CS164 Programming Languages and Compilers **Spring 2024**

Written Assignment 6 Solutions

Assigned: April 10 **Due:** April 19 at 3:00pm

1. Consider the following code:

```
p := a + b
r := 1 * b
s := a * b
x := 0 * r
y := a + b
u := x + 2
w := y * y
v := u + w
z := p ** 2
```

Assume that only `v` and `z` are live at the exit of this block. Apply the following optimization in order, one by one, to this basic block. Show the result of each transformation and the final optimized code.

(a) algebraic simplification
Solution:

```
p := a + b
r := b
s := a * b
x := 0
y := a + b
u := x + 2
w := y * y
v := u + w
z := p * p
```

(b) common sub-expression elimination
Solution:

```
p := a + b
r := b
s := a * b
x := 0
y := p
u := x + 2
w := y * y
v := u + w
z := p * p
```

(c) copy propagation
Solution:

```
p := a + b
r := b
s := a * b
x := 0
```

```
y := p
u := x + 2
w := p * p
v := u + w
z := p * p
```

(d) constant folding/propagation
Solution:

```
p := a + b
r := b
s := a * b
x := 0
y := p
u := 2
w := p * p
v := 2 + w
z := p * p
```

(e) dead code elimination
Solution:

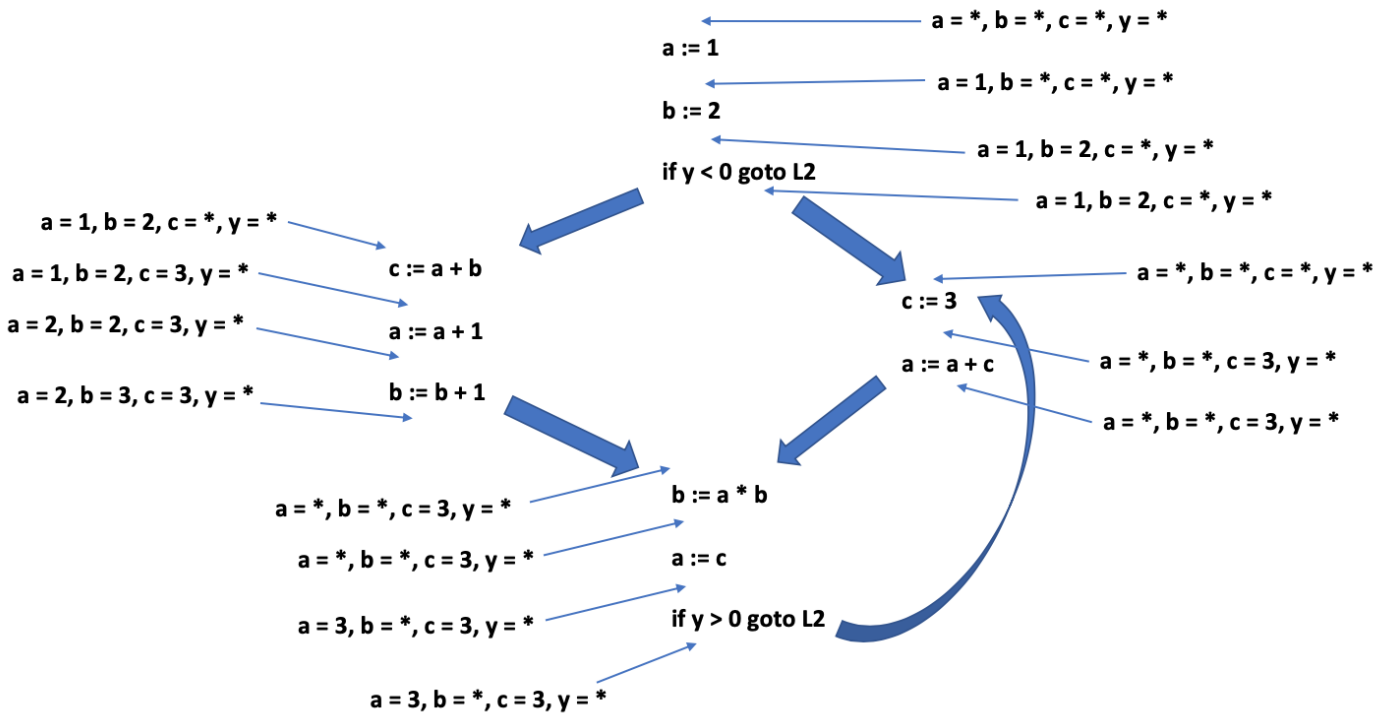
```
p := a + b
w := p * p
v := 2 + w
z := p * p
```

