Math Review Quiz   **PHYS-111**

Review of Algebra

1. Basic Algebra Rules

Basic Rule: In an equation, whatever operation is performed on the left side of the equality must be performed on the right side.

Rules for mathematical operations:

Multiplication: $\left(\dfrac{a}{b}\right)\left(\dfrac{c}{d}\right) = \dfrac{ac}{bd}$ Division: $\dfrac{(a/b)}{(c/d)} = \dfrac{ad}{bc}$ Addition $\dfrac{a}{b} \pm \dfrac{c}{d} = \dfrac{ad \pm bc}{bd}$

Solve the following 25 problems. Show your work on this sheet.

1) Consider 3x - 9 = 24. Solve for x.

2) Let a = 3/4 and b = 7/8
    i.) Find a/b.
    ii.) Find a - b

3) Consider ax - 8 = bx + 2. Solve for x. Can a = b in this solution?

4) Consider a + bx = c.   Solve for x. What value of b must be excluded?

5) Consider $a = \dfrac{1}{1+x}$ Solve for x. What value of a must be excluded?

6) Consider $\dfrac{5}{2x+6} = \dfrac{3}{4x+8}$. Solve for x.

Solve the following equation for y in terms of the other quantities:

7) $A + By^2 = C + Dx^2$

2. Power Rules

$$x^n x^m = x^{n+m} \qquad \dfrac{x^n}{x^m} = x^{n-m} \qquad x^{1/n} = \sqrt[n]{x} \qquad (x^n)^m = x^{nm}$$

Using the above power rules, simplify the following:

8) $3^2 + 3^3 =$

9) $x^2 x^{-8} =$

10) $\dfrac{x^{10}}{x^{-5}} =$

11) $5^{1/3} =$ Use a calculator. Express your answer to two decimal places.

12) $(x^4)^3 =$

3. Factoring Rules

$$ax + ay + az = a(x + y + z) \qquad \text{common factor}$$

$$a^2 + 2ab + b^2 = (a+b)^2 \qquad \text{perfect square}$$
$$a^2 - 2ab + b^2 = (a-b)^2 \qquad \text{perfect square}$$
$$a^2 - b^2 = (a+b)(a-b) \qquad \text{difference of squares}$$

Factor the following and simplify the expression if possible:
13) $8x + 4y - 12 =$
14) $9x^2 - 13x + 4 =$

4. Quadratic Equations

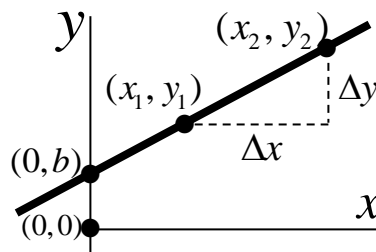$$ax^2 + bx + c = 0 \qquad \text{General quadratic equation (a, b, c are numerical factors)}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Solution for unknown x}$$

Solve the following using the quadratic equation:
15) Solve $2x^2 + 5x - 4 = 0$ for x. How many solutions are there?

5. Linear Equations
Consider the equation $y = mx + b$. A plot of this equation is shown in the figure for m positive.

The slope, m, of this line is expressed as

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

16) Use a ruler to make two legible graphs below. Plot y = ax + b for x = [-3, -2, -1, 0, 1, 2, 3] for each of the following two cases: 1.) a = 3, b = 2   and   2.) a = -2, b = -1

6. Solving Simultaneous Linear Equations

How many equations do you need to solve for two unknowns?

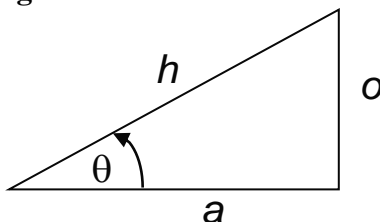17) Solve equations (1) and (2) <u>simultaneously</u> for x and y

$$5x + y = -8 \quad (1)$$
$$2x - 2y = 4 \quad (2)$$

<u>Review of Trigonometry</u>

See Appendix A.5, pg A.13 of the text for trigonometric relations.

