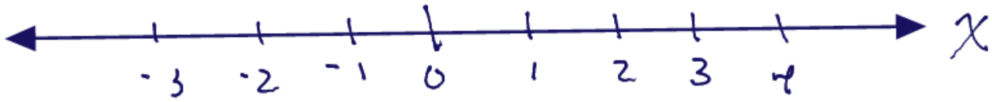
Review



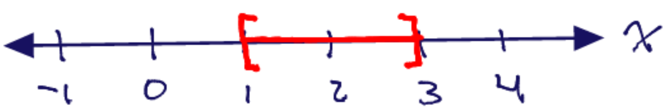
Real number System made up of the real numbers and usual operations $+, -, \times, \div$

Real number

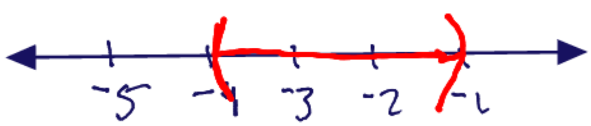
$\mathbb{R} = \{$rational and irrational numbers$\}$

every real number has a place on the real number line (x coordinate, x-axis)

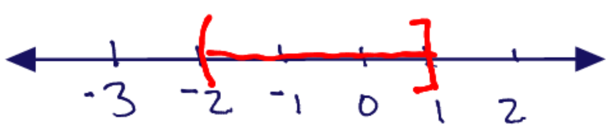
Intervals - a section (∞ or not ∞) of number line.



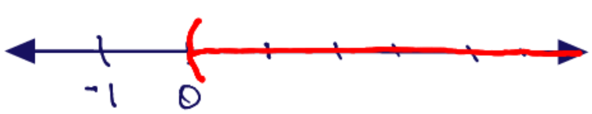
$1 \le x \le 3$ Closed interval
$[1, 3]$



$-4 < x < -1$ Open interval
$(-4, -1)$



$-2 < x \le 1$ Half open interval.
$(-2, 1]$



$0 < x < \infty$ infinite interval
$(0, \infty)$

$(-\infty, 1]$

$-\infty < x \le 1$

Exponents

are a way of representing repeated multiplication

$$b^n = \underbrace{b \cdot b \cdot b \cdot \dots \cdot b}_{n \text{ times}} \qquad n \geq 0 \quad n \in \mathbb{Z}$$

n is integer

b - base n - exponents.

$b^0 = 1 \quad (b \neq 0)$ $\qquad 2^3 = 2 \cdot 2 \cdot 2 = 8$

$b^{-n} = \frac{1}{b^n} \quad b \neq 0$ $\qquad 2^{-4} = \frac{1}{2^4} = \frac{1}{2 \cdot 2 \cdot 2 \cdot 2} = \frac{1}{16}$

what if n is fractional?

$n = \frac{1}{2}, \frac{3}{2}, \frac{7}{3}$

$(2)^{3/2} = (2^{1/2})^3 \qquad (2^{1/2})^2 = 2$

$= (1.414\ldots)^3 = 2.828\ldots$

$(9^{3/2}) = (9^{1/2})^3 = (3)^3 = 27$

$(4^{-5/2}) = (4^{1/2})^{-5} = 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

Laws of exponents

1. $a^m \cdot a^n = a^{m+n}$ $\qquad 3^2 \cdot 3^3 = 3^5$
2. $\frac{a^m}{a^n} = a^{m-n}$ $\qquad \frac{x^{-7}}{x^2} = x^{-7-2} = x^{-9} = \frac{1}{x^9}$
3. $(a^m)^n = a^{nm}$ $\qquad (x^3)^7 = x^{21}$
4. $(ab)^n = a^n b^n$ $\qquad (xy)^3 = x^3 \cdot y^3$
5. $\left(\frac{a}{b}\right)^n = \left(\frac{a^n}{b^n}\right)$ $\qquad \left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$

Note $x^{1/n} = \sqrt[n]{x}$

Ex $x^{1/2} = \sqrt[2]{x}$ - square root

$x^{1/3} = \sqrt[3]{x}$ - Cubed root.

