

Exam 1 - Classical Mechanics - PHYS-330

October 3rd, 2017

PHYS-330 Exam 1

You must answer 3 of the following questions in class, the remaining 2 must be handed in at the start of class the next day. No electronic submissions. Show all of your work if you want full credit. You should work independently on your exam. each question is worth 20 points.

1. a. Consider a particle of mass m moving in a central isotropic field such that,

$$m\ddot{\mathbf{r}} = F(r)\hat{\mathbf{r}}.$$

Show that the above equation can be expressed in polar coordinates as

$$m(\ddot{r} - r\dot{\theta}^2) = F(r), \quad \frac{d}{dt}(mr^2\dot{\theta}) = 0$$

- b. express the first of the two equations in part a. in terms of only r .
c. If $r(\theta) = Ae^{\beta\theta}$, where A and β are constants, determine the time dependent trajectory.
2. A particle of mass m moves in a one dimensional potential given by the infinite series,

$$V(x) = \frac{1}{2}k_1x^2 + \frac{1}{3}k_2x^3 + \frac{1}{4}k_3x^4$$

where the k_i are constants.

- a. Show that the equation of motion is

$$\ddot{x} + \alpha x + \beta x^2 + \gamma x^3 \dots = 0$$

- b. Sketch the possible energy diagrams when the only non-zero constants are i) $\alpha > 0$, and β . ii) $\alpha > 0$ and γ . iii) $\gamma > 0$. Comment on the possible motions in each case.
3. Consider a thin rod of uniform mass density in the shape of an arc of radius a subtended by an angle of $\frac{3\pi}{2}$. Where is the center of mass.
4. A bird flies horizontal to the ground at a speed v_0 . The bird strikes and gets embedded in a stationary vertical rod of length L hinged in the ground. The bird strikes the rod $L/3$ from the top. The mass of the bird is $1/4$ the mass of the rod.
- a. What is the final angular speed of the rod?
- b. What is the ratio of the kinetic energy of the system after the collision to the kinetic energy of the bird before the collision?
5. My son coasts to a stop on his skate board. During this motion he is subject to a quadratic drag force from the air, and a friction force due to the skateboard's contact with the ground.
- a. Write down the equation of motion while my son is in contact with the ground and solve it.
- b. If the force of friction has magnitude 4 N, my son's mass (including skateboard is) 50 kg and his initial velocity is $v_0 = 10.0$ m/s, How long before his speed is 5.0 m/s? How long to come to a full stop? (take the coefficient for quadratic drag to be 0.20)