7. Trigonometry for Right Triangles

$$\sin\theta = \frac{o}{h} \qquad \cos\theta = \frac{a}{h} \qquad \tan\theta = \frac{o}{a}$$

To obtain θ, we use the inverse relations

$$\theta = \sin^{-1}\left(\frac{o}{h}\right) \qquad \theta = \cos^{-1}\left(\frac{a}{h}\right) \qquad \theta = \tan^{-1}\left(\frac{o}{a}\right)$$
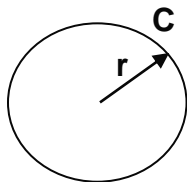
18) Consider $\theta = 30^0$. What is the value of o/h? Set your calculator to <u>degree</u> mode.

19) Consider $o = 0.866$, $h = 1$. What is θ in degrees?

20) In the figure above, $a = 100$m and $\theta = 35.0^0$. Find o.

Angles can be measured in degrees or radians. We consider radians next.

Consider a circle of radius r. The circumference is $C = 2\pi r$ so that $\dfrac{C}{r} = 2\pi$.

$$2\pi \text{ radians} = 360^0 \qquad \pi \text{ radians} = 180^0 \qquad \pi/2 \text{ radians} = 90^0$$

Conversion of degrees to radians $\theta_d = \theta$ in degrees $\quad \theta_r = \theta$ in radians

$$\frac{\theta_r}{\pi} = \frac{\theta_d}{180^0} \qquad\qquad \textbf{Ex. } \theta_d = 30^0 \quad \theta_r = \pi\left(\frac{30^0}{180^0}\right) = \frac{\pi}{6} = 0.5236 \text{ radians}$$

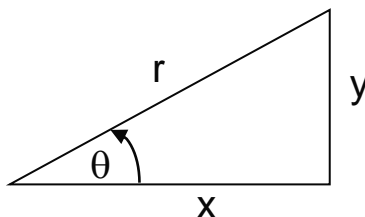**Conversion of radians to degrees** $\theta_d = \theta$ in degrees $\theta_r = \theta$ in radians

$$\frac{\theta_d}{180^0} = \frac{\theta_r}{\pi}$$ **Ex.** $\theta_r = 1.00$ radians $\theta_d = \left(\frac{1.00\,\text{radians}}{\pi}\right)180^0 = 57.2958^0$

21) Consider sinθ = 0.5. What is θ in radians? Note: Set your calculator to radians.

22) In 21, what is the corresponding value of θ in degrees?

Pythagorean Theorem

Consider the right triangle.

23) a.) What is the relation between r, x, and y using the Pythagorean Theorem?

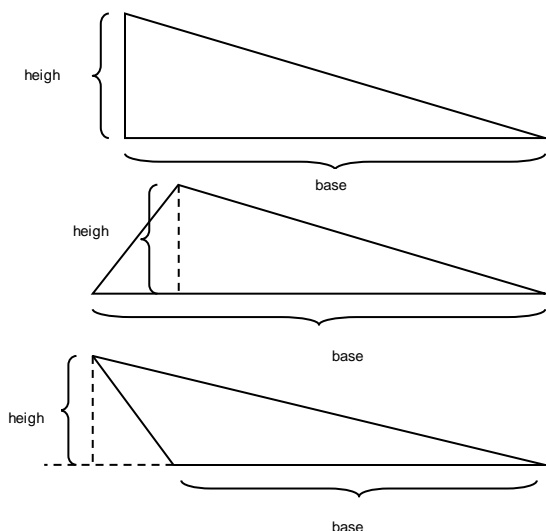   b.) The Pythagorean Theorem only applies for what type of triangles?

24) What is the angle between the sides x and y in degrees?

<u>**Review of Geometry**</u>

The area of a triangle measurement requires correct determination of the base and the height.

$$A = \frac{1}{2} \bullet \text{base} \bullet \text{height}$$

Any side can be the base but once the base is chosen, the height is the length of the perpendicular from the base line to the third vertex.

Note that the height is not necessarily a side of the triangle.

Also, note that in the third triangle, the height line meets the <u>base line</u> at a point external to the triangle itself. That is why the term "base line" and not "base" is used above.

25) The wedge shape below is a right triangle with a horizontal base. The arrow is vertical and perpendicular to the base. The line through the plane is perpendicular to the surface. Prove that the angles marked θ in the figure are equivalent. **Hint:** Consider the complementary angle of θ.