(f) When you've completed part (e), the resulting program will still not be optimal. What optimization can you apply to optimize the result of (e) further? What does this tell you about how one should apply local optimizations?
Solution:
We can eliminate the common sub-expression of $p*p$. Thus, we see that different local optimizations can enable other local optimizations to be applied. We should attempt to do the different optimization techniques until the code does not change, or if we want to compile code in a predefined amount of time, we should continue to do the different techniques until we reach that window of time.

2. Optimize the following code, using global constant propagation (do NOT perform constant folding as that is not part of the global constant propagation algorithm). Draw the CFG and the associated constant propogation dataflow, then write out the optimized code.

```
L0:
  a := 1
  b := 2
  if y < 0 goto L2

L1:
  c := a + b
  a := a + 1
  b := b + 1
  jump L3

L2:
  c := 3
  a := a + c

L3:
  b := a * b
  a := c
  if y > 0 goto L2
```

Solution:
This is the CFG and the associated constant propagation dataflow



The following is the code after doing constant propagation based on the dataflow analysis:

```
L0:
  a := 1
  b := 2
  if y < 0 goto L2

L1:
  c := 1 + 2
  a := 1 + 1
  b := 2 + 1
  jump L3

L2:
  c := 3
  a := a + 3

L3:
  b := a * b
  a := 3
  if y > 0 goto L2
```

3. Consider the following source-level optimizations proposed for the ChocoPy language:

   (a) Replace '`False and e`' with '`False`', where `e` is any ChocoPy expression of type `bool`.

   (b) Replace '`e or True`' with '`True`', where `e` is any ChocoPy expression of type `bool`.

   (c) Replace '`x - 0`' with '`x`', where `x` is any ChocoPy variable of type `int`.

   (d) Replace '`0 % x`' with '`0`', where `x` is any ChocoPy variable of type `int`.

   (e) Replace '`x + 1 >= x`' with '`True`', where `x` is any ChocoPy variable of type `int`.

   It is not important how the optimizations are implemented; you can think of them as search-and-replace over ASTs of semantically valid and well-typed programs.

   For each optimization, answer the following sub-questions:

   (i) Is there any ChocoPy program, whose output will be different if you execute it with and without this optimization? You can use the compiler on chocopy.org to observe a program's output. If your answer is 'yes', then provide a short program (in unoptimized form) that demonstrates this difference, and list its output before and after optimization.

   (ii) Is this optimization sound? Answer 'yes' or 'no'. A compiler optimization is sound if its application does not change the behavior of the program under the language's operational semantics.

   Solutions:

   (a) Replace '`False and e`' with '`False`', where `e` is any ChocoPy expression of type `bool`.

      i. No, there is no difference. Due to short-circuit evaluation, the expression `e` is guaranteed to never be evaluated.

      ii. Yes, this optimization is sound.

   (b) Replace '`e or True`' with '`True`', where `e` is any ChocoPy expression of type `bool`.

      i. Yes, there is a difference, because `e` may have side-effects. For example:

```
x:int = 0
def setx() -> bool:
  global x
  x = 1
  return False

if setx() or True:
  print(x) # Prints 1 without optimization; 0 if optimization is applied
```

      ii. No, this is optimization not sound.

   (c) Replace '`x - 0`' with '`x`', where `x` is a ChocoPy variable of type `int`.

      i. No, there is no difference.

      ii. Yes, this optimization is sound.

   (d) Replace '`0 % x`' with '`0`', where `x` is a ChocoPy variable of type `int`.

      i. Yes, there is a difference. For example:

```
x:int = 0
print(0 % x) # Divide-by-zero error without optimization
             # Prints value 0 if optimization is applied
```

      ii. No, this optimization is not sound.