Ex// Simplify and express with +ve exponents.

a. $\sqrt{9x^6y^2} = (9x^6y^2)^{1/2} = 9^{1/2}x^{\frac{6}{2}}y^{2/2} = 3x^3y$

b. $(x^3y^2)^3 \cdot (3xy)^3 = (3x^3x\,y^2y)^3$
$= 3^3(x^4y^3)^3 = 27x^{12}y^9$

c) $\sqrt[3]{\frac{-27x^6}{8y^3}} = \frac{(-27x^6)^{1/3}}{(8y^3)^{1/3}} = \frac{(-27)^{1/3}x^2}{8^{1/3}y} = \frac{-3x^2}{2y}$

Ex/ rationalize denominator

$$\frac{7x}{2\sqrt{x}} = \frac{7x}{2\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{7\cancel{x}\sqrt{x}}{2\cancel{x}} = \frac{7}{2}\sqrt{x}$$

$$\frac{7x}{\sqrt{x-1}} = \frac{7x\sqrt{x-1}}{\sqrt{x-1}\sqrt{x-1}} = \frac{7x\sqrt{x-1}}{x-1}$$

Polynomials are expressions of a variable x written with increasing powers of x

$$P(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$$

where a_n are the coefficients $a_n \in \mathbb{R}$

degree of $P(x)$ = $\deg P(x) = n$ → highest power.

operations of $+, -, \times, \div$ work with Polynomials.

Ex// $(x^3 - 2x^2 + 7x + 1) + (2x^3 - x^2 - x + 7)$

$= x^3 - 2x^2 + 7x + 1 + 2x^3 - x^2 - x + 7$

group "like terms", those with same power of x.

$= (1+2)x^3 + (-2-1)x^2 + (7-1)x + (1+7)$

$= 3x^3 - 3x^2 + 6x + 8$

when multiplying use the distributative law.

$a(b+c) = ab + ac$

$(a+b)(c+d) = (a+b)c + (a+b)d = ac + bc + ad + bd$

Ex/ $(x^2+1)(x^3 - 2x + 1) = x^5 - 2x^3 + x^2 + x^3 - 2x + 1$

$= x^5 - x^3 + x^2 - 2x + 1$

$(e^t + e^{-t})e^t - e^t(e^t - e^{-t}) = e^{2t} + e^0 - e^{2t} + e^0$

$= 1 + 1$

$= 2$

Note: $(a+b)^2 = a^2 + 2ab + b^2$ NOT $a^2 + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)(a-b) = a^2 - b^2$

Factoring – reverse of distributative law
express poly as product of other polynomials.

Ex. $(2x^3 + x^2) = x^2(2x+1)$

$$2ye^{xy^2} + 4xy^3e^{xy^2} = 2ye^{xy^2}(1+2xy^2)$$

Special case – Polynomials of degree 2.

$$px^2 + qx + r = (ax+b)(cx+d)$$
$$= acx^2 + (ad+bc)x + bd$$

$p = ac \quad q = ad+bc \quad r = bd$

Goal is to find a, b, c, d.

Ex// $x^2 - 2x - 3$
$= (x+1)(x-3)$

$3x^2 - 6x - 24$
$= 3(x^2 - 2x - 8)$
$= 3(x+2)(x-4)$

Ex. $\frac{1}{2}a^2 - a - 12 = \frac{1}{2}(a^2 - 2a - 24)$
$= \frac{1}{2}(a-6)(a+4)$

Roots of Polynomial expression

are values of x such that

$$P(x) = 0$$

Ex// $x^2 + 2x + 1$. for $x = -1$ $\quad x = -1$ is a root

$$(-1)^2 + 2(-1) + 1 = 1 - 2 + 1 = 0$$

$$x^3 - 3x^2 + 2x = 0$$

$x = 0$ $\quad (0)^3 - 3(0)^2 + 2(0) = 0$ root ✓

$x = 1$ $\quad (1)^3 - 3(1)^2 + 2(1) = 1 - 3 + 2 = 0$ root ✓

$x = -1$ $\quad (-1)^3 - 3(-1)^2 + 2(-1) = -1 - 3 - 2 \neq 0$ not root

$x = 2$ $\quad 2^3 - 3(2)^2 + 2(2) = 8 - 12 + 4 = 0$ root ✓

Special cases for polynomial of degree 2.

$$ax^2 + bx + c = 0$$

when $x = \dfrac{-b \mp \sqrt{b^2 - 4ac}}{2a}$

Rational Expressions - fractions involving rational expressions.

