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Shear design procedure for reinforced normal and high-strength concrete beams using artificial neural networks. Part II: beams with stirrups

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Abstract

In order to re-evaluate the current shear procedures of different codes of practice for normal-strength and high-strength beams with web reinforcement an extensive research study was performed. An Artificial Neural Network was developed to predict the shear strength of reinforced beams failing on diagonal tension failure and, based on its results, a parametric study was carried out to study the influence of each parameter affecting the shear strength of beams with web reinforcement. Some important influences are highlighted, for example the non-linear relationship between the amount of stirrups and the shear strength. Finally, new design expressions are proposed, taking into account the observed behaviour for the design of high-strength and normal-strength reinforced concrete beams with shear reinforcement. The new expressions correlate much better with the empirical tests than EC-2 or ACI procedures.

Keywords: Reinforced concrete; High-strength concrete; Shear strength; Structural design; Stirrups; Artificial neural networks

1. Introduction

Since the beginning of the 20th century, when Mörsh and Ritter postulated the earliest truss models, great progress has been made in the analytical solution of shear problems in reinforced concrete. However, most of these highly sophisticated tools [1–3] require considerable simplification to make them suitable for codes of practice. However, for simpler models the neglect of secondary factors can be problematic, what is secondary in one case may be primary in another, so very careful confirmation is always needed. Therefore, this paper aims to reformulate some simple design equations which take account of more complex models that had been shown to give very good correlation with empirical tests.

In addition, high-strength concrete has been increasingly used in the construction industry during the last few years, and any shear procedure should be able to be applied to both normal-strength and high-strength concrete beams. While for beams without web reinforcement the failure shear strength seems not to increase for concrete strengths beyond a certain limit [4], for beams with stirrups some authors consider that the use of HSC tends to prevent shear-compression failure and to ensure a diagonal tension failure instead, thus increasing the effectiveness of shear reinforcement [5].

With the aim of evaluating some current codes of practice [6]-[8], summarized in Table 1, an artificial neural network was developed based on a database with 123 test beams. This relies on the databases developed by Bentz [9] and Kuchma [10]. However, some new tests were added and other were omitted as the reinforcement steel had a very low yielding stress. To summarize, the database includes the beams tested by Refs. [4] and [11–24].

The shear-span-to-depth ratio, a/d, for all of these beam specimens, was greater than 2.49 and all of them failed in tension-shear. Although great efforts were made in order to achieve a homogeneous database (i.e.

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Nomenclature

A_l	area of the longitudinal reinforcement
A_w	area of the shear reinforcement
a/d	shear span to depth ratio
b_w	web width
d	effective depth
d_v	mechanical depth taken to be $0.9{\cdot}d$
f_c	concrete compressive strength
f_{ywd}	design yielding strength of the shear reinforcement
s	spacing of the stirrups
s_x	vertical distance between distributed longitudinal reinforcement
V_{fail}	failure shear strength of the actual test
V_{pred}	predicted shear strength
V_{GSDM}	predicted shear strength by the general shear design method
V_{SSDM}	predicted shear strength by the simplified shear design method
V_{SSVM}	predicted shear strength by the simplified shear verification method
ε_x	longitudinal strain in the web
ρ_l	amount of longitudinal reinforcement
ρ_w	amount of shear reinforcement (MPa)
θ	angle of the compression struts

15% of the beams had $f_c > 80$ MPa) it was not possible due to the properties of the tested beams. For instance, only 10% of the data had an effective depth greater or equal to 600 mm and only 2% of the beams contained an amount of longitudinal reinforcement equal to or less than 1%. However, with the aim of reducing the influence of the database's lack of heterogeneity, the analyses were performed not only on the full set of beams, but also on partial subsets of the database.

2. Artificial neural network

Artificial neural networks (ANNs) are a powerful computation tool able to 'learn' from a set of examples with known inputs and outputs. A brief conceptual introduction together with some good references about engineering applications of the ANNs is given in Ref. [4].

2.1. Data selection

The complete database for members with web reinforcement, described in the introduction, was that used for developing the ANN. Therefore, a total number of 123 test beams were utilized. These data were divided into two sets: a training set containing 104 beams, and a validating set comprised of 19 beams.

