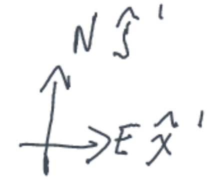
Homework 7



#1 Latitude Saskatoon 52°N (ish)

$$\vec{v}\,' = 400\text{ mph N} = 178.8\text{ m/s N}$$

$$\vec{\omega} = 7.27\times10^{5}\left(\cos 52^\circ\,\hat{j} + \sin 52\,\hat{k}'\right)s^{-1}$$

$$\vec{\omega}\times\vec{v}\,' = -(7.27\times10^{-5})(178.8)\sin 52\,\hat{i}'\ \frac{m}{s^2}$$

$$\frac{F_{cor}}{F_g} = \frac{|-2\cancel{m}\,\vec{\omega}\times\vec{v}\,'|}{\cancel{m}\,g} = \frac{2(7.27\times10^{-5})(178.8)\sin(52)}{9.8}$$

$$\frac{F_{cor}}{F_g} = 0.002$$

Corriolis force is in the $-\vec{\omega}\times\vec{v}\,'$ direction or $+\hat{i}'$ or east.

#2 9.22,

Let S_0 be the inertial frame where $-q$ orbits Q in B

$$m\left(\frac{d^2\vec{r}}{dt^2}\right)_{S_0} = -\frac{kqQ}{r^2}\hat{r} - q\left(\frac{d\vec{r}}{dt}\right)_{S_0}\times\vec{B}$$

or in S frame $\qquad\qquad$ $\dot{r}$ derivative in S frame

$$m\ddot{\vec{r}} - 2m\dot{\vec{r}}\times\vec{\Omega} - m(\vec{\Omega}\times\vec{r})\times\vec{\Omega} = -\frac{kqQ}{r^2}\hat{r} - q(\dot{\vec{r}} + \vec{\Omega}\times\vec{r})\times\vec{B}$$

Choose $\Omega = qB/2m$ Then

$$m\ddot{\vec{r}} = -\frac{kqQ}{r^2}\hat{r} - \frac{q^2}{4m}(\vec{B}\times\vec{r})\times\vec{B}$$

If B weak enough $\quad m\ddot{\vec{r}} = -\frac{kqQ}{r^2}\hat{r}$ – solutions ellipses

9.20 a) $m\ddot{\vec{r}} = \vec{F}_{cp} + \vec{F}_{cor} = m(\vec{\Omega}\times\vec{r})\times\vec{\Omega} + 2m\dot{\vec{r}}\times\vec{\Omega}$

$\vec{r} = (x, y, 0)$ $\vec{\Omega} = (0, 0, \Omega)$

$$\ddot{\vec{r}} = \Omega^2(x, y, 0) + 2\Omega(\dot{y}, -\dot{x}, 0)$$

or $\ddot{x} = \Omega^2 x + 2\Omega\dot{y}$ (1) $\ddot{y} = \Omega^2 y - 2\Omega\dot{x}$ (2)

