Chain Rule.

$y = h(x) = (x^2 + x)^2$. This is the composition of $f(x) = x^2$

and $g(x) = x^2 + x$

Find $h'(x) = \frac{dy}{dx}$

$h(x) = f(g(x))$

when $h = f(g(x))$ then

$$h'(x) = f'(g(x)) \cdot g'(x)$$

$f'(x) = 2x$

$g'(x) = 2x + 1$

Thus $\frac{d}{dx}(x^2 + x)^2 = \frac{d}{dx} f(g(x))$

$$= f'(g(x)) \cdot g'(x)$$

$$= 2(g(x)) \cdot (2x+1)$$

$$= 2(x^2+x) \cdot (2x+1)$$

Ex $h(x) = \sqrt{5x-3}$

$h(x) = f(g(x))$ $f(x) = \sqrt{x}$ $f'(x) = \frac{1}{2\sqrt{x}}$

$g(x) = 5x - 3$ $g'(x) = 5$

$$h'(x) = f'(g(x)) \cdot g'(x)$$

$$= \frac{1}{2\sqrt{g(x)}} \cdot g'(x) = \frac{1}{2\sqrt{5x-3}} \cdot 5 = \frac{5}{2\sqrt{5x-3}}$$

Ex find slope of tangent line to $f(x) = \left(\frac{2x+1}{3x+2}\right)^3$ at $x = 0$

$$f'(x) = 3\left(\frac{2x+1}{3x+2}\right)^2 \cdot \frac{d}{dx}\left(\frac{2x+1}{3x+2}\right) = 3\left(\frac{2x+1}{3x+2}\right)^2 \cdot \left(\frac{(3x+2)\cdot 2 - 3(2x+1)}{(3x+2)^2}\right)$$

$$= 3\left(\frac{2x+1}{3x+2}\right)^2 \cdot \left(\frac{6x+4-6x-3}{(3x+2)^2}\right)$$

$$= 3\left(\frac{2x+1}{3x+2}\right)^2 \cdot \left(\frac{1}{(3x+2)^2}\right)$$

$$= 3\,\frac{(2x+1)^2}{(3x+2)^4}$$

$$f'(0) = 3\,\frac{1}{2^4}$$

$$= \frac{3}{16}$$

#68 pg 195 Amount of digital information created each month globaly is

$f(t) = 400\left(\frac{t}{12}+1\right)^{1.09}$ in billion gigabytes.

starting in 2008 (t - month)

in 2008 $f(0) = 400(1)^{1.09} = 400$

how fast was digital information being created in begining of 2010?

$$f'(t) = 400(1.09)\left(\frac{t}{12}+1\right)^{0.09} \cdot \frac{1}{12} = 36.3\left(\frac{t}{12}+1\right)^{0.09}$$

$$f'(24) = 36.3\left(\frac{24}{12}+1\right)^{0.09} = 36.3(3)^{0.09} = 40.07$$

Ex// $g(t) = (2t+3)^2(3t^2-1)^{-3}$

$$= \left[\frac{d}{dt}(2t+3)^2\right](3t^2-1)^{-3} + (2t+3)^2\left[\frac{d}{dt}(3t^2-1)^{-3}\right]$$

$$= 2(2t+3)(2)(3t^2-1)^{-3} + (2t+3)^2\left[-3(3t^2-1)^{-4}\right]$$

$$= \frac{4(2t+3)}{(3t^2-1)^3} - \frac{3(2t+3)^2}{(3t^2-1)^4}$$

$$= \frac{4(2t+3)(3t^2-1) - 3(2t+3)^2}{(3t^2-1)^4}$$

Ex $g(t) = \dfrac{\sqrt{t+1}}{\sqrt{t^2+1}}$

$$g'(t) = \frac{\sqrt{t^2+1}\cdot\frac{d}{dt}\sqrt{t+1} - \sqrt{t+1}\,\frac{d}{dt}\sqrt{t^2+1}}{t^2+1}$$

$$= \frac{\sqrt{t^2+1}\,\frac{1}{2\sqrt{t+1}} - \sqrt{t+1}\,\frac{t}{\sqrt{t^2+1}}}{t^2+1}$$

$$= \frac{\frac{\sqrt{t^2+1}}{2\sqrt{t+1}} - \frac{t\sqrt{t+1}}{\sqrt{t^2+1}}}{(t^2+1)}$$

$$= \frac{t^2+1-2(t+1)}{2\sqrt{t+1}\,(t^2+1)^{3/2}} = \frac{t^2-2t}{2\sqrt{t+1}\,(t^2+1)^{3/}}$$

Ex $h(x) = \sqrt{x+\sqrt{x^2-1}}$

$$h'(x) = \frac{1}{2}\left(x+\sqrt{x^2+1}\right)^{-1/2} \cdot \frac{d}{dx}\left(x+\sqrt{x^2-1}\right)$$

$$= \frac{1}{2}\left(x+\sqrt{x^2+1}\right)^{-1/2}\left(1+\frac{1}{2}(x^2-1)^{-1/2} \cdot \frac{d}{dx}(x^2-1)\right)$$

$$= \frac{1}{2\sqrt{x+\sqrt{x^2+1}}}\left(1+\frac{1}{2}(x^2-1)^{-1/2}(2x)\right)$$

$$= \frac{1}{2\sqrt{x+\sqrt{x^2+1}}}\left(1+\frac{x}{\sqrt{x^2-1}}\right)$$