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Simulating size effect on shear strength of RC beams without stirrups using neural networks

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Abstract

An artificial neural network (ANN) model was developed using past experimental data on shear failure of slender RC beams without web reinforcements. The neural network model has five input nodes representing the concrete compressive strength (f'_c), beam width (b), effective depth (d), shear span to depth ratio (a/d), longitudinal steel ratio (ρ), five hidden layer nodes and one output node representing the ultimate shear strength ($v_u = V_u/bd$). The model gives reasonable predictions of the ultimate shear stress and can simulate the size effect on ultimate shear stress at diagonal tension failure. The ANN model performs well when compared with existing empirical, theoretical and design code equations. Through the parametric studies using the ANN model, the effects of various parameters such as f'_c, d, ρ and a/d on the shear capacity of RC beams without web reinforcement was shown. This shows the versatility of ANNs in constructing relationships among multiple variables of complex physical processes using actual experimental data for training.

Keywords: RC beam; Diagonal shear; Size effect; Neural network; Modeling

1. Introduction

The prediction of the ultimate shear strength of reinforced concrete (RC) beams is critical especially when this value is used in design. An unconservative value of shear capacity may lead to failure, a classic example is the partial collapse of the Wilkins Air Force Depot warehouse in Ohio, USA in August 1955 [1]. Investigation showed that the beams, which failed in diagonal tension, have depths of 914 mm (36 in.), at the failure location, did not contain stirrups, and had only 0.45% longitudinal bars. Although the American Concrete Institute (ACI) design permitted an allowable shear stress of 0.62 MPa (90 psi) the beams failed at a lower stress of 0.50 MPa (75 psi). A PCA test, on the other hand, showed that a 12-in. beam could resist a shear stress up to 1.0 MPa (150 psi) prior to failure. Why did the beams fail in shear?

Current design methods for shear in RC members are based on almost entirely on the results of tests. Majority of shear tests where the design equations were derived have been carried out on beams of depth less than 300 mm [2]. However, more recent experiments on beams of larger depths have shown that as the size of the beam increases, the intensity of shear stress decreases especially in lightly reinforced beams. This phenomenon that is referred to as "size effect" simply means that the shear strength is not constant for a given compressive strength of concrete but varies with the size of the beam. This observation contradicts the assumption given by some design codes like the simple equation for shear of the ACI which assumes a constant value of shear strength for a given compressive strength of concrete [3]. This phenomenon of size effect gives one explanation for the collapse of some RC structures like the Wilkins Air Force Depot warehouse. Design for shear using our present codes for some types of beams may lead to unconservative results due to size effect.

Researches have been conducted to understand the shear problem, e.g. Refs. [4–10] and more recent

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studies considered the shear effect, e.g. Refs. [1,2,11,12]. Experiments have been conducted to understand shear failure of RC beams with and without stirrups for a wide range of concrete strengths, steel ratios, effective depths, span lengths, subjected to both concentrated and distributed loads. A database of some of these experimental results have been compiled and analyzed by researchers to verify a new theory or to develop a new model on shear failure. Empirical equations of various forms, e.g. Refs. [1,12] were developed by regression analysis of an assumed form of a function relating different parameters which may affect the shear capacity of RC beams using a set of experimental data. Analytical equations based on a hypothesis of the mechanism of shear failure, e.g. Zararis and Papadakis [11], have also been derived to estimate the critical shear intensity in terms of known parameters. Despite the numerous studies over the last 50 years, the problem of shear failure and the prediction of the shear capacity of RC beams still remains an active research area. Kani [5] states that the main obstacle to the shear problem is the large number of parameters involved, some of which may not be known. Krefeld and Thurston [7] explains that the major difficulties in developing a theoretical expression for the shearing strength of RC beams are due primarily to the indeterminacy of the internal force system of a cracked reinforced member, the nonhomogeneity of concrete, and the nonlinearity of its stress–strain diagram.

Most of the empirical and analytical equations for shear were developed using regression analysis of experimental data. To develop such models, the form of the empirical equation must be assumed and then the unknown parameters in the equations are then determined. The main problem with this approach is that it is difficult to determine the form and the number of coefficients of the equation, which will best describe the physical process. With this approach, different expressions have been derived—ranging from simple to complex, linear to nonlinear—depending on the assumptions and details of the experimental data used by the researchers. Because of the restriction of the assumed form of the equation, the model may not be able to capture the interrelationship of the various parameters considered in the model. The empirical models of Zsutty [9] and Mphonde and Frantz [8], for example, cannot capture the size effect, while the other empirical equations by Collins and Kuchma [1] or Niwa et al. [12] usually perform well when tested using their own data , but perform poorly when applied to new data or data not used in the regression analysis.

Recently, researchers have found the potential of artificial neural networks (ANNs) in the modeling of various engineering and natural systems. ANNs have been found very powerful in modeling systems governed by multiple variable interrelationships, especially when the data available are "noisy" or incomplete. One advantage of neural network modeling is that there is no need to know a priori the functional relationship among the various variables involved, unlike in regression analysis. The ANNs automatically construct the relationships for a given network architecture as experimental data are processed through a learning algorithm. This approach is "data driven", meaning that the network adopts to the training data presented to capture the relationship among input and output parameters. For this reason, ANNs should be interesting to engineers and scientists as a tool to support their task related to the modeling and prediction of behavior of engineering and natural systems.