   (e) Replace '`x + 1 >= x`' with '`True`', where `x` is a ChocoPy variable of type `int`.

i. Yes, there is a difference. For example:

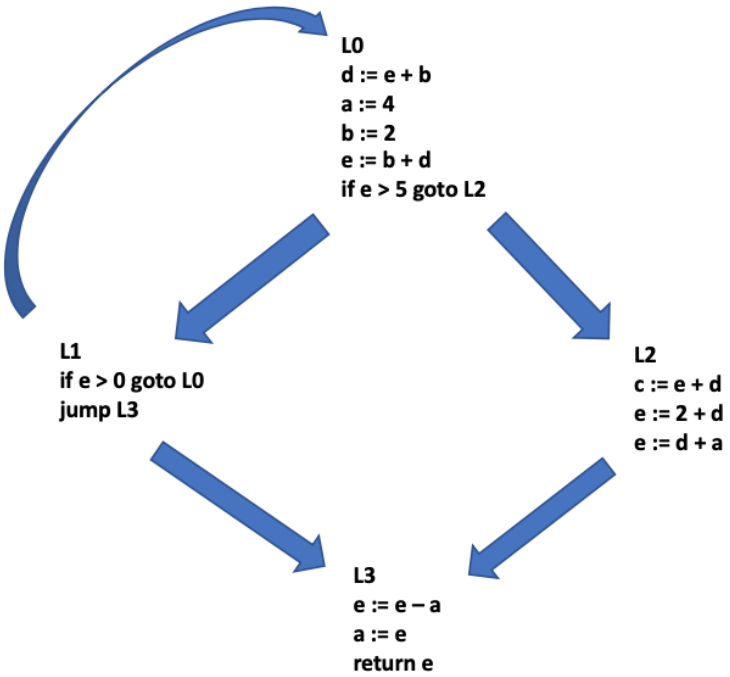
```
x:int = 2147483647  # Largest int = 2**31 - 1
print(x + 1 >= x) # Prints ‘False‘ without optimization
                  # Prints ‘True‘ if optimization is applied
```

ii. Yes, this optimization is sound. Since integer overflow has undefined behavior, there is no violation of the operational semantics here.

4. Consider the following program.

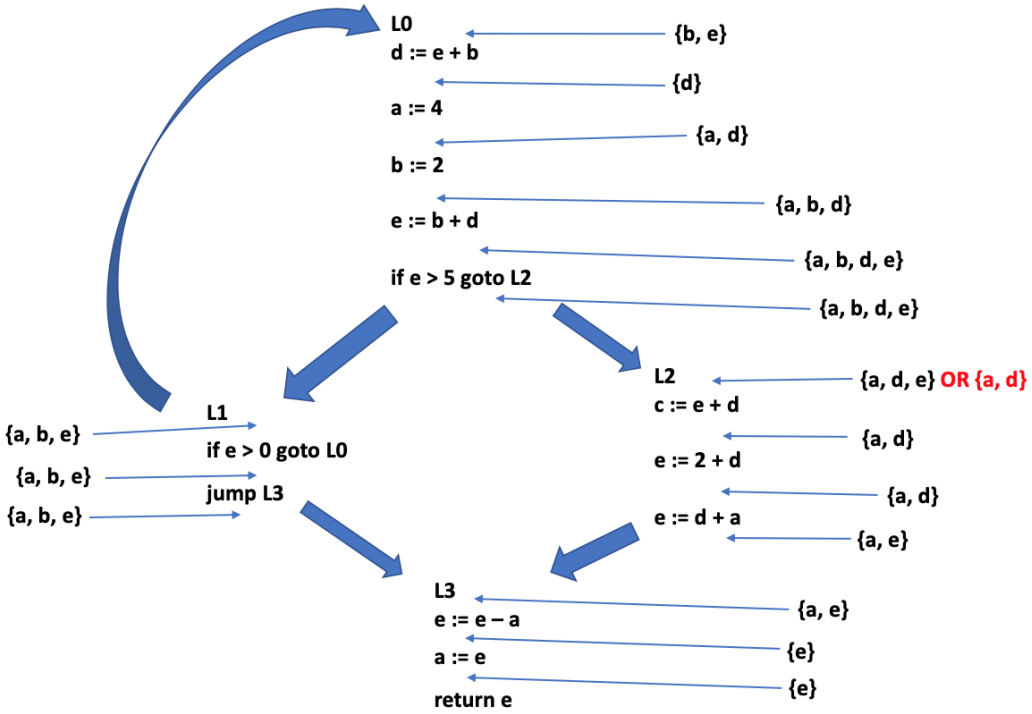
```
L0:
  d := e + b
  a := 4
  b := 2
  e := b + d
  if e > 5 goto L2

L1:
  if e > 0 goto L0
  jump L3

L2:
  c := e + d
  e := 2 + d
  e := d + a

L3:
  e := e - a
  a := e
  return e
```