Ex// $\frac{x^2-2x+1}{x-7}$ $\frac{2x^2y-y^2}{4x}$

we can simplify rational expressions by finding common factors.

$$\frac{x^2-2x+1}{x-1} = \frac{(x-1)(x-1)}{x-1} = x-1$$

$$\frac{(x^2+1)^2(-2)+(2x)2(x^2+1)(2x)}{(x^2+1)^4}$$

$$= \frac{2(x^2+1)[(x^2+1)+4x^2]}{(x^2+1)^4}$$

$$= \frac{2(x^2+1)[3x^2-1]}{(x^2+1)^4}$$

$$= \frac{2[3x^2-1]}{(x^2+1)^3}$$

Ex// Simplify

i) $\frac{1}{4x^2}+\frac{5}{6xy^2}$ find common denominator $12x^2y^2$

$= \frac{3y^2}{12x^2y^2}+\frac{10x}{12x^2y^2}$ simplify

$= \frac{3y^2+10x}{12x^2y^2}$

$\frac{5}{c+2} + \frac{6}{c-3}$ Find common denominator $(c+2)(c-1)$

$= \frac{5}{c+2}\,\frac{(c-3)}{(c-3)} + \frac{6}{c-3}\,\frac{(c+2)}{(c+2)}$

$= \frac{5c-15}{(c+2)(c-3)} + \frac{6c+12}{(c-3)(c+2)} = \frac{11c-3}{(c-3)(c+2)}$

Ex/ $\frac{1+\frac{1}{x+1}}{x-\frac{4}{x}} = \frac{\frac{x+1}{x+1}+\frac{1}{x+1}}{\frac{x^2}{x}-\frac{4}{x}} = \frac{\frac{x+2}{x+1}}{\frac{x^2-4}{x}} = \frac{x+2}{x+1}\cdot\frac{x}{(x+2)(x-2)} = \frac{x}{(x+1)(x-2)}$

Rationalizing Algebraic Expressions.

$(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = (\sqrt{a})^2-(\sqrt{b})^2 = a-b$

$\frac{x+1}{1+\sqrt{x}} = \frac{x+1}{1+\sqrt{x}}\cdot\frac{1-\sqrt{x}}{1-\sqrt{x}} = \frac{(x+1)(1-\sqrt{x})}{1+x}$