With the availability of more experimental data on diagonal shear of RC beams with a wider range of concrete strengths, steel ratios, shear span to depth ratios, and geometrical sizes, especially test data of large beams, this study reanalyzes the new data to develop a neural network model which will help us understand more the diagonal shear problem and the different parameters which affect the shear capacity including size effect. The present study aims to contribute to the continuing research about shear strength prediction of RC beams using new computing technologies. An ANN model was developed to predict the ultimate shear strength of RC beams without stirrups using available data from past experiments. The effects of various factors such as concrete strength, beam dimensions, slenderness, longitudinal steel ratio and other factors on the shear strength were considered. The performance of the ANN model was also compared with existing empirical, theoretical and design code equations. Hopefully, this study will contribute to a better understanding of the influence of various beam parameters on size effect of shear failure and eventually structural design provisions on shear of RC beams will be updated leading to safer design of structures.

2. Shear in slender RC Beams without web reinforcements

2.1. Failure mode

Beams without web reinforcements or stirrups will fail when inclined cracking occurs. This type of failure referred to as diagonal tension failure (Fig. 1) precipitates if the strength of the beam in diagonal tension is lower than its strength in flexure. This behavior is common in slender beams—beams with a/d between 2.5 and 5.5 or L_c/d between 11 and 16 [13]. The shear span, a, is the distance between the point of application of the concentrated load and the face of the support, L_c is the clear beam span if the loading is distributed, and d is the effective beam depth. Diagonal shear fail-

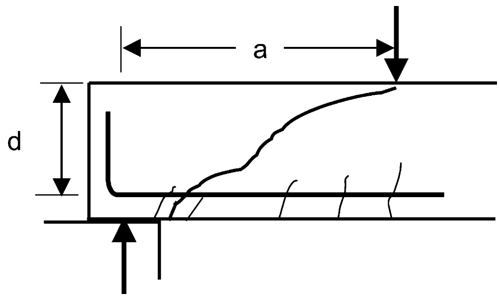

Fig. 1. Diagonal tension failure.

ure starts with the development of a few fine vertical flexural cracks at midspan, followed by the destruction of the bond between the reinforcing steel and surrounding concrete at the support. Thereafter, without ample warning of impending failure, two or three diagonal cracks develop at about 1.5–2d distance from the face of the support. As they stabilize, one of the diagonal cracks widens into a principal diagonal tension crack and extends to the top compression fibers of the beam. This type of brittle failure mode occurs at a relatively small deflection. The shear capacity of the beam is considered to be equal to the inclined cracking shear.

2.2. Design for shear

RC beams are designed against shear failure by providing web reinforcement. The web reinforcement usually takes the form of vertical stirrups that enclose the longitudinal bars along the faces of the beam. Bent-up bars (usually at 45° angles) are another type of web reinforcement. The design for shear based on the ACI code starts with the computation of the shear strength, V_c, of the concrete beam cross-section and the maximum shear, V_u, that the beam will resist. The ACI code assumes that V_c is equal to the shear strength of a beam without stirrups and this is taken equal to the inclined cracking load. The required shear strength to be provided by the web reinforcement is then computed as $V_s = (V_u - \phi\ V_c)/\phi$. The required area of web reinforcement or spacing of the stirrups can then be determined based on V_s. It must be observed that the value of V_c is critical in the design of shear reinforcing because if you overestimate the concrete shear strength, then this will result to a theoretical area of stirrups which will be less than required or a spacing of stirrups which may be larger than the required theoretical spacing.

2.3. Shear strength equations for RC beams without web reinforcements

Various equations have been proposed to predict the shear strength of concrete beams without web reinforcements. Summarized in Table 1 are the ACI design equations, some empirical and theoretical equations for shear which consider the size effect. In this equations, the value of f'_c is in MPa.

The ACI code defines the critical shear strength, V_c or $v_c(= V_c/bd)$ of a concrete beam as the stress at the occurrence of the first inclined crack. The principal diagonal tensile stress controlling the crack is the result of the shearing stress due to the external factored shear V_u and the horizontal flexural stress due to external bending moment M_u. The ACI code provides a simplified and detailed empirical model based on results of extensive tests of failure of a large number of beams without web reinforcement in terms of V_u and M_u. The quantity $V_u d/M_u$ shall not be greater than 1.0 in computing V_c. The shear span to depth ratio, a/d, is often replaced by the quantity $M_u/V_u d$. The ACI equations were formulated based on experimental tests—majority of beams tested then have smaller depths. Hence, the sensitivity of failure shear stress to size and reinforcement ratio was not recognized [1].

Both empirical and theoretical equations were proposed to incorporate the size effect in the shear strength of RC beams without web reinforcements. Collins and Kuchma [1] recommends modifications in the ACI shear provisions by incorporating a crack spacing parameter s_e which contributes to the size effect. The parameter s_e accounts for the influence of the crack spacing s_x and the maximum aggregate size, a. The parameter, s_x, is taken as $0.9d$ for members that have only concentrated reinforcement near the flexural tension face, or as the maximum distance between the layers of longitudinal reinforcement if the member contains intermediate layers of crack control reinforcement.

A re-evaluation of the shear strength equation used in Japan was carried out by Niwa et al. [12] considering the result of large-sized beam tests and a new equation was derived where the nominal shear strength at failure, $v_u(= V_u/bd)$ is inversely proportional to the fourth root of the effective depth.

