant.

since l conserved

$r \times p = l_0$ always Thus $r \perp l_0$ always

$r \rightarrow$ on plane $\perp$ to l_0

#5 $\sum \vec{L}_i = \sum \vec{L}_f$

$$I_1 \vec{\omega}_1 + I_2 \vec{\omega}_2 = (I_1 + I_2)\vec{\omega}$$

$$T_f = \frac{\sum L_f^2}{2 I_f} = \frac{\sum L_i^2}{2 I_f} = \frac{(I_1 \vec{\omega}_1 + I_2 \vec{\omega}_2)^2}{2(I_1 + I_2)}$$

$$= I_1^2 \omega_1^2 + I_2^2 \omega^2 + 2 I_1 I_2 \omega_1 \omega_2$$

$$T_i = \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2$$

$$\frac{T_f}{T_i} = \frac{I_1^2 \omega_1^2 + I_2^2 \omega_2^2 + 2 I_1 I_2 \vec{\omega}_1 \cdot \vec{\omega}_2}{(I_1 + I_2)(I_1 \omega_1^2 + I_2 \omega_2^2)} < 1$$

if $I_1 I_2 (\omega_1^2 + \omega_2^2 - 2 \vec{\omega}_1 \cdot \vec{\omega}_2) > 0$

$\| (\vec{\omega}_1 - \vec{\omega}_2 \| > 0$ unless $\vec{\omega}_1 = \vec{\omega}_2$

Thus $T_f < T_i$

Energy loss.