

Final Exam

Name: _____

Time Limit: 180 min

- [illegible]

1. Evaluate the following limits. If it doesn't exist, explain why.

(a) (4 points) $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}$

(b) (4 points) $\lim_{x \rightarrow \infty} \frac{5x^4 - 3x + 2}{7x^4 + 3x^2 - 1}$

(c) (4 points) $\lim_{x \rightarrow 0^+} \frac{1}{1 + 2^{-1/x}}$

2. (5 points) Use the definition of the derivative to determine the derivative of $f(x) = \frac{-1}{x}$

3. (5 points) Find the values of x for which the function $f(x)$ is discontinuous.

$$f(x) = \begin{cases} -2x + 1 & \text{if } x < 0 \\ x^2 + 1 & \text{if } 0 \leq x \leq 1 \\ \sqrt{x+1} & \text{if } 1 < x < \infty \end{cases}$$

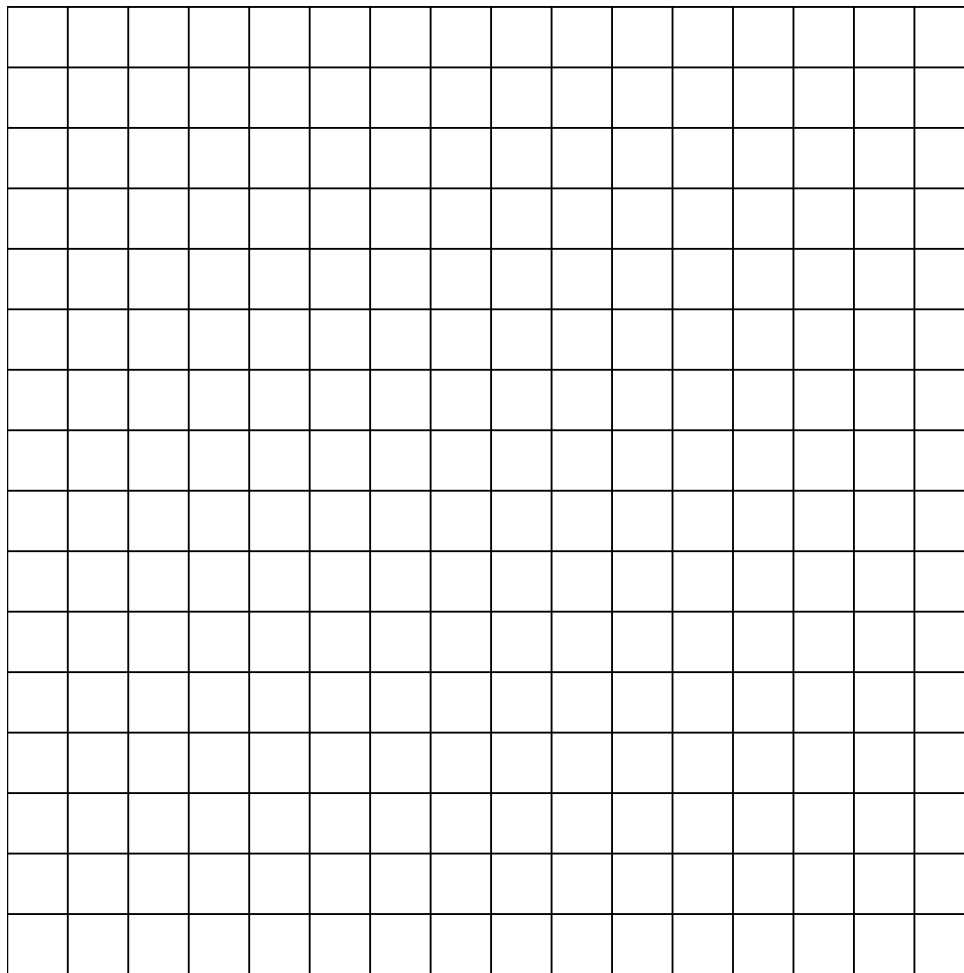
4. For the function $f(x) = x^3 + 2x^2 + x$
- (a) (3 points) Find the critical numbers.

 - (b) (3 points) Find the intervals where $f(x)$ is increasing and decreasing and identify any relative maximum and minimum values.

 - (c) (5 points) Find the regions of concavity.

 - (d) (2 points) Find the x -coordinate of any inflection points.

(e) (5 points) Plot the function.



5. (5 points) Find the equation of the tangent line to the curve $y^2 = x^3(2 - x)$ at the point $(x, y) = (-1, 2)$.

6. Find $\frac{dy}{dx}$ for the following equations.

(a) (5 points) $y = \tan(x^3 + 1)$

(b) (5 points) $y = \arcsin(e^{2x})$

(c) (5 points) $y = (x^2 + 1) \arctan(\sqrt{x})$

7. (5 points) Use Logarithmic differentiation to find the derivative of $y = \frac{(x^3 + 1)^4 \sin^2(x)}{\sqrt[3]{x}}$.
DO NOT SIMPLIFY YOUR ANSWER.

8. A company's cost function and demand function are given by

$$C(x) = 3800 + 5x - \frac{x^2}{1000} \quad \text{and} \quad p(x) = 50 - \frac{x}{100} \quad \text{for } 0 \leq x \leq 1000.$$

(a) (4 points) Compute $C'(300)$ and give an interpretation of the results.

(b) (2 points) Find the revenue function $R(x)$.

(c) (4 points) Find the profit function $P(x)$ and the demand level which maximizes profit.

9. (5 points) Find the horizontal (if any) and vertical (if any) asymptote(s) of the function

$$f(x) = \frac{2x^2 - 6x}{x^2 - 9}$$

10. (10 points) A box with an open top and a square base is to have a volume of 32000 cm^3 . Find dimensions of the box which will minimize the surface area of the box.

11. (10 points) A ladder 10ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6ft from the wall?