Table 1
Shear strength equations for RC beams without web reinforcement

ACI simplified equation [3]	$v_c = \frac{1}{6}\sqrt{f'_c}$
ACI detailed equation [3]	$v_c = \frac{1}{7}\left(\sqrt{f'_c} + 120\rho\frac{V_u d}{M_u}\right) \leq v_{c_{max}} = 0.3\sqrt{f'_c}$
Niwa et al. [12]	$v_u = 0.20\left(100\rho \cdot f'_c\right)^{1/3}(d^{-1/4})[0.75 + 1.4/(a/d)]$
Collins and Kuchma [1]	$v_c = \left(\frac{245}{1275+s_e}\right)\sqrt{f'_c}$ where $s_e = \frac{35s_x}{(a+16)}$
Zararis and Papadakis [11]	$v_u = \left(1.2 - 0.2\frac{a}{d}d\right)\frac{c}{d}f_{ct}$ where $\left(1.2 - 0.2\frac{a}{d}d\right) \geq 0.65; f_{ct} = 0.30\left(f'_c\right)^{2/3}$ and $\left(\frac{c}{d}\right)^2 + 600\frac{\rho}{f'_c}\ \frac{c}{d} - 600\frac{\rho}{f'_c} = 0$

A theory was presented by Zararis and Papadakis [11] under which the diagonal shear failure of RC ordinary (slender) beams without web reinforcement under two-point loading (or one-point load at the middle of the beam) is due to splitting of concrete occurring in a certain region of the shear span. The theory results to an expression where the shear stress, v_u, at failure depends on the ratio (c/d) of the neutral axis depth to the effective depth of the beam times the splitting tensile strength of concrete (f_{ct}). The equation has a correction factor to account for the size effect in slender beams, according to which the size effect depends on the size of the depth, d, and the ratio (a/d).

3. Neural network modeling

3.1. Applications of neural networks

ANNs are useful computing tools for which rules are either unknown or difficult to discover. No priori function is required before an ANN model can be developed. ANNs adapt solutions and are capable of capturing the interrelationships among multiple variables by simply presenting them with data. Because of this capability of ANNs, applications to civil engineering, in general, and concrete structures, in particular, have increased. ANN modeling have been applied in the prediction of capacity of pin-ended RC columns [14], modeling the confined strength and strain of circular columns [15], cost optimization of beam design [16], deflection of externally reinforced RC beams [17], prediction of ultimate shear strength of simply supported deep beams [18], and ordinary and high-performance concrete strength prediction [19,20].

3.2. Network architecture and implementation

An ANN is a collection of simple processing units or neurons connected through links called connections. The topology or architecture of a three-layer feedforward neural network may be presented schematically, as in Fig. 2. The neural network is represented in the form of a directed graph, where the nodes represent the neuron or processing unit, the arcs represent the connections with the normal direction of signal flow is from left to right. The processing units may be grouped into layers of input, hidden and output neurons. The neural network in the figure consists of five input neurons, five hidden neurons and one output neuron. The main tasks of neurons are to receive input from its neighboring units which provide incoming activations, compute an output, and send that output to its neighbors receiving its output. The strength of the connections among the processing units is provided by a set of weights that affect the magnitude of the input that will be received by the neighboring units. The following equations describe the mode of operation of a three-layer feedforward network:

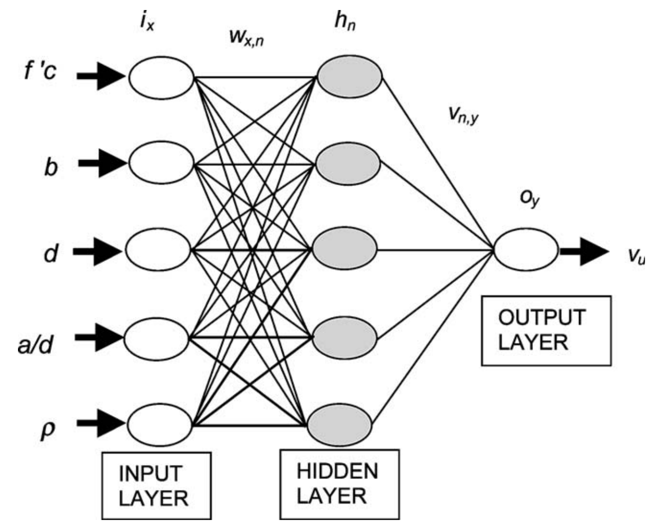

Fig. 2. A three-layer feedforward neural network.

$$h_n = f\left(\sum_{x=1}^{M}(w_{x,n} \cdot i_x) + b_n\right) \tag{1}$$

$$o_y = f\left(\sum_{n=1}^{N}(v_{n,y} \cdot h_n) + b_y\right) \tag{2}$$

where i_x, scaled input value transmitted from the xth input neuron; h_n, activity level generated at the nth hidden neuron; o_y, activity level generated at the yth output neuron; $w_{x,n}$ and $v_{n,y}$, weights on the connections to the hidden and output layers of neurons, respectively; b_n and b_y, weighted biases and f[ ], activation function, in this case, the sigmoid function $f(z) = 1/(1 + e^{-z})$.

3.3. Experimental data

The present research used 155 sets of data, majority of which can be found from test results compiled by Zararis and Papadakis [11]. The database includes test results from 1962 to 1999 for slender beams at various strengths of concrete (low and high), steel ratios, shear span to depth (a/d) ratios, and geometrical sizes (of length and depth). The present study considered test results of beams without web reinforcement under concentrated loading (two-point loading or one-point loading at midspan), and beams with concrete compressive strength (f'_c) up to 55 MPa (8000 psi) only. The data were grouped randomly into two subsets—a training set of 118 data and a testing or validation set of 37 data. The beam parameters available from these experiments are the concrete compressive strength (f'_c), beam width (b), beam height (h), effective depth (d), shear span to depth ratio (a/d), longitudinal steel ratio

(ρ), shear force at failure (V_u). Other beam parameters can be easily derived (e.g. ratio, b/d and shear stress, $v_u = V_u/bd$).

Table 2a and b show the statistical parameters of the two sets of data. The statistical parameters include the mean, standard deviation, coefficient of variance, minimum value and maximum value. The statistics of both training and testing sets are in good agreement meaning both represent almost the same population.

