Case Study: Assigning Regions to Sales Representatives at Pfizer Turkey

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Outline

Outline

Problem Description

The case focuses on the challenges faced by Pfizer Turkey in updating the territories of its Sales Representatives (SRs), a task that has not been undertaken for several years.

SR	Center Brick	Bricks Assigned
1	4	4, 5, 6, 7, 8, 9, 19, 20
2	14	11, 13, 14, 18
3	16	10, 15, 16, 17
4	22	1, 2, 3, 21, 22

Table: Current structure of sales territories

Problem Description

The study has three primary objectives:

1. Minimizing total travel distance.
2. Maintaining workload balance.
3. Minimizing disruption.

Data Description

Index value for each brick

The workload, or the index value, for each brick.

Brick	Index Value
1	0.1609
2	0.1164
3	0.1026
⋮	⋮
22	0.2531

Table: Workload Dataset

Data Description

All-pair distances between bricks

All-pair distances between bricks.

	Brick1	**Brick2**	**Brick3**	...
Brick1	0.00	7.35	13.21	...
Brick2	7.35	0.00	6.58	...
Brick3	13.21	6.58	0.00	...
⋮	⋮	⋮	⋮	⋱

Table: Distance Between Each Brick

Outline

Optimization Problem

- **Input:** M bricks, with locations and demand (index value). All-pair distances. N sales representatives located at different bricks.
- **Output:** An assignment of each brick to a specific sales representative.

Parameters and Decision Variables

$\mathcal{S} = \{1, \ldots, N\}$: Set of sales representatives.
$\mathcal{B} = \{1, \ldots, M\}$: Set of bricks.
Parameters

- d_{ij} : Distance cost between the bricks i and j.
- y_j : The index value for the brick j.
- x_{ij}^0 : The current structure of sales territories.

Decision Variables

$$x_{ij} = \begin{cases} 1 & \text{Sales representative } i \text{ is assigned to work in brick } j \\ 0 & \text{Otherwise} \end{cases}$$

Objective Functions

1. Total traveled distance

$$\sum_{i=1}^{N}\sum_{j=1}^{M} d_{ij}\, x_{ij}$$

2. Disruption between sales representatives

$$\sum_{i=1}^{N}\sum_{j=1}^{M} |x_{ij}^{0} - x_{ij}|\, y_j \quad \longrightarrow \quad \sum_{i=1}^{N}\sum_{j=1}^{M} (x_{ij}^{0} - x_{ij})^2\, y_j$$

Since x and x^0 are binary variables.

Constraints

1. No partial assignment

$$\forall\, j \in \mathcal{B}, \; \sum_{i=1}^{N} x_{ij} = 1$$

2. Workload balance

$$\forall\, i \in \mathcal{S}, \quad L \leq \sum_{j=1}^{M} x_{ij}\, y_j \leq U$$

Summary

Minimizing the Total Distance

$$\text{minimize:} \quad \sum_{i=1}^{N}\sum_{j=1}^{M} d_{ij}\, x_{ij}$$

subject to:

$$\sum_{i=1}^{N} x_{ij} = 1 \quad j \in \mathcal{B}$$

$$\sum_{j=1}^{M} x_{ij}\, y_j \in [L, U] \quad i \in \mathcal{S}$$

$$x_{ij} \in \{0, 1\} \quad i \in \mathcal{S} \quad j \in \mathcal{B}$$

Minimizing the Disruption

$$\text{minimize:} \quad \sum_{i=1}^{N}\sum_{j=1}^{M} (x_{ij}^{0} - x_{ij})^2\, y_j$$

subject to:

$$\sum_{i=1}^{N} x_{ij} = 1 \quad j \in \mathcal{B}$$

$$\sum_{j=1}^{M} x_{ij}\, y_j \in [L, U] \quad i \in \mathcal{S}$$

$$x_{ij} \in \{0, 1\} \quad i \in \mathcal{S} \quad j \in \mathcal{B}$$

Results ($L = 0.8$ and $U = 1.2$)