$\eta = x + iy$ $\ddot{\eta} = \Omega^2\eta - 2i\Omega\dot{\eta}$

guess soln $\eta = e^{-i\omega t}$

$$-\omega^2 = \Omega^2 - 2\Omega\omega$$

has soln if $\omega = \Omega$

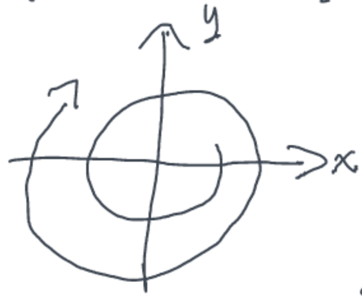
$$\eta = e^{-i\Omega t}$$

second sol of form $te^{-i\Omega t}$

$\eta(t) = e^{-i\omega t}(C_1 + C_2 t)$ C_1, C_2 are constants

c) $\eta(0) = C_1$ $\dot{\eta}(0) = C_2 - i\Omega C_1$

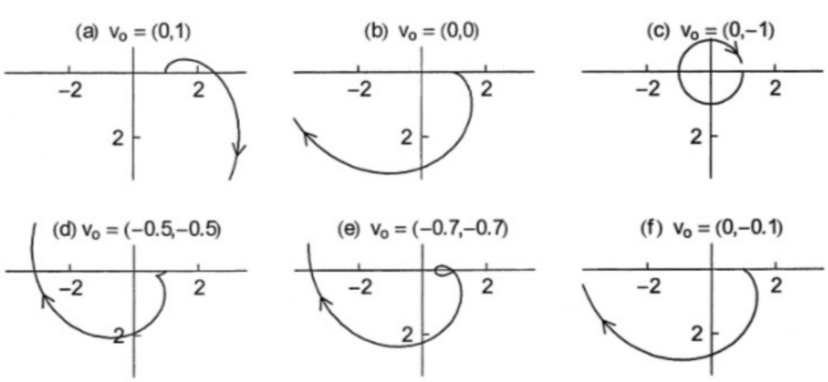
$$\eta(t) = e^{-i\Omega t}[x_0 + v_{x0}t + i(v_{y0} + \Omega x_0)t]$$



For large t exclude term without t factor

$x(t) = tA\cos(\Omega t - \delta)$ $y(t) = -tA\sin(\Omega t - \delta)$

spiral expanding.

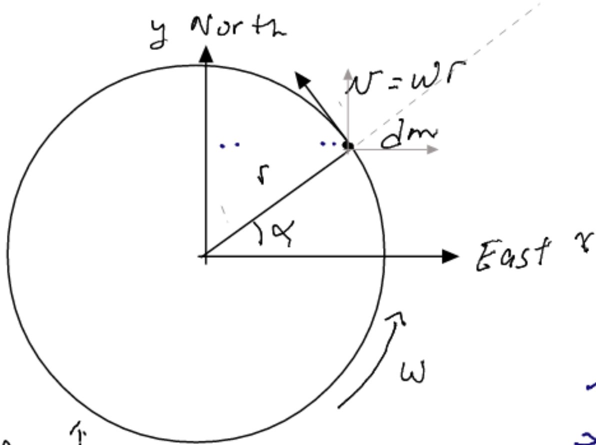


9.28 a) you should get $t = 34.9$ and $R = 16.4\,Km$

b) $y = -32m$ (32 m south of target)

at $50°S$ shell lands 32m North of target.

9.37

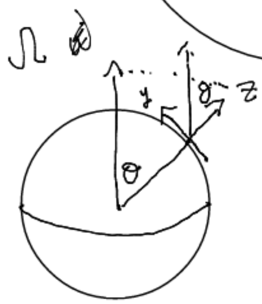


$$dm = 2\, dm = \frac{m\, d\alpha}{2\pi}$$

$$d\vec{F} = 2\, dm\; \vec{v} \times \vec{\omega}$$

$$\vec{v} = \omega r(-\sin\alpha, \cos\alpha, 0)$$

$$\vec{\omega} = (0, \sin\theta, \cos\theta)\Omega$$



$$\vec{r} = r(\cos\alpha, \sin\alpha, 0)$$

$$d\Gamma_{cor} = \vec{r} \times d\vec{F} = 2m\, \vec{r} \times (\vec{v} \times \quad) = 2dm[\vec{v}(\vec{r}\cdot\Omega) - \Omega(\vec{r}\cdot\vec{v})]$$

$$d\Gamma_{cor} = 2m\, \omega r^2 \Omega(-\sin^2\alpha, \sin\alpha\cos\alpha, 0)\sin\theta$$

$dm = \frac{m\, d\alpha}{2\pi}$

$$\Gamma_{cor} = \int_0^{2\pi} \frac{m\omega}{\pi} r^2 \Omega(-\sin^2\alpha, \sin\alpha\cos\alpha, 0)\sin\theta\, d\alpha$$

$$= \frac{m\omega r^2}{\pi}\Omega(-\pi, 0, 0)\sin\theta$$

$$= -m\omega r^2 \Omega \sin\theta\, \hat{x}$$