3.4. Network data preparation

Preprocessing of data by scaling was carried out to improve the training of the neural network. To avoid the slow rate of learning near the end points specifically of the output range due to the property of the sigmoid function which is asymptotic to values 0 and 1, the input and output data were scaled between the interval 0.1 and 0.9. The linear scaling equation: $y = (0.8/\Delta)\,x + (0.9 - 0.8x_{max}/\Delta)$ was used in this study for a variable limited to minimum (x_{min}) and maximum (x_{max}) values given in Table 3 with $\Delta = x_{max} - x_{min}$.

3.5. Neural network simulations

Using the experimental data on beam tests for shear, a three-layer feedforward neural network was developed using the backpropagation learning algorithm with momentum. Details of the backpropagation procedure can be found in the literature, e.g. Ref. [21]. The minimum number of hidden layers is considered, and in this case one hidden layer is sufficient to produce an acceptable model.

Determining the network architecture is one of the most important tasks in the development of ANN models. It requires the selection of the input parameters and output parameters which will restrict the number of the input and output nodes of the network. Based on the experimental data, various combinations of input and output parameters can be considered in developing the ANN models. Four types of architectures based on input and output parameters were considered as summarized in Table 4. Models M1A and M1B have four input nodes representing f'_c, b/d, a/d and ρ, while models M2A and M2B have five input nodes representing f'_c, b, d, a/d and ρ. The "A" models are shear force (V_u) predictors while the "B" models are shear stress ($v_u = V_u/bd$) predictors. A model is labeled based on the number of input nodes, hidden layer nodes and output nodes (e.g. M2B-551 has five input nodes, five hidden layer nodes and one output node in Fig. 2).

ANN simulations were conducted for the different ANN architectures with the number of hidden layer nodes varied using 118 training data. The values of the learning parameter, momentum parameter, noise, number of cycles were applied to a specific network architecture and the behavior of the error is observed. In most of the simulations, the values of 0.05–0.06 for the learning parameter, 0.008–0.01 for the momentum parameter, 0.01 for noise were used. The stopping criteria used are 500–4000 for total number of cycles or a value of 0.001 for the error tolerance. The different ANN models were compared with respect to the following error metrics, MAE, RMSE and R, using the 37 test data.

Table 5 presents the values of the error metrics for the various ANN models with varying hidden layer nodes. The comparison of error metrics is given for both shear stress and shear force. For the shear force predictors, M1A and M2A models, the shear stress is

Table 2
Statistics of experimental data

	f'_c (MPa)	b (cm)	d (cm)	h (cm)	a/d	ρ (%)	b/d	V_u (kN)	v_u (MPa)
(a) *Training data*									
No. of data	118	118	118	118	118	118	118	118	118
Mean	29.31	17.99	34.41	38.99	3.83	1.83	0.67	74.57	1.23
Standard deviation	6.53	9.69	26.05	28.92	1.50	0.68	0.41	66.01	0.28
COV	0.22	0.54	0.76	0.74	0.39	0.37	0.61	0.89	0.23
Min. value	10.50	5.00	7.00	8.00	2.35	0.59	0.14	7.30	0.56
Max. value	52.60	61.20	120.00	125.00	9.05	3.10	2.73	358.40	2.11
(b) *Test data*									
No. of data	37	37	37	37	37	37	37	37	37
Mean	29.07	17.07	32.67	36.54	3.64	1.89	0.66	68.72	1.27
Standard deviation	6.03	8.95	23.62	25.35	0.99	0.80	0.40	54.05	0.30
COV	0.21	0.52	0.72	0.69	0.27	0.42	0.60	0.79	0.24
Min. value	16.20	6.00	10.60	12.10	2.92	0.74	0.14	9.80	0.70
Max. value	41.70	61.10	109.70	122.00	7.00	3.36	2.25	235.00	1.84

Table 3
Minimum and maximum values for scaling data

Variable	Minimum	Maximum
f'_c (MPa)	10.0	55.0
b (cm)	5.0	65.0
d (cm)	7.0	120.0
b/d	0.1	2.8
a/d	2.5	9.5
ρ (%)	0.0	4.0
V_u (kN)	7.0	380.0
v_u (MPa)	0.5	2.5

derived from the model output, V_u as $v_u = V_u/bd$. On the other hand, for the shear stress predictors, M2A and M2B models, the shear force is derived from the model output, v_u, as $V_u = v_u bd$. The error metrics based on the predicted and derived values are then computed.

The best results of the M1A models for the shear force predictions are MAE = 27.949 kN, RMSE = 32.145 kN and a Pearson coefficient R= 0.807. These errors are relatively large. Moreover, the derived shear stress for M1A models are not satisfactory with the best error metrics, MAE = 1.159 MPa, RMSE = 2.164 MPa and R= 0.425. M1B models perform better than the M1A models. The minimum MAE and RMSE values for shear stress are 0.124 and 0.166, respectively—about 10% of the M1A values—and the best R value is 0.842, almost twice that of the M1A models. The R values for the derived shear force are also satisfactory with values greater than 0.90. The error values for the derived shear stress particularly on the R values for M1A models show that deriving the shear stress from the predicted shear force produces very poor predictions. A shear stress predictor model is more superior than a shear force predictor model.

M2A models with five input nodes performed better than the M1A models with the errors drastically reduced—a reduction of about 75% for the MAE and RMSE and improved values of R greater than 0.98. This shows that an ANN model with five input nodes and the parameters b and d, represented independently as in M2A models and not combined as one parameter, b/d in M1A models, performed better. Comparing M2A and M1B models, on the other hand, show that M1B models yielded better estimates for shear stress. M2B models performed best among the three sets of ANN models. Comparing M1B and M2B models, it can be seen that the MAE and RMSE values for the predicted shear stress were reduced by about 18% and the R values significantly improved from about 0.84 to 0.91 for shear stress and from about 0.94 to 0.99 for the derived shear force. A comparison among the four architectures show that the M2B models have the most superior performance with respect to the error metrics.