SR	Center Brick	Disruption	Total Distance	Bricks Assigned
1	4	1.038	64.37	4, 5, 6, 7, 8, 9, 12, 19, 20
2	14	1.045	4.53	11, 13, 14, 18
3	16	1.115	6.57	10, 15, 16, 17
4	22	0.803	76.130	1, 2, 3, 21, 22

Table: Structure of Sales Territories when minimizing the total distance

SR	Center Brick	Disruption	Total Distance	Bricks Assigned
1	4	0.951	19.30	4, 5, 6, 7, 8, 15
2	14	1.168	7.82	10, 13, 18
3	16	0.874	37.09	9, 11, 12, 16, 17, 18
4	22	1.007	124,740	1, 2, 3, 19, 20, 21, 22

Table: Structure of Sales Territories when minimizing the disruption.

Results ($L = 0.8$ and $U = 1.2$)

Objective function	Disruption	Total Distance
Current Structure (Baseline)	0.0	187.41
Minimize the total distance	1.20	**154.60**
Minimize the disruption	**0.169**	188.95

Table: Mono-Objective Optimization Results

Outline

Multi-Objective Optimization

$$\text{minimize:} \quad \left\{ \sum_{i=1}^{N}\sum_{j=1}^{M} d_{ij}\, x_{ij},\ \sum_{i=1}^{N}\sum_{j=1}^{M}(x_{ij}^{0} - x_{ij})^{2}\, y_j \right\}$$

subject to:

$$\sum_{i=1}^{N} x_{ij} = 1 \quad j \in \mathcal{B}$$

$$\sum_{j=1}^{M} x_{ij}\, y_j \in [L, U] \quad i \in \mathcal{S}$$

$$x_{ij} \in \{0, 1\} \quad i \in \mathcal{S} \quad j \in \mathcal{B}$$

Weighted Sum Method

- Consider a weight vector w
- Solve the weighted sum

$$x^{\star} = \arg\min_{x \in \mathcal{X}} w_1 \, f_1(x) + w_2 \, f_2(x)$$

- $(f_1(x^{\star}), f_2(x^{\star}))$ is a supported non-dominated solutions.

The ε-Constraint Method

Algorithm 1: The ε-Constraint Method

1 **Initialization:**

- Determine x^1, an optimal solution for f_1 $(\varepsilon_2 = \infty)$.
- $\mathrm{ND} = \{x^1\}$
- $\varepsilon = f_2(x^1) - \epsilon$

while $\min_{x \in \mathcal{X}} \left\{ f_1(x) \ : \ f_2(x) \leq \varepsilon \right\}$ **do**

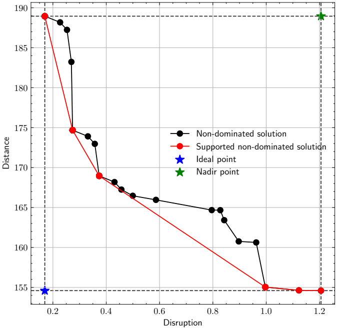
Denote x^{nd}, an optimal solution of the program

$\mathrm{ND} = \mathrm{ND} \cup \{x^{\mathrm{nd}}\}$

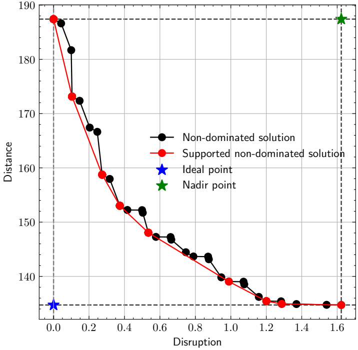
$\varepsilon = f_2(x^{\mathrm{nd}}) - \epsilon$

Results

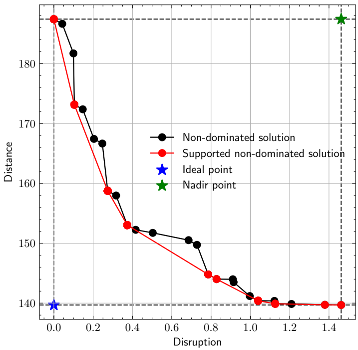
Set of supported non-dominated solutions and the set of non-dominated solutions ($L = 0.8,\ U = 1.2$).