(a) Construct a control-flow graph for the program
Solution:



(b) Perform Liveness Analysis on the program (use the method described in lecture on 11/19). Do not do any copy propagation or constant folding but eliminate dead code based on your liveness analysis.

Solution:

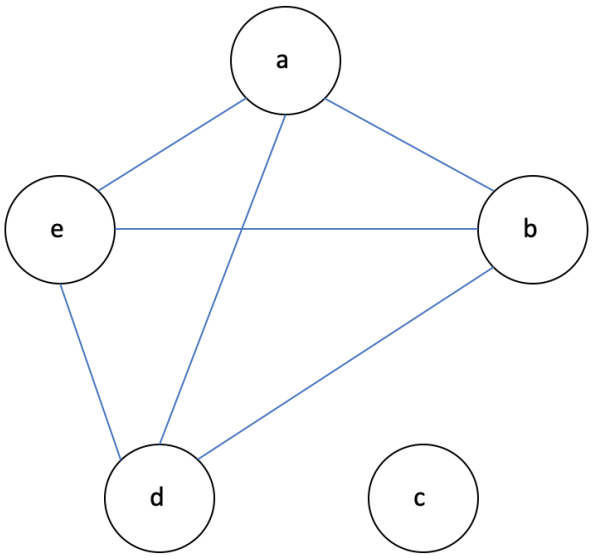


*Note: For $In(c := e + d)$, {a, d} is also an acceptable answer because it may have been unclear based on the lecture slides whether e should have been included since c is not live after the statement.

Code after Dead Code Elimination (1 line eliminated in L3, 2 lines eliminated in L2):

```
L0:
  d := e + b
  a := 4
  b := 2
  e := b + d
  if e > 5 goto L2

L1:
  if e > 0 goto L0
  jump L3

L2:
  e := d + a

L3:
  e := e - a
  return e
```

(c) Construct a RIG (Register Interference Graph) for the original program above, without the optimization done in part b. (Hint: a, b, c, d, e should be the nodes in your graph.)
Solution:



(d) Find a coloring scheme with 3 colors for the RIG using the heuristic introduced in class. Break possible ties in alphabetical order. You will find that you get stuck at a certain point and will need to spill a certain register into memory. Decide this node alphabetically.

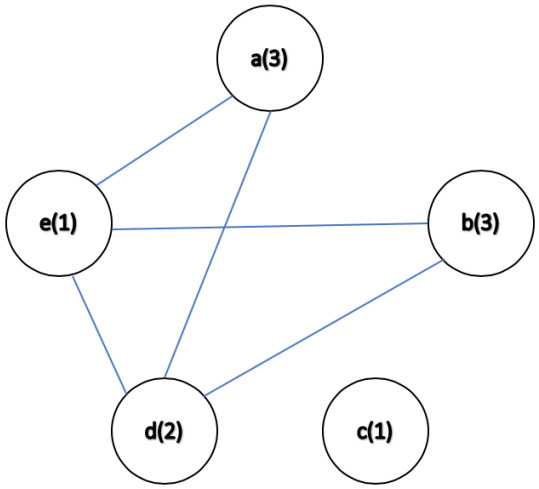
Solution:

1. Push c onto stack.
2. Spill a (No longer edge between a and b)
3. Push a onto stack
4. Push b onto stack
5. Push d onto stack
6. Push e onto stack
7. Pop e off stack, and give it color 1
8. Pop d off stack, and give it color 2
9. Pop b off stack, and give it color 3
10. Pop a off stack, and give it color 3
11. Pop c off stack, and give it color 1

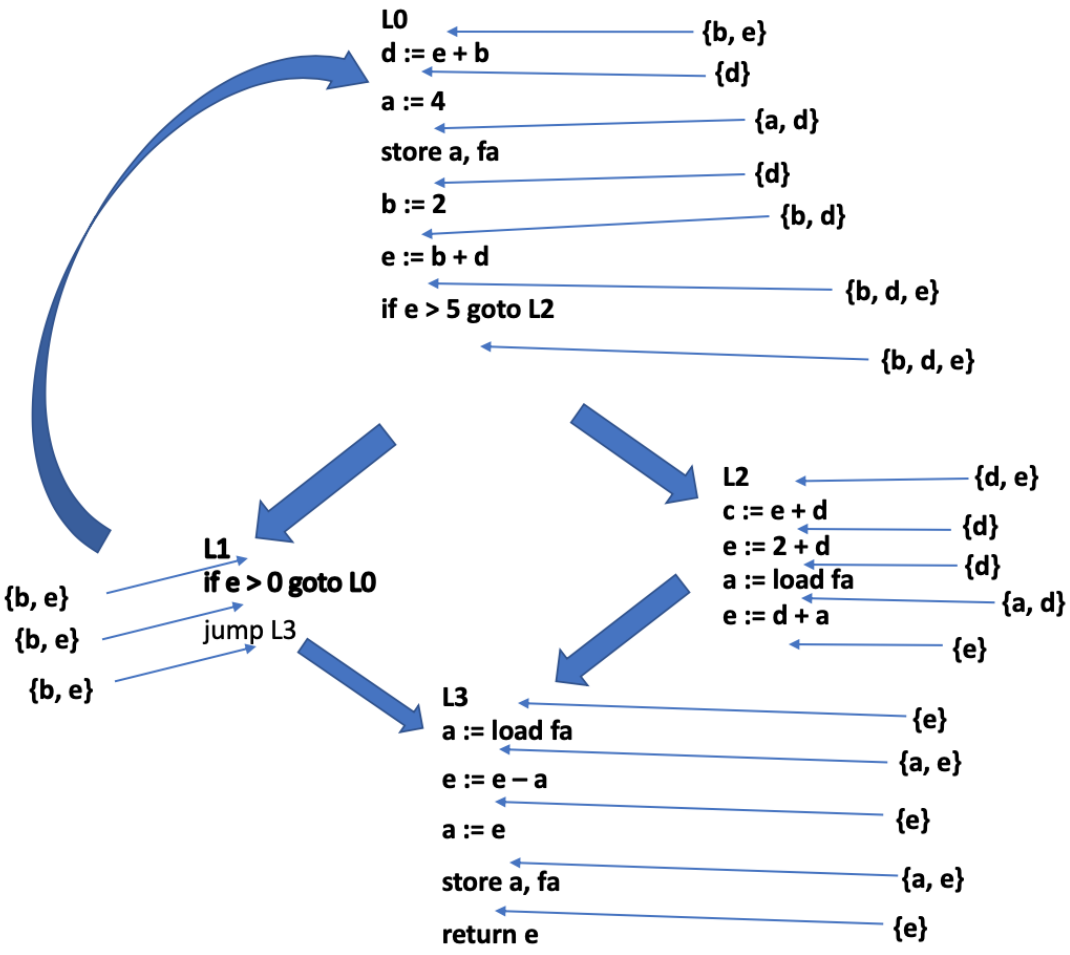
Detailed Explanation:
Since c has no edges, we can first push c.
Of the remaining temporaries (a, b, d, e) they all have 3 edges and thus we need to spill one of them; we will decide this alphabetically and thus we choose to spill temporary a.
As we see in the next part, when we spill a it will cause there to no longer be an edge between a and b.
Thus now temporary a and b have 2 edges which is less than the number of colors. Thus we can push a onto the stack, and then push b onto the stack.
The remaining graph only has temporaries d and e which now both only have 1 edge. Thus we push d onto the stack and then we push e onto the stack.
Now we can begin popping from the stack and coloring the nodes in RIG accordingly. First, we pop off temporary e, and give it color 1. Next, we pop off temporary d, which we give color 2 since it is connected to temporary e which has color 1.
Next we pop off temporary b, which we give color 3 since it is connected to e which has color 1 and to d which has color 2.
Then we pop off temporary a, we must give it color 3 since it is connected to e which has color 1 and to d which has color 2.