Table 4
ANN architecture and model parameters

Model	No. of input nodes	Input parameters	Output parameter
M1A	4	f'_c, b/d, a/d, ρ	V_u
M1B	4	f'_c, b/d, a/d, ρ	v_u
M2A	5	f'_c, b, d, a/d, ρ	V_u
M2B	5	f'_c, b, d, a/d, ρ	v_u

Table 5
Error metrics of ANN models

ANN	Shear stress (MPa)			Shear force (kN)		
Model	MAE	RMSE	R	MAE	RMSE	R
M1A-441	1.413	2.837	0.425	28.649	34.244	0.805
M1A-451	1.159	2.164	0.353	27.949	32.145	0.807
M1A-461	1.280	2.410	0.346	30.361	36.399	0.802
M1B-441	0.126	0.172	0.828	8.721	19.516	0.935
M1B-451	0.125	0.165	0.842	8.428	18.037	0.943
M1B-461	0.124	0.166	0.842	8.380	17.873	0.944
M2A-441	0.200	0.286	0.584	7.607	10.150	0.984
M2A-451	0.200	0.315	0.476	7.104	10.110	0.983
M2A-461	0.164	0.235	0.723	6.605	9.490	0.985
M2B-541	0.103	0.140	0.902	4.529	6.086	0.994
M2B-551	0.101	0.137	0.906	4.517	5.983	0.994
M2B-561	0.102	0.137	0.907	4.622	6.059	0.994

Based on the analysis of errors of the different ANN models, the M2B-551 model had the minimum values for MAE and RMSE both for shear stress and shear force and is thus selected for further validation. The model has to be tested on its performance with respect to other empirical models and its ability to simulate real physical processes is also verified by parametric studies.

4. Performance of the neural network model

4.1. Connection weights

The M2B-551 model (Fig. 2) has five input nodes representing the concrete compressive strength (f'_c), beam width (b), effective depth (d), shear span to depth ratio (a/d), longitudinal steel ratio (ρ), five hidden layer nodes and one output node representing the ultimate shear strength ($v_u = V_u/bd$). The connection weights for this model are shown in Table 6. Zero biases were set in the developing the model for simplicity. Using a causal inference procedure by Garson [22], the relative importance of the input variables to the output can be determined. It can be seen that the effective depth of the beam, d, is the most important

Table 6
Connection weights of M2B-551 model

Hidden nodes	Input layer					Output Layer
	f'_c	b	d	a/d	ρ	
1	−0.841836	−0.520036	2.295805	−0.740027	−0.576550	1.944814
2	−0.136782	3.614227	−2.695393	0.805690	3.364672	5.030095
3	−0.244926	0.276289	−0.023932	0.961188	−0.461074	−0.790688
4	−2.127035	−1.374138	0.223555	1.701900	−1.371270	−2.199725
5	−1.386961	3.659177	6.639413	0.337491	1.268700	−5.171329
Rel. impt. (%)	14.48	21.26	25.20	19.78	19.29	

parameter for the model, followed by the beam width, b, and the shear span to depth ratio, a/d. The parameters b and d are related to the shearing area while the shear span to depth ratio (a/d) is related to the beam slenderness which influences mode of failure. Using the computed weights in Table 6, the performance of the M2B-551 model can be validated with respect to errors and its ability to capture the inter-relationship of the different parameters used in the model as will be shown in the succeeding sections.

4.2. Model predictions

The predictions of the M2B-551 model as compared to the experimental values are shown in Fig. 3 for both training and test data. The Pearson product moment correlation coefficient R for training data is 0.9125 and for test data is 0.9058. The M2B-551 model has a mean percentage error of about 7.6% and a maximum percentage error of about 20% for the test data. The histogram showing the distribution of the percentage error for both training and test data is shown in Fig. 4. About 76% of the predictions of the ANN model lie within the ±10% error. There are a few large misses which have an error outside the ±10% range.

4.3. Comparison with empirical and theoretical equations

The M2B-551 model predictions for the test data are compared with the predictions of the empirical equations by Collins and Kuch (assuming a maximum aggregate size of 25 mm), Niwa et al. and the theoretical equation by Zararis and Papadakis. Fig. 5 presents the comparison of the predictions of the neural network model with the different models which consider size effect. The M2B-551 model predictions lie above and below the target line—the line where the predicted value is equal to the experimental value. The M2B-551 model with an R equal to 0.9058 compares well with Niwa et al. (R= 0.9283) and Zararis and Papadakis (R= 0.9315). The model by Collins and Kuchma, however, has an R= 0.6891—this low value may be due to the assumed value of the maximum aggregate. Moreover, the model of Collins and Kuchma is for the prediction of V_c, the shear stress at inclined cracking not at ultimate failure.

4.4. Comparison with ACI design code equations

Fig. 6 presents the M2B-551 model predictions compared with the three ACI equations for the test data.