Workload Balance Constraint Relaxation

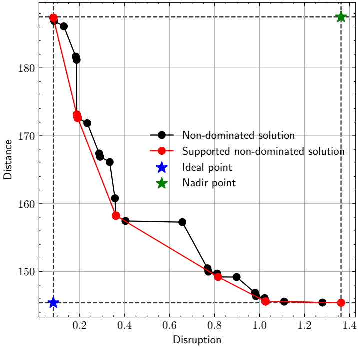


(a) $L = 0.0$ and $U = 2.0$

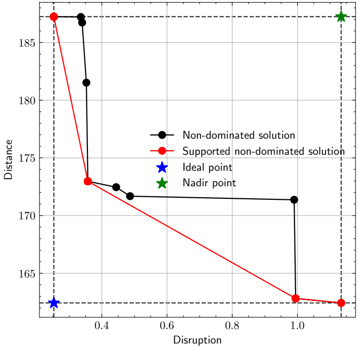


(b) $L = 0.6$ and $U = 1.4$

Workload Balance Constraint Relaxation



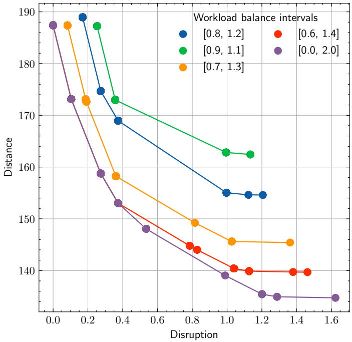
(c) $L = 0.7$ and $U = 1.3$



(d) $L = 0.9$ and $U = 1.1$

Sensitivity Analysis of the Workload Balance Constraint

As we relax the workload balance constraint more, both the disruption and the total traveled distance decrease.



Outline

Assigning a brick to multiple Sales Representatives

Decision Variables

$$x_{ij} = \begin{cases} 1 & \text{Sales representative } i \text{ is assigned to work in brick } j \\ 0 & \text{Otherwise} \end{cases}$$

$$x^{c}_{ij} \in \mathbb{R}_{+}$$

Constraint

$$x_{ij} = \begin{cases} 1 & \text{if } x^{c}_{ij} > 0 \\ 0 & \text{if } x^{c}_{ij} = 0 \end{cases} \longrightarrow \begin{cases} x^{c}_{ij} \geq \epsilon - M \cdot (1 - x_{ij}) \\ x^{c}_{ij} \leq M \cdot x_{ij} \end{cases}$$

Assigning a brick to multiple Sales Representatives

The New Mathematical Formulation

$$\text{minimize:} \quad \left\{ \sum_{i=1}^{N} \sum_{j=1}^{M} d_{ij}\, x_{ij},\ \sum_{i=1}^{N} \sum_{j=1}^{M} (x_{ij}^{0} - x_{ij}^{c})^2\, y_j \right\}$$

subject to:

$$\sum_{i=1}^{N} x_{ij}^{c} = 1 \quad j \in \mathcal{B}$$

$$\sum_{j=1}^{M} x_{ij}^{c}\, y_j \in [L, U] \quad i \in \mathcal{S}$$

$$x_{ij}^{c} \geq \epsilon - M \cdot (1 - x_{ij}) \quad i \in \mathcal{S} \quad j \in \mathcal{B}$$

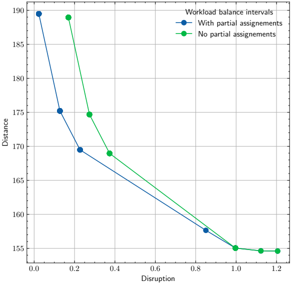
$$x_{ij}^{c} \leq M \cdot x_{ij} \quad i \in \mathcal{S} \quad j \in \mathcal{B}$$

$$x_{ij}^{c} \in \mathbb{R}_{+} \quad x_{ij} \in \{0, 1\} \quad i \in \mathcal{S} \quad j \in \mathcal{B}$$

Assigning a brick to multiple Sales Representatives

Results

Allowing sales representatives to share bricks results in reduced travel distances and less disruption compared to when partial assignments between them are not permitted.



