

## Written Assignment 5 Solutions

Assigned: March 20

Due: April 10 at 3:00pm

1. Consider the following ChocoPy program:

```
x:int = 1
class A(object):
    a1:int = 0
    a2:int = 1
    def __init__(self:"A"):
        global x
        x = x + 1

    def foo(self:"A", x:int) -> int:
        def bar() -> int:
            return x
        return bar()
bar : A = None
bar = A()
bar.a1
```

- (a) Give an inverted evaluation tree, using operational semantics judgments, for the right side of the assignment on the second to last line:  $A()$ . You may assume that the environments and store  $G$ ,  $E$ , and  $S$  are available. We have provided a template for the inverted tree, leaving blanks in lemmas (i), (ii), and (iii) for you to fill out.

$$\begin{array}{l}
 \text{class}(A) = (a1 = 0, a2 = 1, \text{__init__} = \text{def } \text{__init__}(\text{self} : "A"), \\
 \quad \text{foo} = \text{def } \text{foo}(\text{self} : "A", x : \text{int}) \rightarrow \text{int}) \\
 l_{a1}, l_{a2}, l_{\text{__init__}}, l_{\text{foo}} = \text{newloc}(S, 4) \\
 v_0 = A(a1 = l_{a1}, a2 = l_{a2}, \text{__init__} = l_{\text{__init__}}, \text{foo} = l_{\text{foo}}) \\
 G, G, S \vdash 0 : \text{int}(0), S, - \\
 G, G, S \vdash 1 : \text{int}(1), S, - \\
 G, G, S \vdash \text{def } \text{__init__}(\text{self} : "A")\{\dots\} : v_{\text{__init__}}, S, - \quad \text{from lemma (i)} \\
 G, G, S \vdash \text{def } \text{foo}(\text{self} : "A", x : \text{int}) \rightarrow \text{int}\{\dots\} : v_{\text{foo}}, S, - \quad \text{from lemma (ii)} \\
 S_1 = S[\text{int}(0)/l_{a1}][\text{int}(1)/l_{a2}][v_{\text{__init__}}/l_{\text{__init__}}][v_{\text{foo}}/l_{\text{foo}}] \\
 l_{\text{self}} = \text{newloc}(S_1) \\
 E' = E_{\text{__init__}}[l_{\text{self}}/\text{self}] \\
 S_2 = S_1[v_0/l_{\text{self}}] \\
 G, E', S_2 \vdash x = x + 1 : -, S_3, - \quad \text{from lemma (iii)} \\
 \hline
 G, E, S \vdash A() : v_0, S_3, -
 \end{array}$$

(i)

$$\begin{array}{l} \text{x declared to be global in } \text{\_init\_} \\ E_{\text{\_init\_}} = G \\ v_{\text{\_init\_}} = (\text{self}, \text{x} = \text{x} + 1, E_{\text{\_init\_}}) \\ \hline G, G, S \vdash \text{def } \text{\_init\_}(\text{self} : \text{"A"})\{\dots\} : v_{\text{\_init\_}}, S, - \end{array}$$

(ii)

$$\begin{array}{l} \text{bar} = \text{def bar}() \rightarrow \text{int is a nested function defined in foo} \\ E_{\text{foo}} = G \\ v_{\text{foo}} = (\text{self}, x, \text{bar} = \text{bar}, \text{return bar}(), E_{\text{foo}}) \\ \hline G, G, S \vdash \text{def foo}(\text{self} : \text{"A"}, x : \text{int}) \rightarrow \text{int}\{\dots\} : v_{\text{foo}}, S, - \end{array}$$

(iii)

$$\begin{array}{l} E'(x) = l_x \\ S_2(l_x) = \text{int}(1) \\ \hline G, E', S_2 \vdash x : \text{int}(1), S_2, - \\ G, E', S_2 \vdash 1 : \text{int}(1), S_2, - \\ 1 + 1 = 2 \\ \hline G, E', S_2 \vdash \text{x} + 1 : \text{int}(2), S_2, - \\ E'(x) = l_x \\ S_3 = S_2[\text{int}(2)/l_x] \\ \hline G, E', S_2 \vdash \text{x} = \text{x} + 1 : -, S_3, - \end{array}$$

- (b) Give an inverted evaluation tree, using operational semantics judgments, for the last line: **bar.a1**. You may assume as before that the environments and store  $G$ ,  $E$ , and  $S$  are available.

$$\begin{array}{l} v_{\text{bar}} = A(a1 = l_{a1}, a2 = l_{a2}, \text{\_init\_} = l_{\text{\_init\_}}, \text{foo} = l_{\text{foo}}) \\ G, E, S \vdash \text{bar} : v_{\text{bar}}, S, - \quad \text{from lemma (i)} \\ S(l_{a1}) = \text{int}(0) \\ \hline G, E, S \vdash \text{bar.a1} : \text{int}(0), S, - \end{array}$$

