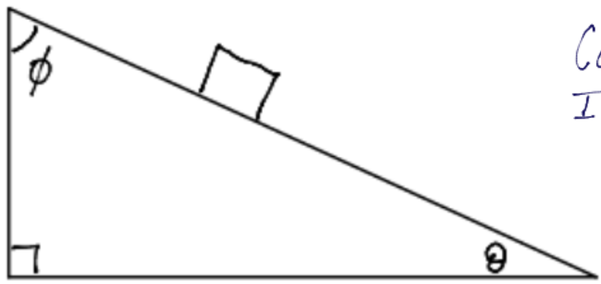
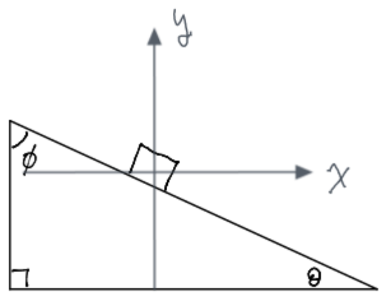
Motion of an object on an inclined Plane



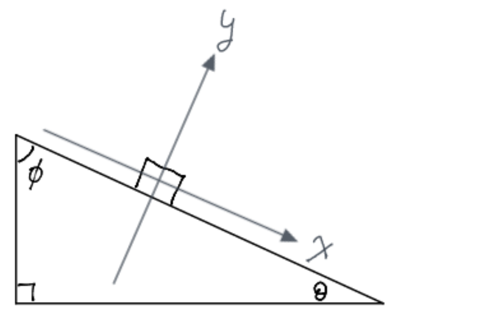
Consider an object sliding on an Inclined plane (without friction)

Choice of coordinate system



we are "used" to this coordinate system orientation

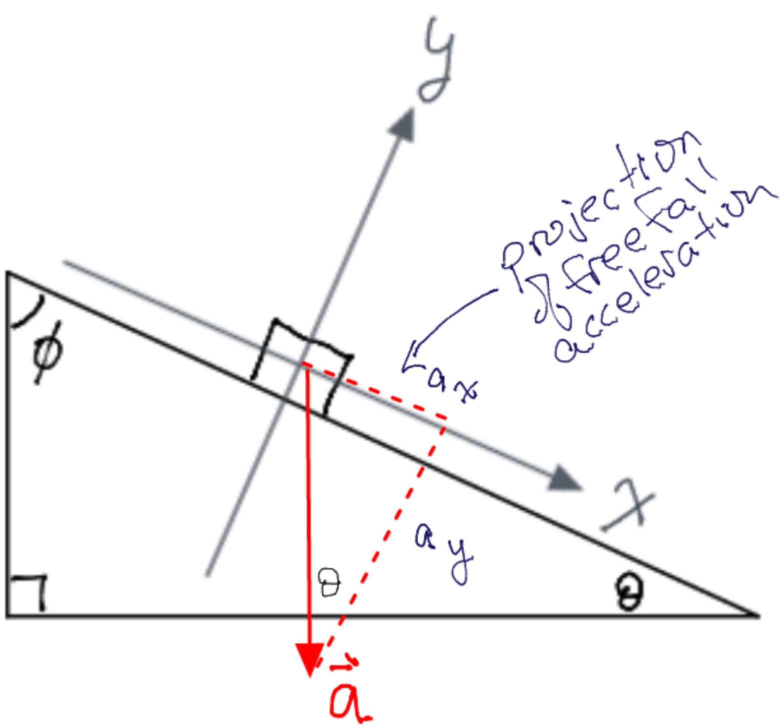
or

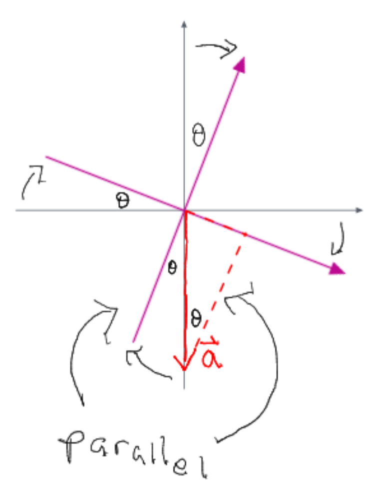


when motion is constrained to move in a line, choose one coordinate to be in that direction.

acceleration of object

$\vec{a}$ = {projection of the freefall acceleration vector in the direction of incline, down incline}

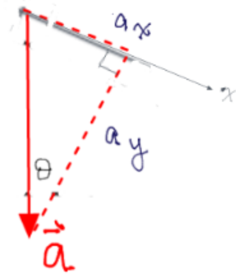




a_x = projection of $\vec{a}$ onto incline →
= component of vector $\vec{a}$ in x direction

$$a_x = |\vec{a}| \sin\theta \qquad |\vec{a}| = g$$

$$a_x = g \sin\theta$$



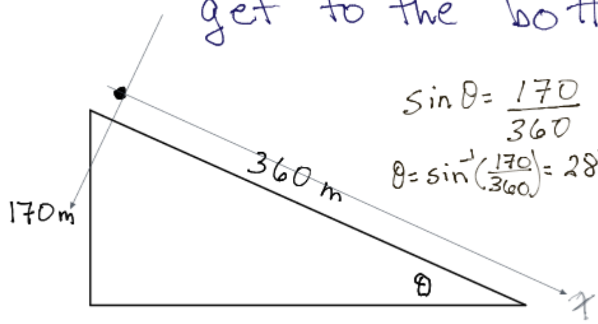
when $\theta = 0$ No incline
$a_x = g \sin 0 = 0$

when $\theta = 90$ Free fall
$a_x = g \sin 90 = g$ No constraint

Ex// A skier starts at rest from the top of a hill and travells 360m while dropping a vertical distance of 170m.

a) what is the speed of the skier at the bottom

b) How much time does it take to get to the bottom.



$\sin\theta = \frac{170}{360}$

$\theta = \sin^{-1}\left(\frac{170}{360}\right) = 28°$

Given

$x_0 = 0$ m $\qquad x_f = 360$m

$v_{x0} = 0$ m/s $\qquad v_{xf} = ?$

$a_x = ?$ $\qquad t_f = ?$

$\vec{a} = \{9.8\,m/s^2, \text{centre of earth}\}$

$$a_x = g \sin\theta = (9.8\,m/s^2)\left(\frac{170\,m}{360\,m}\right)$$

$$a_x = 4.6\,m/s^2$$

This is linear motion with constant acceleration

Thus $(v_{xf})^2 = (v_{xi})^2 + 2a_x\Delta x \qquad \Delta x = x_f - x_i = 360 - 0 = 360$m

$$v_{xf} = \sqrt{(v_{xi})^2 + 2a_x\Delta x}$$

$$= \sqrt{0^2 + 2(4.6\,m/s^2)(360\,m)}$$

$$= 58\,m/s$$

∴ The speed at the bottom is 58 m/s

Also $x_f = x_i + v_{xi}t_f + \frac{1}{2}a_x t_f^2$

since $x_i = 0$ $\quad v_{xi} = 0$ $\qquad x_f = \frac{1}{2}a_x t_f^2$

or $t_f = \sqrt{\frac{2x_f}{a_x}}$

Thus $t_f = \sqrt{\frac{2(360\,m)}{4.6\,m/s^2}} = 13\,s$

∴ The time to get to the bottom is 13 seconds