Inequalities Properties $a<b,\ b<c \Rightarrow a<c$

$a<b \Rightarrow a+c < b+c$

$c<0,\ a<b \Rightarrow ac > ab$

$c>0,\ a<b \Rightarrow ac < bc$

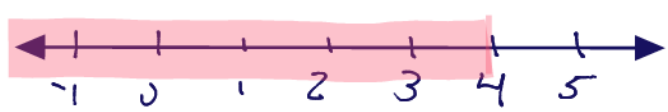
Ex $11q+5 \le 49$

$11q+5-5 \le 49-5$

$11q \le 44$

$\frac{11q}{11} \le \frac{44}{11}$

$q \le 4$

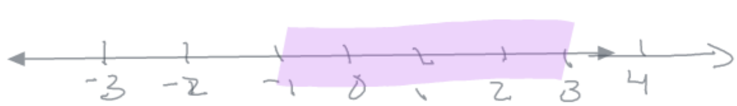


$q \le 4$ $(-\infty, 4]$

Ex $0 \le x+1 \le 4$

$0 \le x+1$ $x+1 \le 4$

$x \ge -1$ $x \le 3$



$[-1, 3]$ $-1 \le x \le 3$

Absolute value of a number a

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Note $|-a| = |a|$

$|a \cdot b| = |a| \cdot |b|$,

$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$|a+b| \leq |a| + |b|$

Evaluate i) $|\pi - 6| - 3$ Note $\pi - 6 < 0$

$|\pi - 6| - 3 = -(\pi - 6) - 3 = 3 - \pi$

ii) $|2\sqrt{3} - 3| - |\sqrt{3} - 4|$ $\qquad 2\sqrt{3} - 3 < 0$

$\qquad\qquad\qquad\qquad\qquad\qquad \sqrt{3} - 4 < 0$

$= -(2\sqrt{3} - 3) + (\sqrt{3} - 4)$

$= -\sqrt{3} - 7$

Cartesian Coordinate System

Any point on the xy plane is represented by an ordered pair of numbers

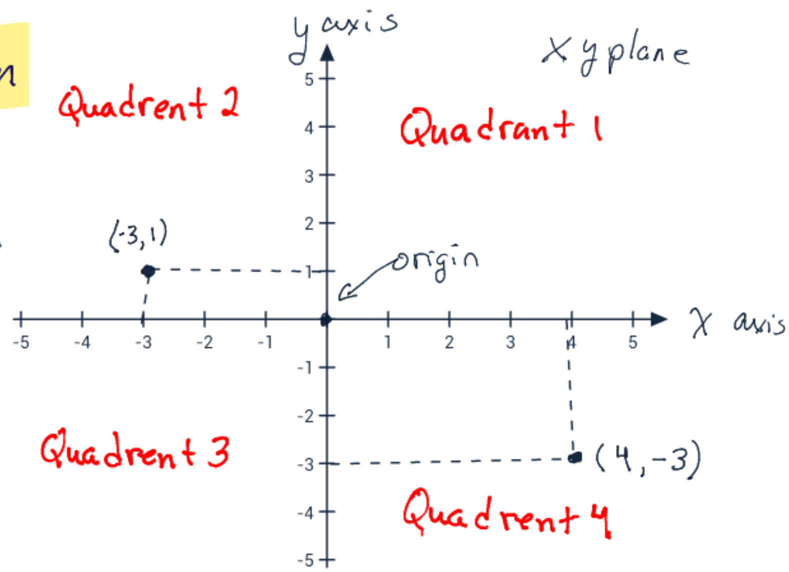
(x, y)

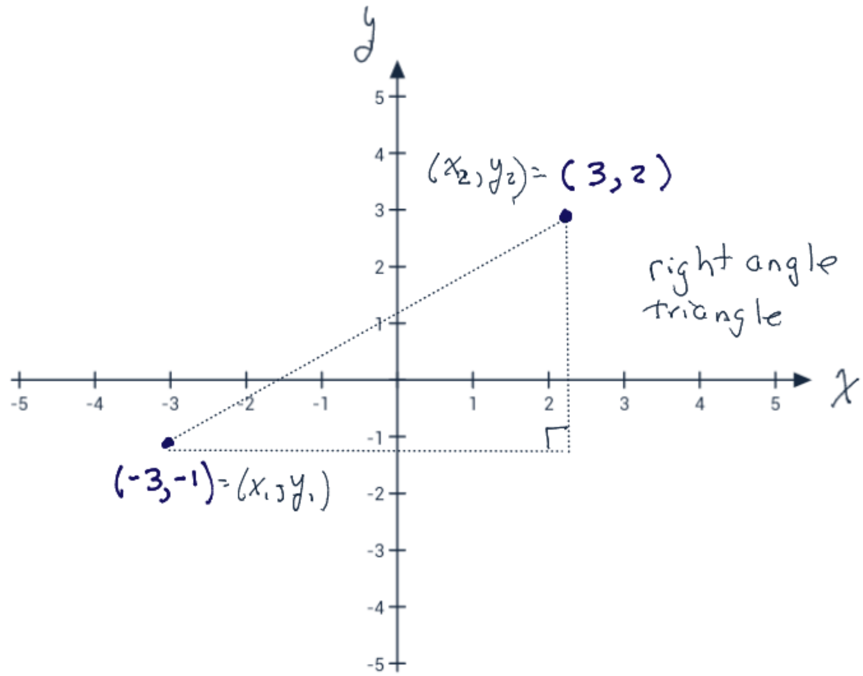
position in x direction

x-Coordinate

position in y direction

y-Coordinate





Distance Between two Points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(3-(-3))^2 + (2-(-1))^2}$$