(i)

$$E(bar) = l_{bar}$$

$$S(l_{bar}) = v_{bar}$$

$$v_{bar} = A(a1 = l_{a1}, a2 = l_{a2}, \_init\_ = l_{\_init\_}, foo = l_{foo})$$

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$$G, E, S \vdash \mathbf{bar} : v_{bar}, S, \_$$

2. Consider the following ChocoPy program:

```
x:int = 0
y:int = 0
def f1() -> int:
    global y
    def f2() -> int:
        return x + 1
    y = 2
    return f2()
f1()
```

Assume that the environments and store  $G$ ,  $E$ , and  $S$  are available. Write the inverted evaluation tree for the expression  $\mathbf{f1}()$ . We have provided a template for the inverted tree, leaving blanks in lemmas (i), (iii), (v), (vi) and (vii) for you to fill out.

$$\begin{array}{l}
 b_{f1} = y = 2; \text{ return } f2() \\
 E_{f1} = E[G(y)/y] \\
 S(E(f1)) = (f2 = \text{def } f2() \rightarrow \text{int}, b_{f1}, E_{f1}) \\
 l_{f2} = \text{newloc}(S) \\
 E' = E_{f1}[l_{f2}/f2] \\
 b_{f2} = \text{return } x + 1 \\
 v_{f2} = (b_{f2}, E_{f2}) \\
 G, E', S \vdash \text{def } f2() \{...\} : v_{f2}, S, - \quad \text{from lemma (i)} \\
 S_1 = S[v_{f2}/l_{f2}] \\
 G, E', S_1 \vdash b_{f1} : -, S_2, \text{int}(1) \quad \text{from lemma (ii)} \\
 \hline
 G, E, S \vdash \mathbf{f1}() : \text{int}(1), S_2, -
 \end{array}$$

(i)

$$\begin{array}{l}
 b_{f2} = \text{return } x + 1 \\
 E_{f2} = E \\
 v_{f2} = (b_{f2}, E_{f2}) \\
 \hline
 G, E, S \vdash \text{def } f2() \{...\} : v_{f2}, S, -
 \end{array}$$

(ii)

$$\begin{array}{l}
 G, E', S_1 \vdash y = 2 : -, S_2, - \quad \text{from lemma (iii)} \\
 G, E', S_2 \vdash \text{return } f2() : -, S_2, \text{int}(1) \quad \text{from lemma (iv)} \\
 \hline
 G, E', S_1 \vdash b_{f1} : -, S_2, \text{int}(1)
 \end{array}$$

(iii)

$$\begin{array}{l} G, E', S_1 \vdash 2 : \text{int}(2), S_1, - \\ E'(y) = l_y \\ S_2 = S_1[\text{int}(2)/l_y] \end{array}$$

---

$$G, E', S_1 \vdash y = 2 : -, S_2, -$$

(iv)

$$G, E', S_2 \vdash \text{f2}() : \text{int}(1), S_2, - \quad \text{from lemma (v)}$$

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$$G, E', S_2 \vdash \text{return f2}() : -, S_2, \text{int}(1)$$

(v)

$$\begin{array}{l} b_{f2} = \text{return } x + 1 \\ S_2(E'(f_2)) = (b_{f2}, E_{f2}) \\ G, E_{f2}, S_2 \vdash b_{f2} : -, S_2, \text{int}(1) \quad \text{from lemma (vi)} \end{array}$$

---

$$G, E', S_2 \vdash \text{f2}() : \text{int}(1), S_2, -$$

(vi)

$$\begin{array}{l} G, E', S_2 \vdash x : \text{int}(0), S_2, - \quad \text{from lemma (vii)} \\ G, E', S_2 \vdash 1 : \text{int}(1), S_2, - \\ G, E', S_2 \vdash x + 1 : \text{int}(1), S_2, - \end{array}$$

---

$$G, E', S_2 \vdash \text{return } x + 1 : -, S_2, \text{int}(1)$$

(vii)

$$\begin{array}{l} E'(x) = l_x \\ S_2(l_x) = \text{int}(0) \end{array}$$

---

$$G, E', S_2 \vdash x : \text{int}(0), S_2, -$$

3. Suppose we introduce the **switch** expression into ChocoPy:

**switch**  $e$  :  $b_1$  **case**  $e_1$ ,  $b_2$  **case**  $e_2$ , ...,  $b_n$  **case**  $e_n$ ,  $b_{n+1}$  **default**

where  $e$  and all of the  $e_i$  and  $b_i$  are expressions, and  $n \geq 1$ .