Managing Increased Brick Demand

What is the percentage increase in demand that would necessitate hiring a new sales representative?

Managing Increased Brick Demand

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Algorithm 3: Find the percentage that would necessitate hiring a new sales representative

Initialization:

- $p \leftarrow 0$
- $\mathrm{FS} \leftarrow \{p\}$

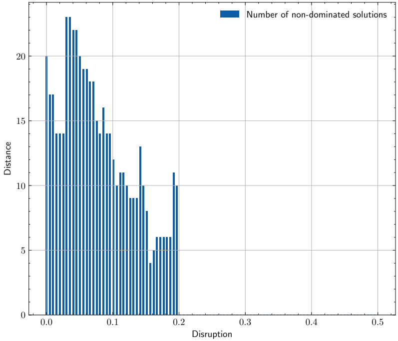
while The program still has feasible solutions **do**

  $\mathrm{FS} \leftarrow \mathrm{FS} \cup \{p\}$

  $p \leftarrow p + \epsilon$

Managing Increased Brick Demand

What is the percentage increase in demand that would necessitate hiring a new sales representative?



Managing Increased Brick Demand

Now, we have a 20% increase in demand. Where should the office of the newly recruited sales representative be located?

Managing Increased Brick Demand

Now, we have a 20% increase in demand. Where should the office of the newly recruited sales representative be located?
We need to add a new decision variable:

$$c_k = \begin{cases} 1 & \text{The new SR office is at the brick } k \\ 0 & \text{Otherwise} \end{cases}$$

Subject to

$$\sum_{j=1}^{M} c_j = 1$$

Managing Increased Brick Demand

The New Mathematical Formulation

minimize: $$\left\{ \sum_{i=1}^{N}\sum_{j=1}^{M} d_{ij}\, x_{ij} + \sum_{j=1}^{M}\left(\sum_{k=1}^{M} c_k\, D_{jk}\right) x_{5j}\, ,\ \sum_{i=1}^{N}\sum_{j=1}^{M}(x_{ij}^{0} - x_{ij})^2\, y_j \right\}$$

subject to:

$$\sum_{j=1}^{M} c_j = 1$$

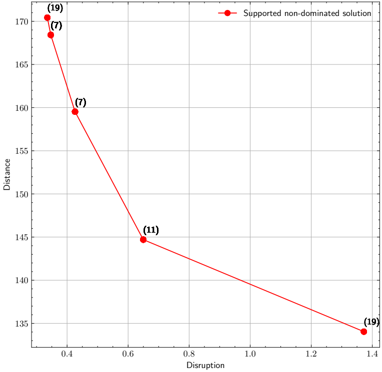
$$\sum_{i=1}^{N} x_{ij} = 1 \quad j \in \mathcal{B}$$

$$\sum_{j=1}^{M} x_{ij}\, y_j \in [L, U] \quad i \in \mathcal{S}$$

$$c_j \in \{0,1\} \quad x_{ij} \in \{0,1\} \quad i \in \mathcal{S} \quad j \in \mathcal{B}$$

Managing Increased Brick Demand

Results



Relocating Sales Representatives Offices

This is a generalization of the problem of determining the optimal location for placing the office of a new hire.

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This is a generalization of the problem of determining the optimal location for placing the office of a new hire.
Now, we need to increase the size of the decision variable c

$$c_{ik} = \begin{cases} 1 & \text{The SR } i \text{ office is at the brick } k \\ 0 & \text{Otherwise} \end{cases}$$