Finally, we pop off temporary c, which we give color 1 (though any of the colors works here).

Resulting RIG:



(e) Construct a CFG for the spilled program.
Solution:
Optionally, you could delete the dead code determined in part b.


L0
d := e + b
a := 4
store a, fa
b := 2
e := b + d
if e > 5 goto L2
{b, e}
{d}
{a, d}
{d}
{b, d}
{b, d, e}
{b, d, e}
L1
if e > 0 goto L0
jump L3
{b, e}
{b, e}
{b, e}
L2
c := e + d
e := 2 + d
a := load fa
e := d + a
{d, e}
{d}
{d}
{a, d}
{e}
L3
a := load fa
e := e – a
a := e
store a, fa
return e
{e}
{a, e}
{e}
{a, e}
{e}

5. In this problem, we will construct a dataflow analysis for an extended form of the intermediate language described in lecture. Recall that we defined an intermediate language whose instructions were:

```
instr:
| id := id op id          (binary operation)
| id := op id             (unary operation)
| id := id                (copy)
| push id                 (push)
| id := pop               (pop)
| if id relop id goto L   (conditional jump)
| L:                      (label)
| jump L                  (unconditional jump)
```

Now let us add the following new kinds of instructions:

```
instr:
...
| id := new T
| id := call id, m
```

The instruction `id := new T` constructs a new instance of class `T`, and assigns it to `id`; the equivalent in ChocoPy code would be `id = T()`. The instruction `id1 := call id2, m` calls the method `m` on the class object stored in `id2`, and assigns it to `id1`; it is the equivalent of `id1 = id2.m(...)`. (Note that the arguments for `m` are located on the stack, so the intermediate-language instruction itself does not need to refer to the arguments.)

As mentioned in lecture, it is sometimes possible to statically determine which exact instance of `m` will be called by any `id := call id, m` instruction. For example, in the following ChocoPy code, it is easy to see that the `f` method called is necessarily the version defined in `B`, instead of the version defined in `A`:

```
class A(object):
    def f(self: "A"):
        print("A:f")
class B(A):
    def f(self: "B"):
        print("B:F")
obj : A = None
result : object = None
obj = B()
result = obj.f()
```

The resulting IL (intermediate language) representation of the top-level code could be:

```
obj := new B
push obj
result := call obj, f
```

From this IL we may immediately verify the observation that `B::f` is called instead of `A::f`. Since `obj` is assigned a `B` value, and there are no intervening assignments to `obj`, then the last line `result := call obj, f` necessarily calls the version of `f` defined in class `B`. In other words, we know that the last line will always call the `B` version of `f` since we know that just before the last IL instruction, the dynamic type of `obj` is in the set {`B`}, and since this set is a singleton, this just means that we know exactly what the dynamic type of `obj` is.

Of course we may not be able to statically determine in all cases which instance of a method will be called at a particular program point. Consider the following IL, which uses the same class definitions as before; also assume that the register `i` has some unspecified integer value:

```
    if i == 0 goto i_is_zero
    jump i_is_not_zero
i_is_not_zero:
    obj := new A
    jump make_the_call
i_is_zero:
    obj := new B
    jump make_the_call
make_the_call:
    result := call obj, f
```