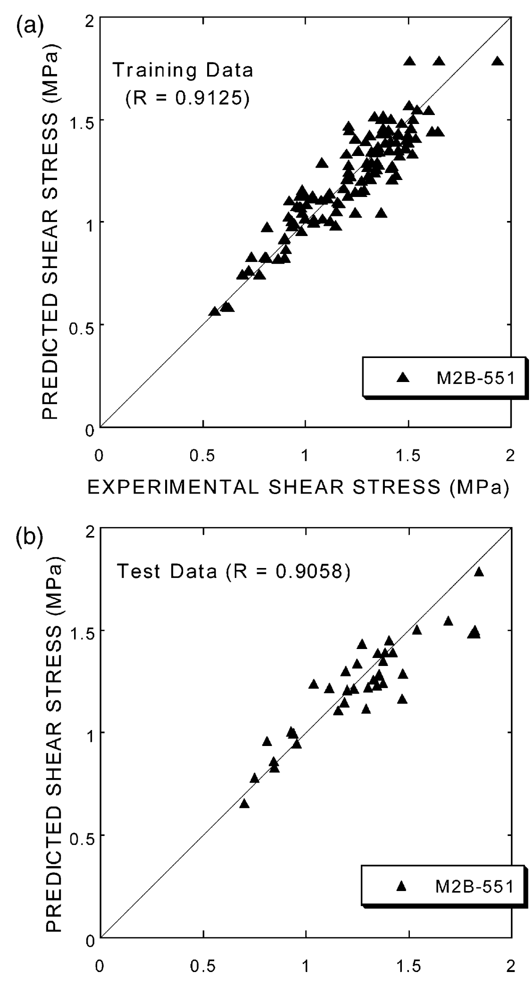

Fig. 3. Neural network shear stress predictions (a) training data, (b) test data.

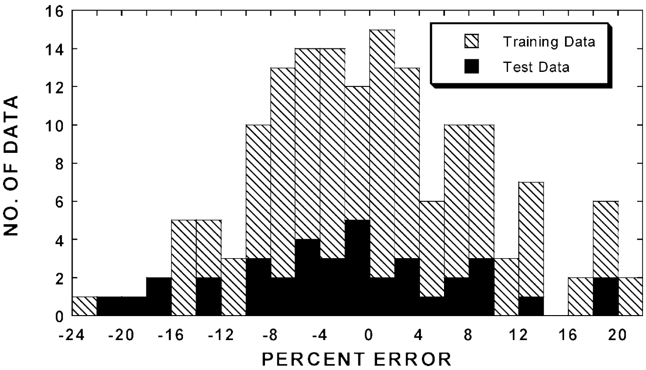

Fig. 4. Histogram of percent error of shear stress predictions.

The ACI simplified and detailed equations gave conservative predictions in most cases while the ACI maximum equation overestimate the shear stress at failure in almost all cases. Fig. 7, on the other hand, compares the prediction of the M2B-551 model and the ACI detailed equation for the shear strength with respect to the effective depth of the beam for both training and test data. The ACI predictions at smaller depths are conservative by as much as 50% in some cases and the predictions at larger depths are greater than the experimental values overestimating by as much as 40% of the experimental values. The M2B-551 model, on the other hand, gave predictions lying at $1.0 \pm 20\%$ of experimental values, covering the range from shallow to larger depths of the beam. If the neural network model will be used in design, then an appropriate factor of safety may be applied to the predicted value. The value of the factor of safety, however, has still to be evaluated since the neural network model can still be improved by considering more experimental data especially in the region of larger depths and at high strength concrete.

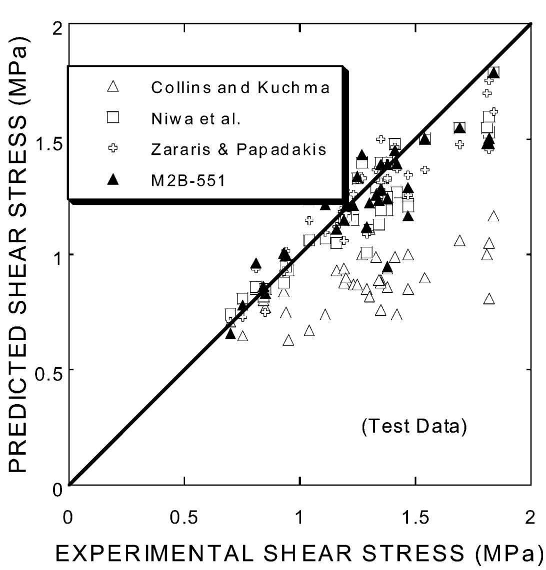

Fig. 5. Comparison of shear strength predictions.

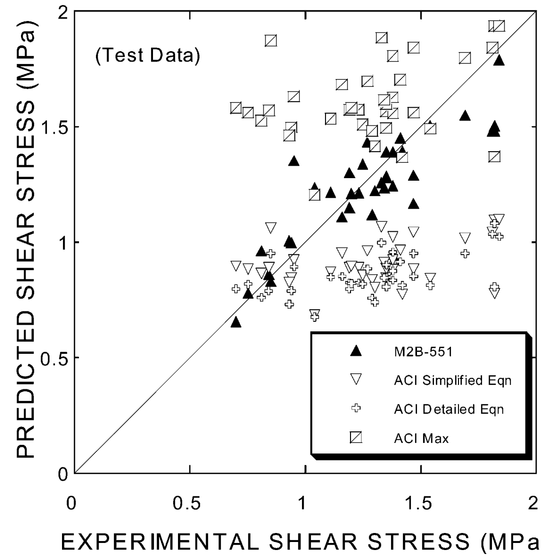

Fig. 6. Comparison of shear strength predictions with ACI code.

5. Parametric studies on size effect

One advantage of neural network models is that parametric studies can be easily done by simply varying one input parameter and all other input parameters are set to constant values. Through parametric studies, we can verify the performance of the M2B-551 model in simulating the physical behavior of an RC beam without web reinforcement particularly the size effect with respect to varying parameters. The results of parametric studies which follow trends that are consistent with experimental results will be our qualitative evidence that the ANN model has learned to reproduce the physical process of shear failure of RC beams without web reinforcement.

