

Problem 1

$$i) \frac{35n}{35n^2} = \frac{1}{n}$$

$$ii) \frac{45}{25} \frac{x^2}{x} = \frac{9}{5} x$$

$$\begin{aligned} iii) \frac{x-8}{x^2+x-72} \\ = \frac{x-8}{(x+9)(x-8)} \\ = \frac{1}{x+9} \end{aligned}$$

$$\begin{aligned} iv) \frac{p^2-3p-54}{p-9} \\ = \frac{(p-9)(p+6)}{(p-9)} \\ = p+6 \end{aligned}$$

$$\begin{aligned} v) \frac{56N-72}{32N} \\ = \frac{8(7N-9)}{8 \cdot 4N} \\ = \frac{7N-9}{4N} \end{aligned}$$

$$\begin{aligned} vi) \frac{N^2+7N-30}{9N^2+90N} \\ = \frac{(N+10)(N-3)}{9N(N+10)} \\ = \frac{N-3}{9N} \end{aligned}$$

$$\begin{aligned} vii) \frac{5m^2-57mn+70n^2}{2m^2-16mn-40n^2} \\ = \frac{(m-10n)(5m-7n)}{2(m^2-8mn-20n^2)} \\ = \frac{(m-10n)(5m-7n)}{2(m-10n)(m+2n)} \\ = \frac{5m-7n}{2(m+2n)} \end{aligned}$$

$$\begin{aligned} a+b &= -57 \\ ab &= 350 \\ a &= -50 \\ b &= -7 \\ 5m^2-50mn-7mn+70n^2 \\ (5m)(m-10n)-7n(m-10n) \\ (m-10n)(5m-7n) \end{aligned}$$

$$\begin{aligned} viii) \frac{xy+3x-2y-6}{y^2+y-6} \\ = \frac{x(y+3)-2(y-3)}{(y+3)(y-2)} \\ = \frac{(y+3)(x-2)}{(y+3)(y-2)} \\ = 1 \end{aligned}$$

$$\begin{aligned} ix) \frac{2x^3+16x^2+24x}{x^2-x-6} \\ = \frac{2x(x^2+8x+12)}{(x-3)(x+2)} \\ = \frac{2x(x+2)(x+6)}{(x-3)(x+2)} \\ = \frac{2x(x+6)}{(x-3)} \end{aligned}$$

$$\begin{aligned}
 x) & \frac{ax - ay + bx - by}{ax - ay - bx + by} \\
 &= \frac{a(x-y) + b(x-y)}{a(x-y) - b(x-y)} \\
 &= \frac{a+b}{a-b}
 \end{aligned}$$

Problem 2

$$\begin{aligned}
 i) & \frac{5y^2}{3} \cdot \frac{9x}{10y} \\
 &= \frac{3yx}{2}
 \end{aligned}$$

$$\begin{aligned}
 ii) & \frac{9y^2}{8} \cdot \frac{32x}{27y} \\
 &= \frac{4yx}{3}
 \end{aligned}$$

$$\begin{aligned}
 iii) & \frac{4x^2y}{2z^2} \cdot \frac{6xz^3}{20y^4} \\
 &= \frac{3x^3z}{5y^3}
 \end{aligned}$$

$$\begin{aligned}
 iv) & \frac{x+4}{3x+4y} \cdot \frac{9x^2-16y^2}{2x^2+3x-20} \\
 &= \frac{x+4}{3x+4y} \cdot \frac{(3x+4y)(3x-4y)}{(x-4)(2x+5)} \\
 &= \frac{(x+4)(3x-4y)}{(x-4)(2x+5)}
 \end{aligned}$$

$$\begin{aligned}
 v) & \frac{3x-6}{5x} \cdot \frac{x^3}{5x-10} \\
 &= \frac{3(x-2)}{5x} \cdot \frac{x^3}{5(x-2)} \\
 &= \frac{3x^2}{25}
 \end{aligned}$$