Subject to

$$\sum_{j=1}^{M} c_{ij} = 1 \quad i \in \mathcal{S}$$

Relocating Sales Representatives Offices

The New Mathematical Formulation

$$\text{minimize: } \left\{ \sum_{i=1}^{N} \sum_{j=1}^{M} \left(\sum_{k=1}^{M} c_{ik} \, D_{jk} \right) x_{ij} \, , \, \sum_{i=1}^{N} \sum_{j=1}^{M} (x_{ij}^0 - x_{ij})^2 \, y_j \right\}$$

subject to:

$$\sum_{j=1}^{M} c_{ij} = 1 \quad i \in \mathcal{S}$$

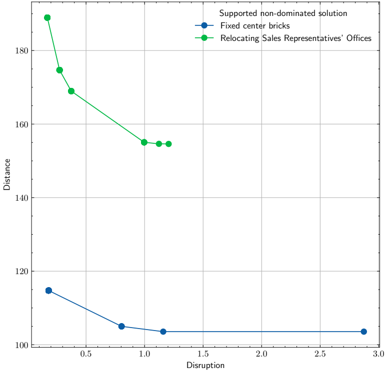
$$\sum_{i=1}^{N} x_{ij} = 1 \quad j \in \mathcal{B}$$

$$\sum_{j=1}^{M} x_{ij} \, y_j \in [L, U] \quad i \in \mathcal{S}$$

$$c_{ij} \in \{0, 1\} \quad x_{ij} \in \{0, 1\} \quad i \in \mathcal{S} \quad j \in \mathcal{B}$$

Relocating Sales Representatives Offices

Results



Relocating Sales Representatives Offices

Results

SR	Center Brick	Disruption	Total Distance	Bricks Assigned
1	4	1.007	56.846	1, 2, 3, 19, 20, 21, 22
2	14	0.834	34.094	4, 5, 6, 7, 8, 9, 12
3	16	1.131	6.237	10, 11, 15, 16
4	22	1.029	6.385	13, 14, 17, 18

Table: Structure of Sales Territories when minimizing the total traveled distance

SR	Center Brick	Disruption	Total Distance	Bricks Assigned
1	8	0.951	19.686	4, 5, 6, 7, 8, 15
2	6	1.168	27.500	10, 13, 14
3	10	0.874	52.111	9, 11, 12, 16, 17, 18
4	16	1.007	192.579	1, 2, 3, 19, 20, 21, 22

Table: Structure of Sales Territories when minimizing the disruption

Relocating Sales Representatives Offices

Results

Decision	Objective function	Disruption	Total Distance
Baseline	Current Structure	0.0	187.41
Minimize the total distance	Fixed center bricks	**1.20**	154.60
	Relocating SRs Offices	3.537	**103.56**
Minimize the disruption	Fixed center bricks	**0.169**	**188.95**
	Relocating SRs Offices	**0.169**	291.87

Table: Mono-Objective Optimization Results

Sales Representatives' Preferences

$$\text{minimize} \left\{ w_1 \sum_{i=1}^{N} \sum_{j=1}^{M} d_{ij}\, x_{ij},\ + w_2 \sum_{i=1}^{N} \sum_{j=1}^{M} r_{ij}\, x_{ij} \right\} \tag{1}$$

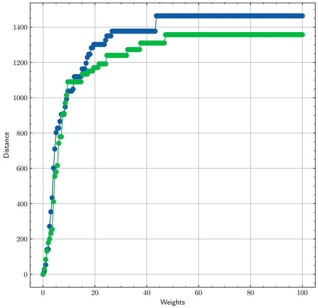
subject to:

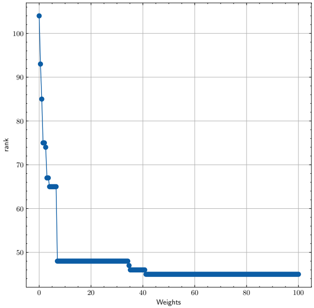
$$\sum_{i=1}^{N} x_{ij} = 1 \quad j \in \mathcal{B}$$

$$\sum_{j=1}^{M} x_{ij}\, y_j \in [L, U] \quad i \in \mathcal{S}$$

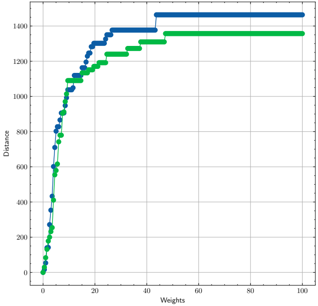
$$x_{ij} \in \{0, 1\} \quad i \in \mathcal{S} \quad j \in \mathcal{B}$$