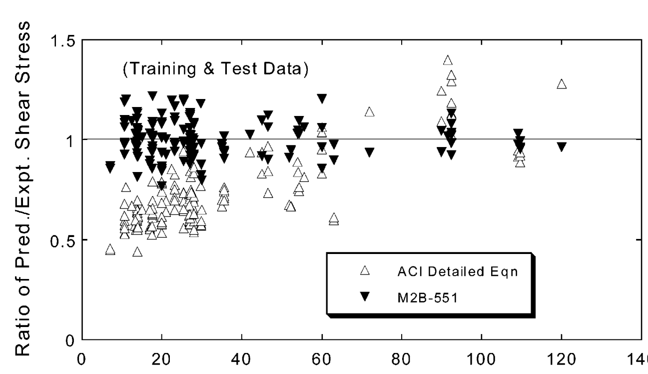

Fig. 7. Ratio of ANN and ACI predictions to experimental shear stress.

5.1. Size effect and the parameters f_c' and a/d

Can the neural network model capture the size effect in shear stress at failure? Fig. 8 presents the predicted shear stress of the M2B-551 model for varying effective depth, d, for a compressive strength of concrete, $f_c' = 28$ MPa (4000 psi). The shear span to depth ratio (a/d) was varied at 2.5–5.0. The other input parameters were set to constant values with b= 15 cm and ρ= 2.75%. Superimposed in the figure are the data from Kani [5] for the tests with the following range of values: f_c' (24–30 MPa), b (15.1–17.7 cm), a/d (2.35–6.05) and ρ(2.59–2.87%). The size effect is clearly demonstrated in the figure where the shear stress decreases as the depth increases. The decrease in shear stress is almost 50% from d= 20 to d= 100 cm. The shear stress at failure is not constant. The decreasing trend of the shear stress with increasing depth is consistent with the experimental results by Kani [5]. The shear capacity also depends on the shear span to depth ratio—a shorter beam (a/d= 2.5) has a larger shear stress than a longer beam (a/d= 5.0).

Fig. 9 present the behavior of the shear stress at failure for three values of compressive strength of concrete at a/d= 5.0, respectively. The neural network model can realistically simulate the effect of the compressive strength of concrete. The beam with a higher compressive strength (e.g. $f_c' = 35$ MPa) has a larger shear capacity than a beam with a lower compressive strength (e.g. $f_c' = 21$ or 28 MPa).

5.2. Size effect and longitudinal steel ratio

Is the size effect influenced by the longitudinal steel ratio? Fig. 10 shows the effect of the amount of longitudinal steel on the size effect. It is observed that the rate of decrease of the shear stress for the lightly reinforced beams (e.g. ρ= 0.5% and 1.0%) is larger than for beams with higher longitudinal steel ratio (e.g. ρ = 2.0% and 3.0%). The shear stress for ρ= 0.5% decreased by about 50% from 0.85 to 0.4 MPa, while the shear stress for ρ= 3.0% decreased only by about 20% from 1.31 to 1.04 MPa. The value of the shear strength using the ACI simplified equation for f_c' = 21 MPa (3000 psi) is also shown in the figure as 0.76 MPa. Observed that the ACI shear strength values at larger depths for the lightly reinforced beams (ρ= 0.5% and 1.0%) are greater than the neural network predictions. Size effect is more pronounced in lightly reinforced beams.

5.3. Varying compressive strength of concrete

What is the behavior of the shear strength with varying compressive strength of concrete? Fig. 11 shows an almost linear relationship between the shear stress and f_c' with the other parameters constant. The Pearson correlation coefficient between V_u/bd and f_c' is equal to 1.0 for all four curves. The shorter beam (a/d= 2.5) with a higher longitudinal steel ratio (ρ= 2.75%) has the largest shear capacity among the four cases shown in the figure.

Is the relationship between the shear strength and the compressive strength of concrete influenced by the

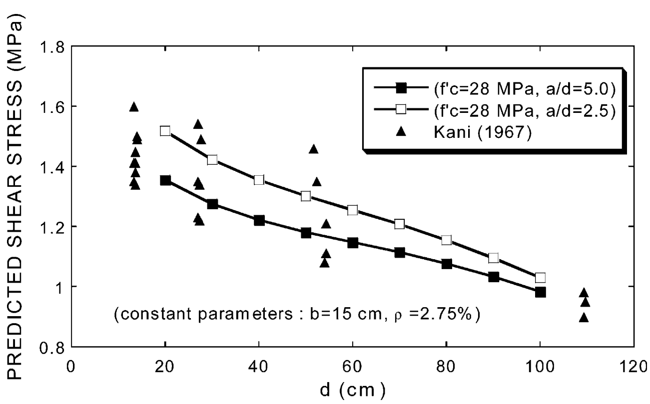

Fig. 8. Size effect for $f_c' = 28$ MPa and varying d and a/d.

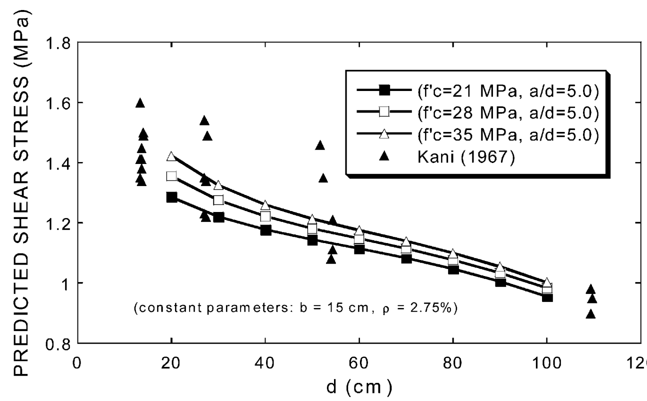

Fig. 9. Size effect for a/d= 5.0 and varying d and f_c'.