$$\begin{aligned}
 vi) & \frac{5t^3}{4t-8} \cdot \frac{6t-12}{10t} \\
 &= \frac{5t^3}{4(t-2)} \cdot \frac{6(t-2)}{10t} \\
 &= \frac{3t^2}{4}
 \end{aligned}$$

$$\begin{aligned}
 vii) & \frac{y^2-16}{2y+6} \cdot \frac{y+3}{y-4} \\
 &= \frac{(y-4)(y+4)}{2(y+3)} \cdot \frac{y+3}{y-4} \\
 &= \frac{y+4}{2}
 \end{aligned}$$

$$\begin{aligned}
 viii) & \frac{x^2-16}{x^2} \cdot \frac{x^2-4x}{x^2-x-12} \\
 &= \frac{(x-4)(x+4)}{x^2} \cdot \frac{x(x-4)}{(x-4)(x+3)} \\
 &= \frac{(x+4)(x-4)}{x(x+3)}
 \end{aligned}$$

$$\begin{aligned}
 ix) & \frac{y^2-10y+9}{y^2-1} \cdot \frac{y+4}{y^2-5y-36} \\
 &= \frac{(y-1)(y-9)}{(y-1)(y+1)} \cdot \frac{y+4}{(y-9)(y+4)} \\
 &= \frac{1}{y+1}
 \end{aligned}$$

$$\begin{aligned}
 x) & \frac{4x^2-9y^2}{8x^3-27y^3} \cdot \frac{4x^2+6xy+9y^2}{4x^2+6xy+9y^2} \\
 &= \frac{(2x-3y)(2x+3y)}{(2x-3y)(4x^2+6xy+9y^2)} \\
 &= \frac{2x+3y}{4x^2+6xy+9y^2}
 \end{aligned}$$

Problem 3

$$\begin{aligned} \text{i)} \quad 28p^2q^4 \div \frac{4pq^4}{5r} \\ = 28p^2q^4 \times \frac{5r}{4pq^4} \\ = 35pr \end{aligned}$$

$$\begin{aligned} \text{ii)} \quad \frac{u^5x}{y} \div \frac{4x^2}{y^4} \\ = \frac{u^5x}{y} \times \frac{y^4}{4x^2} \\ = \frac{u^4y^3}{x} \end{aligned}$$

$$\begin{aligned} \text{iii)} \quad \frac{16a^7}{3b^5} \div \frac{8a^3}{6b} \\ = \frac{16a^7}{3b^5} \times \frac{6b}{8a^3} \\ = \frac{4a^4}{b^4} \end{aligned}$$

$$\begin{aligned} \text{iv)} \quad \frac{3y+15}{y^7} \div \frac{y+5}{y^2} \\ = \frac{3y+15}{y^7} \times \frac{y^2}{y+5} \\ = \frac{3}{y^5} \end{aligned}$$

$$\begin{aligned} \text{v)} \quad \frac{3x^2+4x+1}{3x^2-5x-2} \div \frac{x^2-2x-3}{-5x^2+25x-30} \\ = \frac{(x+1)(3x+1)}{(x-2)(3x+1)} \times \frac{5(x-2)(x-3)}{(x-3)(x+1)} \\ = 5 \end{aligned}$$

$$\begin{aligned} \text{vi)} \quad \frac{y^2-9}{y^2} \div \frac{y^5+3y^4}{y+2} \\ = \frac{(y-3)(y+3)}{y} \times \frac{y+2}{y^4(y+3)} \\ = \frac{(y-3)(y+2)}{y^5} \end{aligned}$$

$$\begin{aligned} \text{vii)} \quad \frac{x^2-16}{x^2-10x+25} \div \frac{3x-12}{x^2-3x-10} \\ = \frac{(x-4)(x+4)}{(x-5)(x-5)} \times \frac{(x-5)(x-2)}{3(x-4)} \\ = \frac{(x+4)(x-2)}{(x-5)3} \end{aligned}$$