We want the value of the entire **switch** expression to be the evaluation of the expression associated with the case which matches the value of the expression just after the **switch** keyword. In the following example:

```
x : str = "test2"
switch x: 1 case "test", 2 case "test2", 3 default
```

the value of `x` is “test2”, which matches with the second case, and thus the whole `switch` expression evaluates to 2. In the scenario that none of the cases match then the whole `switch` expression evaluates to the body of the default case. The `switch` expression can have 1 or more cases (since  $n \geq 1$ ), but must have a default case.

To be more formal, this hypothetical `switch` expression is evaluated in the following way: we want to determine if  $e == e_1$ , and if so, then the result of the `switch` expression is the value of  $b_1$ . If not, then if  $e == e_2$ , the result of the `switch` expression is the value of  $b_2$ . Evaluation proceeds in this manner, checking  $e$  against each  $e_i$  using the `==` operator. If none of the cases match, then the result of the `switch` expression is the value of  $b_{n+1}$ . We require the following properties to hold:

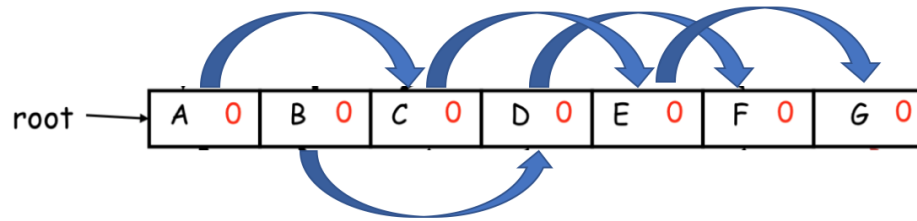
- If the  $i^{\text{th}}$  case of the `switch` expression matches, and no previous case matches, then the expressions  $e, e_1, \dots, e_i, b_i$  are evaluated in this order, each exactly once, and the value of the `switch` expression is the value of  $b_i$ .
- If none of the cases match, then the expressions  $e, e_1, \dots, e_n, b_{n+1}$  are evaluated in this order, each exactly once, and the value of the `switch` expression is the value of  $b_{n+1}$ .

Define operational semantics rule(s) for evaluating any `switch` expression.

$$\begin{array}{c}
G, E, S \vdash e : v, S_1, - \\
\text{for some } i \text{ where } 1 \leq i \leq n \\
G, E, S_1 \vdash e_1 : v_1, S_2, - \\
v_1 \neq v \\
\vdots \\
G, E, S_i \vdash e_i : v_i, S_{i+1}, - \\
v_i = v \\
G, E, S_{i+1} \vdash b_i : v_{b_i}, S_{i+2}, - \\
\hline
G, E, S \vdash \text{switch } e : b_1 \text{ case } e_1, b_2 \text{ case } e_2, \dots, b_n \text{ case } e_n, b_{n+1} \text{ default : } v_{b_i}, S_{i+2}, -
\end{array}$$
  

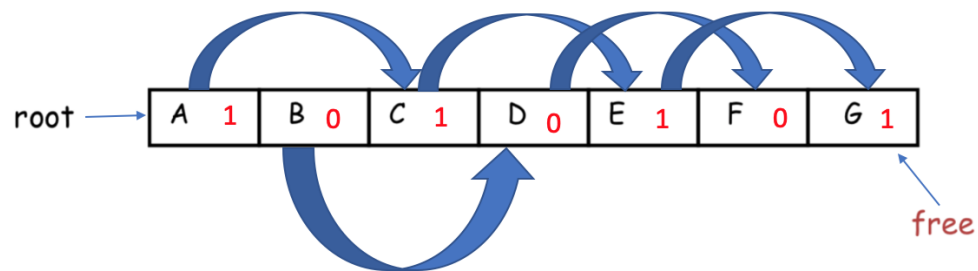
$$\begin{array}{c}
G, E, S \vdash e : v, S_1, - \\
G, E, S_1 \vdash e_1 : v_1, S_2, - \\
v_1 \neq v \\
\vdots \\
G, E, S_n \vdash e_n : v_n, S_{n+1}, - \\
v_n \neq v \\
G, E, S_{n+1} \vdash b_{n+1} : v_{b_{n+1}}, S_{n+2}, - \\
\hline
G, E, S \vdash \text{switch } e : b_1 \text{ case } e_1, b_2 \text{ case } e_2, \dots, b_n \text{ case } e_n, b_{n+1} \text{ default : } v_{b_{n+1}}, S_{n+2}, -
\end{array}$$

4. (a) This question is about the mark-and-sweep garbage collection strategy. The following figure shows the state of the heap before garbage collection, where the red zeros indicate that the mark bits are reset on each object:

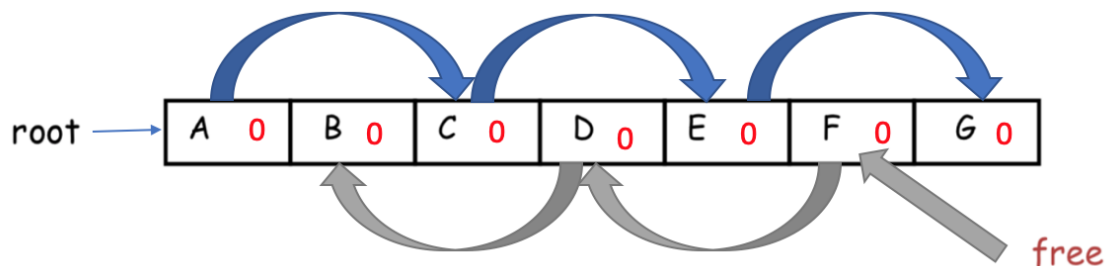


Recall that the mark-and-sweep strategy is to perform the mark operation, and then the sweep operation. Show the state of the heap after each operation. Explicitly write the mark bits.

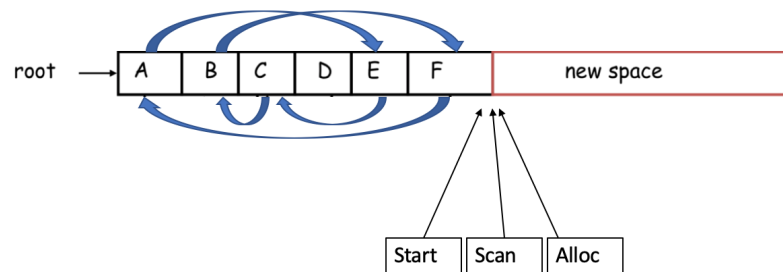
After mark:



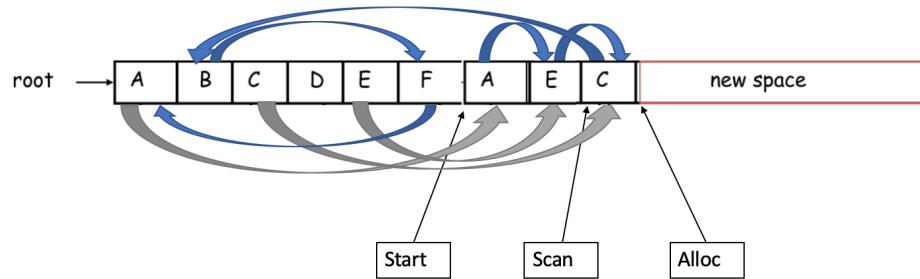
After sweep:



- (b) This question is about the stop-and-copy garbage collector. The following figure shows the state of the heap before garbage collection:



Show the state of the heap immediately after the first two objects have been both copied and scanned.



Answer: A will be copied thus there will be a forwarding node to the new location of A, and the alloc pointer will move forward. Then we will scan A, and thus will alloc space for E. We will add a forwarding pointer from the old E to the new E. We are done scanning A so the scan pointer can be moved forward. We then scan E and thus add C to the new space and move the alloc pointer forward, and we add a forwarding pointer from old C to new C. We then move scan pointer forward since we are done scanning E. Thus we have reached the point where the first two objects are copied and scanned.

- (c) Write a ChocoPy program such that reference counting would not free an object on the heap which should be freed.

Answer: Student answers will vary but we are looking for a cyclical structure of unreachable objects which will prevent reference counting from correctly garbage collecting. One simple example is illustrated here with a Node class that represents nodes in a linked list that contains integers with a first and next attribute.

```
class Node(object):
    first: int = 1
    next: Node = None
x: Node = None
x = Node()
x.next = x
x = Node()
```

- (d) Consider the following pseudocode of a program which allocates a large number of fixed-size objects, given `total_bytes` and `object_size` as inputs:

```
cumulative := 0
while cumulative < total_bytes:
    obj := allocate_object_of_size(object_size)
    # this obj is never used
    cumulative += object_size
```

- i. Suppose `total_bytes` is much larger than the available memory size and `object_size` is small enough that many objects of this size can fit in memory. The program therefore allocates a large number of small objects. Between the mark-and-sweep and stop-and-copy algorithms, which garbage collection strategy would be more efficient, and why?

Answer: stop-and-copy, since it will only copy at most one reachable object, whereas mark-and-sweep will have to iterate through almost all small objects which were allocated in order to add them to the free list.

- ii. Now suppose `total_bytes` is much larger than the available memory size and `object_size` is large enough that only a few objects of this size can fit in memory. The program therefore allocates a small number of large objects. Between the mark-and-sweep and stop-and-copy



algorithms, which garbage collection strategy would be more efficient, and why?

Answer: mark-and-sweep, since it will only have to mark and sweep a few large objects, while stop-and-copy may have to actually copy a large object (maybe several GB!) to the new-space each time memory fills up due to the allocations.