$$d = \sqrt{(6)^2 + (3)^2}$$

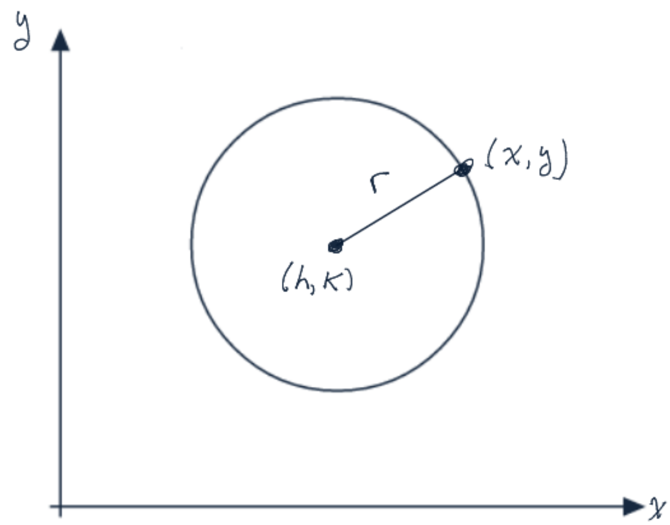
$$= \sqrt{45}$$

$$= 6.71$$

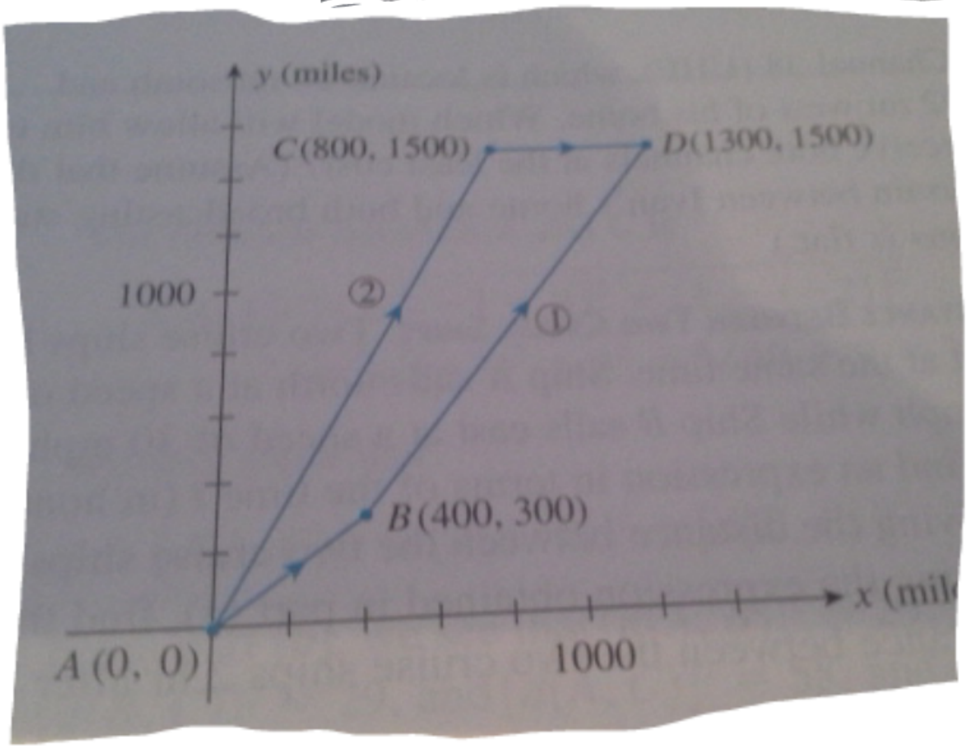
For Circle of radius r with centre at (h, k)

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

$$r^2 = (x-h)^2 + (y-k)^2$$



38. OPTIMIZING TRAVEL TIME Towns A, B, C, and D are located as shown in the following figure. Two highways link town A to town D. Route 1 runs from Town A to Town D via Town B, and Route 2 runs from Town A to Town D via Town C. If a salesman wishes to drive from Town A to Town D and traffic conditions are such that he could expect to average the same speed on either route, which highway should he take to arrive in the shortest time?