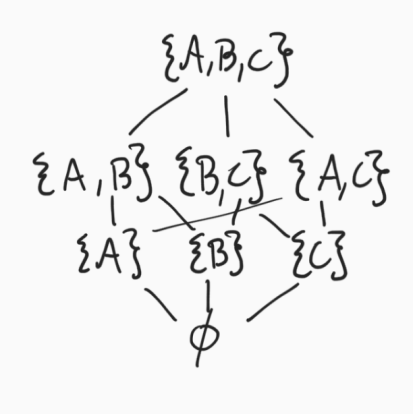
In this IL example, the instance of `f` called (i.e. `A::f` or `B::f`) depends on the value if `i`, which may not be statically determinable; if `i == 0` then we call the `B` version of `f`, and if `i != 0` then we call the `A` version of `f`. We may view this as related to the following data: we know that just before the last IL instruction, the dynamic type of `obj` is in the set $\{\texttt{A}, \texttt{B}\}$; since this set is *not* a singleton, then we don't know exactly which version of `f` will be called.

We now want to define a dataflow analysis which will provide (conservative) data on what the possible dynamic types are for any variable at any program point. For this analysis, our *value domain*, i.e. the collection of abstract values we can assign to any variable at any program point, will be the collection of *any set of classes*. For example, $\{\texttt{A}, \texttt{B}\}$ is an element of our value domain, as is $\{\texttt{B}\}$ and also $\emptyset$, the empty set (which may represent the possible dynamic types of an unassigned variable).

For any IL instruction p and register x, let $T_{in}(x, p)$ be a conservative approximation for the dynamic types of x just before p, and let $T_{out}(x, p)$ be a conservative approximation for the dynamic types of x just after p. This means that if x can have dynamic type T just before the instruction p, then we must have $T \in T_{in}(x, p)$; and similarly, if x can have dynamic type T just after the instruction p, then we must have $T \in T_{out}(x, p)$.

(a) Should this analysis be a forward or backward dataflow analysis?
It should be a forward dataflow analysis, since like constant propagation we want to pass type information forward through expressions and statements.

(b) For an IL program which has (only) the classes `A`, `B`, and `C` defined, where `B` and `C` are subclasses of `A`, draw the resulting value-domain lattice. Make sure that $\emptyset$ is at the bottom and that $\{\texttt{A}, \texttt{B}, \texttt{C}\}$ is at the top.
Solution:



(c) Now we must define the transfer functions of this analysis.

i. Suppose that $p_1, \ldots, p_n$ are the direct predecessors of the IL instruction s. What should $T_{in}(x, s)$ be in terms of $T_{out}(x, p_1), \ldots, T_{out}(x, p_n)$? If you use the `lub` operator of our value domain, then you must define it. (It has a relatively simple form.)
For $C_1, \ldots, C_n$ sets of classes, we let $lub(C_1, \ldots, C_n) = C_1 \cup \cdots \cup C_n$. Then

$$T_{in}(x, s) = lub(T_{out}(x, p_1), \ldots, T_{out}(x, p_n)).$$

ii. Suppose that p is the IL instruction `x := y`. What should $T_{out}(r, p)$ be in terms of $T_{in}(r, p)$, for any register r? (Hint: split the analysis into two cases: $r =$ `x` or $r \neq$ `x`.)
We have $T_{out}(\texttt{x}, p) = T_{in}(\texttt{y}, p)$, and $T_{out}(r, p) = T_{in}(r, p)$ for every other register r.

iii. Suppose that p is the IL instruction `x := new T`. What should $T_{out}(r, p)$ be in terms of $T_{in}(r, p)$, for any register r? (Hint: again, split the analysis into cases.)
We have $T_{out}(\texttt{x}, p) = \{\texttt{T}\}$, and $T_{out}(r, p) = T_{in}(r, p)$ for every other register r.

iv. Suppose that p is the IL instruction `x := call y, m`, and suppose that the dynamic type of the return value of m is known to be U. What should $T_{out}(r, p)$ be in terms of $T_{in}(r, p)$, for any register r?
We have $T_{out}(\texttt{x}, p) = \{U\}$, and $T_{out}(r, p) = T_{in}(r, p)$ for every other register r.

