13.5 $\rho = R \quad z = c\phi$ only one generalized coord. ϕ.

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m(\dot{z}^2 + R^2\dot{\phi}^2)$$

$$= \frac{1}{2}m(c^2+R^2)\dot{\phi}$$

$$U = mgz = mgc\phi$$

generalized momentum $p = \dfrac{\partial T}{\partial \dot{\phi}} = m(c^2+R^2)\dot{\phi}$

$$\mathcal{H} = \frac{p^2}{2m(c^2+R^2)} + mgc\phi$$

eqn of motion

$$\dot{\phi} = \frac{\partial \mathcal{H}}{\partial p} = \frac{p}{m(c^2+R^2)} \qquad \dot{p} = -\frac{\partial \mathcal{H}}{\partial \phi} = -mgc$$

$$\text{or } \ddot{z} = c\ddot{\phi} = -g\frac{c^2}{c^2+R^2} = -g\sin^2\alpha.$$

Where $\tan\alpha = \dfrac{c}{R} \quad \sin\alpha = \dfrac{c}{\sqrt{c^2+R^2}}$

13.20 a) $U(\vec{r}) = -\int \vec{F}\cdot d\vec{r} = -\vec{F}\cdot\vec{r}$

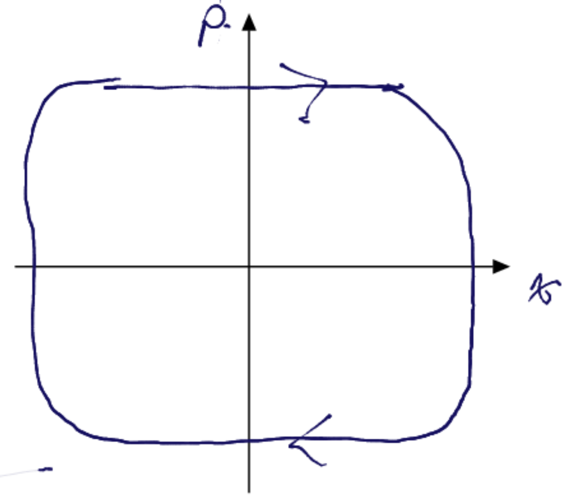
$$\mathcal{H} = \frac{p^2}{2m} - \vec{F}\cdot\vec{r} = \frac{p_x^2 + p_y^2}{2m} - F_x x - F_y y$$

b) if x-axis $\parallel$ to $\vec{F}$ then $F_y = 0$ and y-ignorable.

c) If neither x or y $\parallel$ to $\vec{F}$ then neither x or y are ignorable.

13.26 $\quad U = -\int F\,dx = \frac{1}{4}kx^4$

$$\mathcal{H} = \frac{p^2}{2m} + \frac{1}{4}kx^4 = E$$



Flattened ellipse

13.25 $\quad$ Note $\quad q = \sqrt{2P}\sin Q \qquad p = \sqrt{2P}\cos Q$

$$\frac{\partial \mathcal{H}}{\partial Q} = \frac{\partial \mathcal{H}}{\partial q}\frac{\partial q}{\partial Q} + \frac{\partial \mathcal{H}}{\partial p}\frac{\partial p}{\partial Q} = -\dot{p}(\sqrt{2P}\cos Q) - \dot{q}(\sqrt{2P}\sin Q)$$

$$= -(\dot{p}p + \dot{q}q) = \frac{1}{2}\frac{d}{dt}(p^2 + q^2) = \frac{d}{dt}P(\cos^2 Q + \sin^2 Q) = \dot{P}$$

$$\frac{\partial \mathcal{H}}{\partial P} = \frac{\partial \mathcal{H}}{\partial q}\frac{\partial q}{\partial P} + \frac{\partial \mathcal{H}}{\partial p}\frac{\partial p}{\partial P}$$

$$= -\dot{p}\left(\frac{1}{\sqrt{2P}}\sin Q\right) + \dot{q}\left(\frac{1}{\sqrt{2P}}\cos Q\right)$$

$$= -\dot{p}\frac{q}{2P} + \dot{q}\frac{p}{2P} = \frac{p^2}{2P}\frac{d}{dt}\left(\frac{q}{p}\right) = \frac{p^3}{2P}\frac{d}{dt}\tan Q = \frac{p^3}{2P}\dot{Q}\sec^2 Q$$

$$= \dot{Q}$$