Route 1

$d = \sqrt{800^2 + 1500^2} + 500$

$= 1700 + 500 = 2200$ miles

Route 2

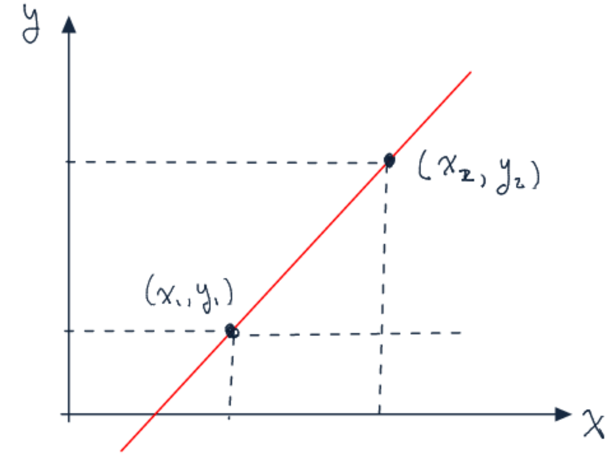
$d = \sqrt{400^2 + 300^2} + \sqrt{(1300-400)^2 + (1500-300)^2}$

$= 500 + 1500$

$= 2000$ miles

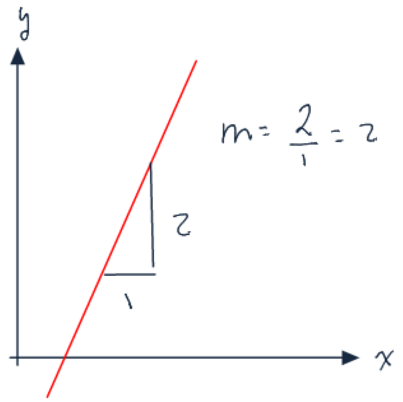
Choose route 2 !!

Lines and Cartesian Coordinates

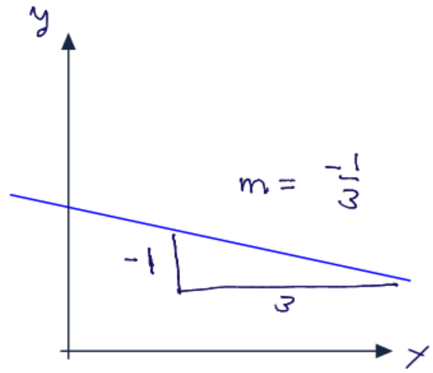


The slope of a line tells us how one variable with respect to another.

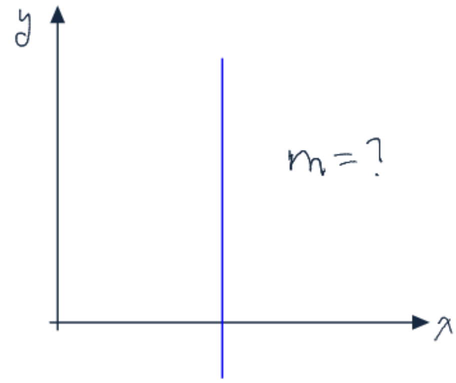
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



$m = \frac{2}{1} = 2$



$m = \frac{-1}{3}$



$m = ?$

Ex. find the slope of the line passing through the points $(1, 2)$ and $(-3, 7)$

$$m = \frac{-3 - 1}{7 - 2} = -\frac{4}{5}$$

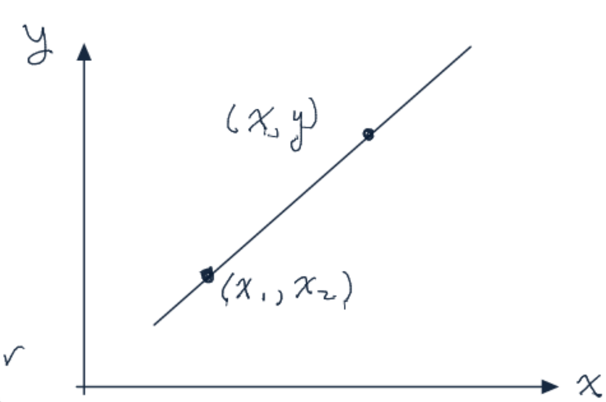
* only require two points to define a slope *