(d) Here are the transfer functions for the other IL instructions:

- $T_{out}(\texttt{x}, \texttt{x := y op z}) = \{t_{\texttt{op}}(T_{\texttt{y}}, T_{\texttt{z}})\}$, where $T_{\texttt{y}}$ is the statically-known type of `y` (and $T_{\texttt{z}}$ the statically-known type of `z`), and $t_{\texttt{op}}(T, U)$ for two primitive types T and U is defined to be the resulting type of evaluating `op` on a value of type T and a value of type U. For example, $t_{+}(\texttt{int}, \texttt{int}) = \texttt{int}$, while $t_{+}(\texttt{str}, \texttt{str}) = \texttt{str}$ and $t_{==}(\texttt{object}, \texttt{[object]}) = \texttt{bool}$ since the `==` operator on any two values always returns a `bool`. We also have $T_{out}(r, \texttt{x := y op z}) = T_{in}(r, \texttt{x := y op z})$ for $r \neq$ `x`.
- Similarly define $T_{out}(\texttt{x}, \texttt{x := op y}) = \{t_{\texttt{op}}(T_{\texttt{y}})\}$, where now $t_{\texttt{op}}(T)$ is defined to be the resulting type of evaluating `op` on a value of type T. As before, $T_{out}(r, \texttt{x := op y}) = T_{in}(r, \texttt{x := op y})$ for $r \neq$ `x`.
- For p an instruction `push id` or `if id relop id goto L`, we simply have $T_{out}(x, p) = T_{in}(x, p)$ for any register x.

Now consider the following full analysis algorithm:

```
Initialize Tin(x,p) and Tout(x,p) to {} for every register x and instruction p.
while there is any Tin(x,p) or Tout(x,p) not satisfying our transfer functions:
    update Tin(x,p) and Tout(x,p) using our transfer functions
```

Explain why this algorithm is guaranteed to terminate. (Hint: make the claim that any time we update $T_{in}(x, p)$ or $T_{out}(x, p)$, the new value is at least as high in our value domain lattice as is the previous value, that is, if v was the original value and v' the new value, then $v \subseteq v'$. Then argue that our value domain lattice has finite height, and explain why these two statements imply that the algorithm terminates.)

Our value domain lattice has finite height since any chain in the lattice is a strictly increasing sequence of subsets of classes, and there are only finitely many classes in any given program, so any chain has length at most the number of classes. Now it suffices to show that any update operation cannot reduce the "level" of any set of classes (i.e. cannot move the set lower in the lattice), since then we know that the algorithm has a fixed point – since the T_{in} and T_{out} of each variable at each program point can only strictly increase finitely many times – so that the algorithm terminates.

Now just note that the transfer rules ensure that if we can reach program point q after program point p, then $T_{in}(x, p) \subseteq T_{out}(x, p) \subseteq T_{in}(x, q) \subseteq T_{out}(x, q)$ for any variable x; that is, we can only increase the set of known dynamic types as we move through the program points. Also, when we update a

T_{in} set, assuming that the predecessors' T_{out} sets have not gotten smaller, then the new T_{in} set also does not get smaller, since it is the union of the predecessors' T_{in} sets. When we update a T_{out} set, assuming that T_{in} has not been reduced from the previous update, then we can verify case-by-case (looking at each transfer function) that the new T_{out} is also not reduced. (Indeed, the new T_{out} set is either always the same singleton set, or is updated to be the value of some T_{in} assignment.)

This kind of argument is somewhat annoying to formalize, so it is sufficient to give a more informal argument, like the one above.

(e) Now let's see how we can actually use the information produced by our dataflow analysis for the purpose of optimization. Suppose that after running our analysis, we know that for an IL instruction `x := call y, m`, we have $T_{in}(\texttt{y}, \texttt{x := call y, m}) = C$ for some collection of classes C. Under what conditions on C can we replace the dynamic dispatch of this call by a dispatch to a statically determined target function? (Hint: you may assume that you statically know whether or not m is overwritten, or merely inherited, by each class $T \in C$.)

We can replace the dynamic dispatch by a static dispatch if all classes in C inherit from some common superclass T which implements method m, and for which all the classes in C inherit (and do not override) the T-implementation of m. If C is a singleton, then this is necessarily the case, since we may let $T = C$.