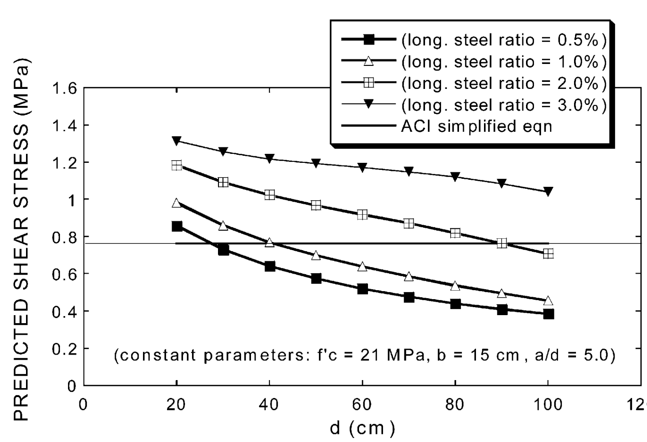

Fig. 10. Size effect and longitudinal steel ratio.

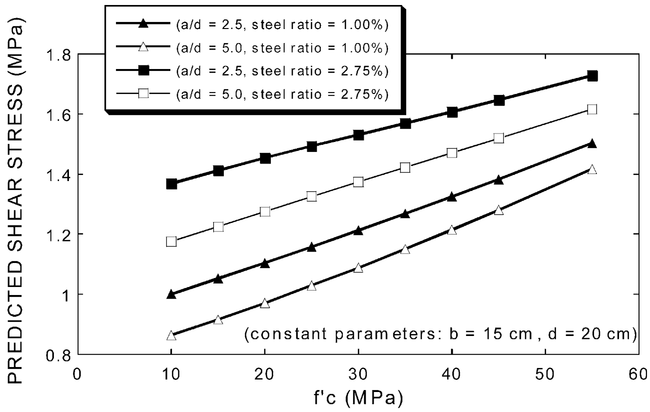

Fig. 11. Predicted shear stress and effect of f_c', a/d and ρ.

other parameters? Fig. 12 also shows a similar relationship which is almost linear between shear stress and concrete compressive strength, for $a/d = 5.0$. The rate of increase of the shear strength is influenced by the depth, d. The shear strength of the RC beam with $d = 20$ cm increased about 63% from 0.87 (at $f_c' = 10$ MPa) to 1.42 MPa (at $f_c' = 55$ MPa), while the shear strength of the RC beam with $d = 100$ cm increased only by about 16% from 0.43 (at $f_c' = 10$ MPa) to 0.50 MPa (at $f_c' = 55$ MPa. The increase in the shear capacity with respect to f_c' is significant in beams with smaller depths.

It is also interesting to note that the Pearson correlation coefficients R between V_u/bd and f_c' for $a/d = 5.0$ are about 1.0, indicating a linear relationship for this case. However, the R values decrease with increasing depth—$R = 1.0$ for $d = 20$ cm, $R = 0.99$ for $d = 50$ cm and $R = 0.94$ for $d = 100$ cm for $a/d = 5.0$. The effective depth of the beam and the shear span to depth ratio may have an influence on the relationship between V_u/bd and f_c'. A linear relationship R between V_u/bd and f_c' is observed for beams with smaller depths.

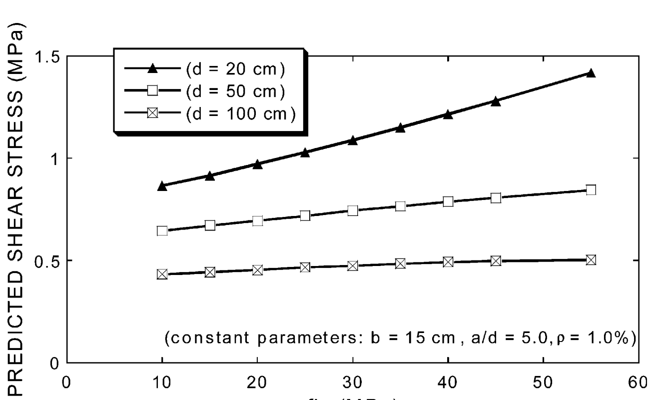

Fig. 12. Predicted shear stress for $a/d = 5.0$ and varying f_c' and d.

6. Conclusion

An ANN model with five input nodes, five hidden layer nodes and one output node was developed for predicting the shear stress at failure of slender RC beams without web reinforcements. The M2B-551 model gave predictions with an error of ± 20% of experimental values, covering the range from shallow to larger depths of the beam. Although there is still a need to improve the accuracy of the predictions, the present ANN model successfully simulates the effects of various parameters on the shear capacity of RC beams without web reinforcement. This shows the versatility of ANNs in constructing relationships among multiple variables of complex physical processes using actual experimental data for training.

Through parametric studies, the behavior of shear stress, V_u/bd at failure due to various factors is observed. A beam with a higher compressive strength of concrete, f_c', has a larger shear capacity than a beam with a lower compressive strength. However, the increase in the shear capacity with respect to f_c' is found to be significant in beams with smaller depths. The size effect is clearly demonstrated where the shear stress decreases with the increase in depth. The shear stress at failure is not constant. The decreasing trend of the shear stress with increasing depth is consistent with the experimental results. The shear capacity also depends on the shear span to depth ratio—a shorter beam has a larger shear stress than a longer beam. The effect of the amount of longitudinal steel on the size effect is also observed. The rate of decrease of the shear stress for the lightly reinforced beams is larger than for beams with higher longitudinal steel ratio.

If an ANN model which has wide application is desired, there should be an increase of the training database which is distributed and varied. Only when we have sufficient number of data that we can develop ANNs which can completely model the complex interactions among the multiple variables. The present ANN model can still be improved and extended by using more experimental data with a wider range of values—for example, extending the data up to $a/d <$ 2.5 to include deep beams which fail under shear compression and including higher strength concrete. More refined neural network training algorithms can also be used to reduce the error of the models. This study shows the potential of ANNs as a tool to support the task of civil and structural engineers in the modeling and prediction of behavior of engineering and natural systems.

Acknowledgements

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