$$\begin{aligned} \text{viii)} \quad \frac{y^2-16}{y^2-8y+16} \div \frac{3y-18}{y^2-y-12} \\ = \frac{(y-4)(y+4)}{(y-4)(y-4)} \times \frac{(y-4)(y+3)}{3(y-6)} \\ = \frac{(y+4)(y+3)}{3(y-6)} \end{aligned}$$

$$\begin{aligned} \text{ix)} \quad \frac{a^3+4a}{a^2-16} \div \frac{a^2+8a+15}{a^2+a-20} \\ = \frac{a(a^2+4)}{(a-4)(a+4)} \times \frac{(a+5)(a-4)}{(a+5)(a+3)} \\ = \frac{a(a^2+4)}{(a+4)(a+3)} \end{aligned}$$

$$\begin{aligned} \text{x)} \quad \frac{x^3+8y^3}{2x^2+5xy+2y^2} \div \frac{x^3-2x^2y+4xy^2}{8x^2-2y^2} \\ = \frac{(x+2y)(x^2-2xy+4y^2)}{(x+2y)(2x+y)} \times \frac{2(2x+y)(2x-y)}{x(x^2-2xy+4y^2)} \\ = \frac{2(2x-y)}{x} \end{aligned}$$

$$\begin{aligned} m+n &= 5 \\ mn &= 4 \end{aligned}$$

Problem 4

$$i) \frac{x-4}{3} + \frac{5x}{3}$$

$$= \frac{6x-4}{3}$$

$$ii) \frac{8}{x} + \frac{x+9}{x}$$

$$= \frac{x+17}{x}$$

$$iii) \frac{2}{3x} + \frac{4}{x}$$

$$= \frac{2}{3x} + \frac{12}{3x}$$

$$= \frac{14}{3x}$$

$$iv) \frac{6}{5x^3y} - \frac{1}{2x^2y^3}$$

$$= \frac{12xy^2}{10x^3y^3} - \frac{5xy^2}{10x^3y^3}$$

$$= \frac{7xy^2}{10x^3y^3}$$

$$= \frac{7}{10x^2y}$$

$$v) 2x - \frac{x}{y}$$

$$= \frac{2xy}{y} - \frac{x}{y}$$

$$= \frac{2xy-x}{y}$$

$$vi) \frac{x-1}{x+2} - \frac{x+3}{x-4}$$

$$= \frac{(x-1)(x-4) - (x+3)(x+2)}{(x+2)(x-4)}$$

$$= \frac{\cancel{x^2} - 5x + 4 - \cancel{x^2} - 5x - 6}{(x+2)(x-4)}$$

$$= \frac{-10x-2}{(x+2)(x-4)}$$

$$vii) \frac{x-1}{x-2} - \frac{x^2+4x-4}{x^2+4x-12}$$

$$= \frac{x-1}{x-2} - \frac{x^2+4x-4}{(x-2)(x+6)}$$

$$= \frac{(x-1)(x+6) - x^2 - 4x + 4}{(x-2)(x+6)}$$

$$= \frac{\cancel{x^2} + 5x - 6 - \cancel{x^2} - 4x + 4}{(x-2)(x+6)}$$

$$= \frac{x-2}{(x-2)(x+6)} = \frac{1}{x+6}$$

$$viii) -\frac{x-2}{x^2-2x-8} - \frac{x-1}{x^2-4}$$

$$= -\frac{x-2}{(x-4)(x+2)} - \frac{x-1}{(x-2)(x+2)}$$

$$= -\frac{(x-2)(x-2) - (x-1)(x-4)}{(x-4)(x+2)(x-2)}$$

$$= -\frac{-x^2+4x-4 - x^2+5x-4}{(x-4)(x+2)(x-2)}$$

$$= \frac{-2x^2+9x-8}{(x-4)(x+2)(x-2)}$$

$$ix) \frac{x+1}{x^2+6x+9} + \frac{x-4}{x^2-9}$$

$$= \frac{x+1}{(x+3)(x+3)} + \frac{x-4}{(x-3)(x+3)}$$

