Problem 1. Use the definition of the derivative to find the derivative of the following

i) $f(x) = 2$

ii) $f(x) = x^2$

iii) $g(x) = 2x^2 - x$

iv) $h(x) = \dfrac{2}{x}$

v) $f(x) = \sqrt{x}$

Problem 2. Find the slope of the tangent line to the function $f(x) = 3x - x^2$ at the point $(2, 11)$.

Problem 3. Let

$$f(x) = \begin{cases} x^2 & \text{if} x \leq 1 \\ ax + b & \text{if} x > 1 \end{cases}$$

Find the values of a and b such that f is continuous and has a derivative at $x = 1$

Problem 4. find the derivative of the following functions.

i) $f(x) = 4x^2 + x$

ii) $g(x) = \dfrac{5}{x}$

iii) $f(t) = 2t^3 + \sqrt{t^3}$

iv) $h(x) = (x^2 - x)(1/x - 2)$

v) $f(x) = \frac{(x^2-x)}{(x^3+1)}$

vi) $g(w) = (w^3 - w^2 + w - 1)(w^2 + 1)$

vii) $f(x) = \dfrac{3}{3x + 1}$

viii) $g(x) = \dfrac{\sqrt{x} + 1}{x^2 + 1}$

ix) $f(x) = \dfrac{x}{x^2 - 4} - \dfrac{x - 1}{x^2 + 4}$

Problem 5. The jnumber of bacteria $N(t)$ in a certain culture t min after an experiment bactericde is introduced is given by

$$N(t) = \frac{10000}{1 + t^2} + 2000$$

Find the rate of change of the number of bacteria in the culture 1 min and 2 min after the bactericide is introduced. What is the population of the bacteria in the culture 1 min and 2 min after the bactericide is introduced?

Problem 6. Suppose the distance s (in feet) covered by a car moving along a straight road after t seconds is given by the function $s = f(t) = 2t^2 + 48t$

i) Calculate the average velocity of the car over the time interval $[20, 21]$.

ii) Calculate the velocity of the car at $t = 20$