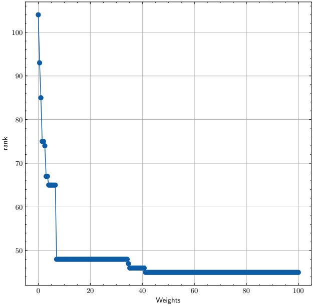
Sales Representatives' Preferences





Sales Representatives' Preferences

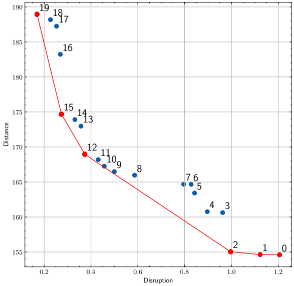




The Weighted Sum method alone is insufficient to model the preferences of sales representatives.

Decision Maker' Preferences

Set of non-dominated solutions for the basic model, where $L = 0.8$ and $U = 1.2$.



Decision Maker' preferences: $4 \succ 2 \succ 17 \succ 1 \succ 18 \succ 0 \succ 19$.

Optimization Problem

$$u(a) = \sum_{i=1}^{n} u_i(g_i(a))$$
$$u_0(a) = u(a) + \sigma(a), \quad \forall a \in \mathcal{A}_0$$

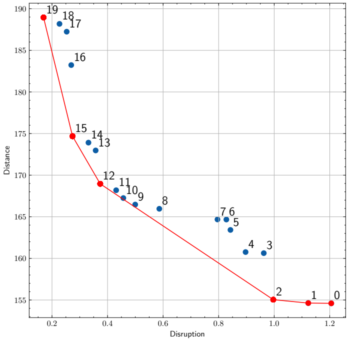
$$\text{minimize} \sum_{a \in \mathcal{A}_0} \sigma(a)$$

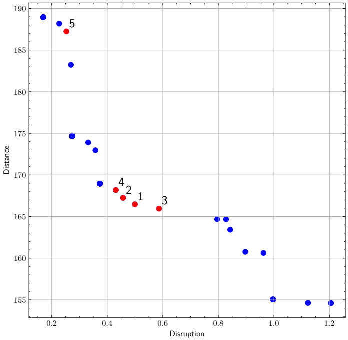
subject to:

$$u(a) + \sigma(a) - u(b) - \sigma(b) = 0 \quad \text{if} \quad a \sim b$$
$$u(a) + \sigma(a) - u(b) - \sigma(b) > 0 \quad \text{if} \quad a \succ b$$
$$u_i(u_l^i) - u_i(u_{l+1}^i) > 0, \quad \forall i, \quad \forall l$$
$$u_i(u_0^i) = 0, \quad \forall i$$
$$\sum_{i=1}^{n} u_i(u_L^i) = 1,$$
$$u_i(u_l^i), \quad \forall i, \quad \forall l$$

Decision Maker' Preferences

In the figure on the right, the red dots are ranked from best to worst.





Outline

Recommendations

1. If the primary concern of the decision maker is to minimize travel distances in order to reduce expenses, then relocating the offices of sales representatives should be considered. However, it is essential to acknowledge that such a decision may lead to significant disruptions among the sales representatives.
2. Hiring a new sales representative is unnecessary unless there is a demand increase exceeding 20%.
3. If there's a 20% increase in demand, hiring a new sales representative is crucial, and situating their office in the eleventh territory can optimize efficiency.

Recommendations

4. Taking into account the decision maker's preferences will aid in prioritizing certain solutions over others. This approach ensures interpretability.
5. Currently, we are operating under the assumption that the demand for each brick is constant. If the decision maker wishes to incorporate non-stationarity, such as seasonal demand, it is recommended to include workload balance as the third objective function and consider L and U as decision variables. The workload balance should always be kept close to 1.