* can be any two points on the line *

* two lines are parallel if and only if their slopes * are the same

Equation of a line

since any two points on a line can calculate a slope, choose one with values (x_1, y_1) and the other as variables (x, y)



Then slope is $m = \frac{y - y_1}{x - x_1}$ point slope

or $y - y_1 = m(x - x_1) \Rightarrow$ Equation of a line

Ex// find equation of the line passing through $(1, 3)$ and $(4, -1)$

first $m = \frac{-1 - 3}{4 - 1} = \frac{-4}{3}$

Then $y - 3 = -\frac{4}{3}(x - 1)$

$$y - 3 = -\frac{4}{3}x + \frac{4}{3}$$

$$y + \frac{4}{3}x - 3 - \frac{4}{3} = 0$$

$$y + \frac{4}{3}x - \frac{13}{3} = 0$$

$$3y + 4x - 13 = 0$$

Perpendicular lines

given two lines with slopes m_1 and m_2, they are perpendicular if $m_1 = \frac{-1}{m_2}$

Ex for line above find the $\perp$ line through $(1, 3)$

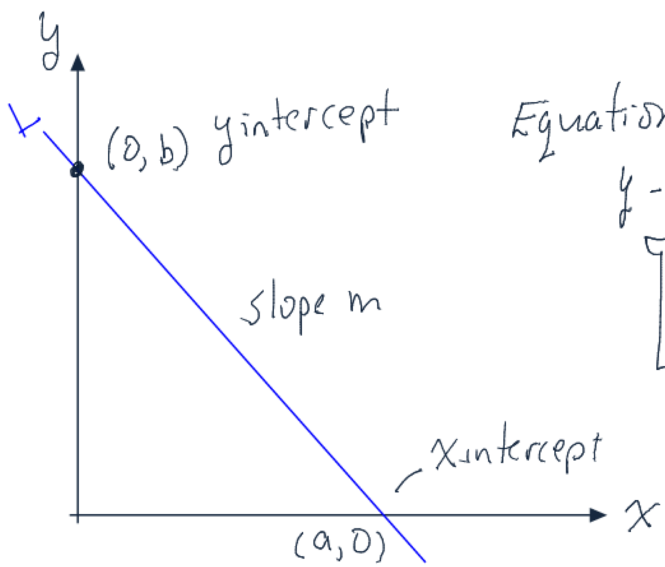
$$y - 3 = +\frac{3}{4}(x - 1)$$

$$y - 3 = \frac{3}{4}x - \frac{3}{4}$$

$$y - \frac{3}{4}x - 3 + \frac{3}{4} = 0$$

$$4y - 3x - 12 + 3 = 0$$

$$4y - 3x - 9 = 0$$



Equation of line

$$y - b = m(x - 0)$$

$m =$

$$\boxed{y = mx + b}$$

· b - yintercept
m - slope

Slope intercept eqn of a line

Ex Find the yintercept for the line passing through the points $(3, 1)$, $(1, -2)$

$$m = \frac{-2-1}{1-3} = \frac{-3}{-2} = \frac{3}{2}$$

$$y - 1 = \frac{3}{2}(x-3)$$

$$y = \frac{3}{2}x - \frac{9}{2} + 1$$

$$y = \frac{3}{2}x - \frac{7}{2}$$

yintercept is $-\frac{7}{2}$.

Ex Sketch the graph of line represented by

$$3x - 4y - 12 = 0$$

y intercept when $x = 0$ $\quad -4y - 12 = 0 \Rightarrow y = -3$

x intercept when $y = 0$ $\quad 3x - 12 = 0 \Rightarrow x = 4$