$$= \frac{(x+1)(x-3) + (x-4)(x+3)}{(x+3)(x+3)(x-3)}$$

$$= \frac{x^2-2x-3 + x^2-x-12}{(x+3)(x+3)(x-3)}$$

$$= \frac{2x^2-3x-15}{(x+3)(x+3)(x-3)}$$

$$x) \frac{1}{x+1} - \frac{x}{x-2} + \frac{x^2+2}{x^2-x-2}$$

$$= \frac{1}{x+1} - \frac{x}{x-2} + \frac{x^2+2}{(x-2)(x+1)}$$

$$= \frac{x-2 - x(x+1) + x^2+2}{(x-2)(x+1)} = \frac{\cancel{x}-2 - \cancel{x^2}-x + \cancel{x^2}+2}{(x-2)(x+1)}$$

$$= 0$$

Problem 5

$$\begin{aligned} i) \quad & \left(\frac{1}{x} + \frac{1}{y} \right) \div (x^2 - y^2) \\ &= \frac{y+x}{xy} \times \frac{1}{(x-y)(x+y)} \\ &= \frac{1}{xy(x-y)} \end{aligned}$$

$$\begin{aligned} ii) \quad & \left(\frac{1}{x-1} - \frac{1}{x+1} \right) \div \frac{1-x^2}{x} \\ &= \frac{x+1-x-1}{(x-1)(x+1)} \times \frac{x}{(1-x)(1+x)} \\ &= \frac{-2x}{(1-x^2)^2} \end{aligned}$$

$$\begin{aligned} iii) \quad & \frac{1}{x-1} - \frac{1}{x+1} \div \frac{1-x^2}{x} \\ &= \frac{1}{x-1} - \frac{1}{x+1} \times \frac{x}{(1-x)(1+x)} \\ &= \frac{-1}{1-x} - \frac{1}{1+x} \times \frac{x}{(1-x)(1+x)} \\ &= \frac{-(1+x)^2 - x}{(1+x)^2(1-x)} \end{aligned}$$

$$\begin{aligned} iv) \quad & \frac{1}{x-1} - \frac{1}{x+1} \cdot \frac{1-x^2}{x} \\ &= \frac{1}{x-1} - \frac{1}{1+x} \cdot \frac{(1-x)(1+x)}{x} \\ &= \frac{1}{x-1} - \frac{1-x}{x} \\ &= \frac{x + (1-x)(1-x)}{x(x-1)} \end{aligned}$$

$$\begin{aligned} v) \quad & \left(\frac{1}{x-1} - \frac{1}{x+1} \right) \cdot \frac{1-x^2}{x} \\ &= - \left(\frac{1}{x-1} - \frac{1}{x+1} \right) \cdot \frac{x^2-1}{x} \\ &= - \left(\frac{x+1-(x-1)}{x} \right) \\ &= - \frac{2}{x} \end{aligned}$$

$$\begin{aligned} vi) \quad & \frac{\frac{1}{1-x} - \frac{1}{x}}{\frac{1}{1-x^2}} \\ &= \frac{x-1+x}{x(1-x)} \times (1-x^2) \\ &= \frac{2x-1}{x(1-x)} (1-x^2) = \frac{(2x-1)(1+x)}{x} \end{aligned}$$

$$\begin{aligned} vii) \quad & \frac{1 - \frac{7}{y} + \frac{12}{y^2}}{1 + \frac{1}{y} - \frac{20}{y^2}} \\ &= \frac{y^2 - 7y + 12}{y^2 + y - 20} \\ &= \frac{(y-3)(y-4)}{(y-4)(y+5)} = \frac{(y-3)}{(y+5)} \end{aligned}$$