The input parameters used were the effective depth (d); the web slenderness factor d/b_w; where b_w is the web width; the shear span-to-depth ratio a/d; the reinforcement ratio of longitudinal tensile steel ($\rho_l = A_w f_{yw}/b_w s$); the concrete cylinder compressive stress (f_c); and the amount of transverse reinforcement

Table 1
Summary of current code of practice shear procedures

Code	Predicted failure shear strength	Comments
2002 Final draft of the Eurocode 2 [6]	$V_{Rd,s} = \frac{A_{sw}}{s} z\, f_{ywd} \cot\theta$	Do not consider concrete contribution $1 \le \cot\theta \le 2.5$
AASHTO LRFD 2000 [7]	$V_c = \beta\sqrt{f_c'}\, b_w d_v\, V_s = \frac{A_v f_y}{s} d_v \cot\theta$	β and θ are listed in a table as a function of the longitudinal strain in the web and the non-dimensional shear
ACI 318-02 Eq. 11–3[8]	$V_c = \left(\frac{\sqrt{f'_c}}{6}\right) b_w d V_s = \frac{A_{sw}}{s} z \quad f_{ywd}$	$f_c < 70$ MPa
ACI 318-02 Eq. 11–5 [8]	$V_c = \left(0{,}16\sqrt{f_c} + 17\rho_l \frac{V \cdot d}{M}\right) b_w d V_s = \frac{A_{sw}}{s} z \quad f_{ywd}$	$f_c < 70$ MPa, $Vd/\mathrm{M} \le 1$

Table 2
Artificial Neural Networks for beams with web reinforcement. Ranges of parameters in the database

Parameter	Minimum	Maximum
d (mm)	198	925
d/b	0.792	4.5
ρ_l (%)	0.50	5.80
ρ_w (MPa)	0.33	3.57
f_c (MPa)	21	125.2
a/d	2.49	5.00
V (kN)	63.28	1172.19

expressed in MPa (ρ_w). The output value is the failure shear strength (V_{fail}). Table 2 summarizes the ranges of the different variables.

2.2. Topology of the artificial neural network, learning process and validation

The trial and error process took more than 65 attempts. The optimum solution was obtained after 3000 epochs with a neural network containing nine hidden units. The average $V_{\text{test}}/V_{\text{fail}}$ ratio was equal to 1.00 for the training set and 1.01 for the validating set. The coefficients of variation were 10.96 and 13.38%, respectively.

3. Parametrical analyses based on the ANN results

Once the artificial neural network had been trained, a parametric analysis was used to study the influence of the different parameters having an effect on the shear strength. The parametric study used entailed many difficulties for three basic reasons: the amount of transverse reinforcement interacts considerably with the other parameters, the number of tested beams was not very high and there was a lack of information for some groups of beams, and comparison can only be made with the shear strength independently of the concrete and steel contributions.

3.1. Influence of the amount of transverse web reinforcement, ρ_w

Logically, the amount of web reinforcement has a very important influence on the failure shear strength. Historically, from the first models of Ritter and Mörsch to variable-angle-truss-models with or without concrete contribution, linearity has been assumed to exist between the amount of shear reinforcement and the ultimate beam response. Nevertheless, some authors pointed out that for high-strength concrete beams stirrups seemed to be more effective.

The ANN predicts a non-linear response based on the amount of web reinforcement. The greater number of stirrups the less effective they are, as can be seen in Fig. 1. For highly reinforced concrete beams calculated by, for example, final draft of the Eurocode 2 this effect leads to unconservative results, compared to the ANN predictions. The AASHTO LRFD procedure [7] takes into account the strain compatibility conditions and its predictions correlate better with the ANN results.

The beam specimens from Fig. 1 had the following characteristics: $d = 350$ mm, $b_w = 200$ mm, $\rho_l = 3\%$ and $a/d = 3$. The parameters f_c and ρ_w varied as indicated in the plots. As was mentioned earlier, the influence of the stirrups depends on the concrete compressive strength. For HSC beams that are not highly reinforced, this influence is higher than for normal strength concrete beams, according to the ANN results. This is the reason for the steeper slope of the $f_c = 100$ MPa graph. Neither the ACI318-02 [8], EC-2 [6], or AASHTO [7] procedures reproduce this behavior.

3.2. Size effect. Influence of the effective depth, d

It has been traditionally considered that the size effect disappears when stirrups are provided. However,

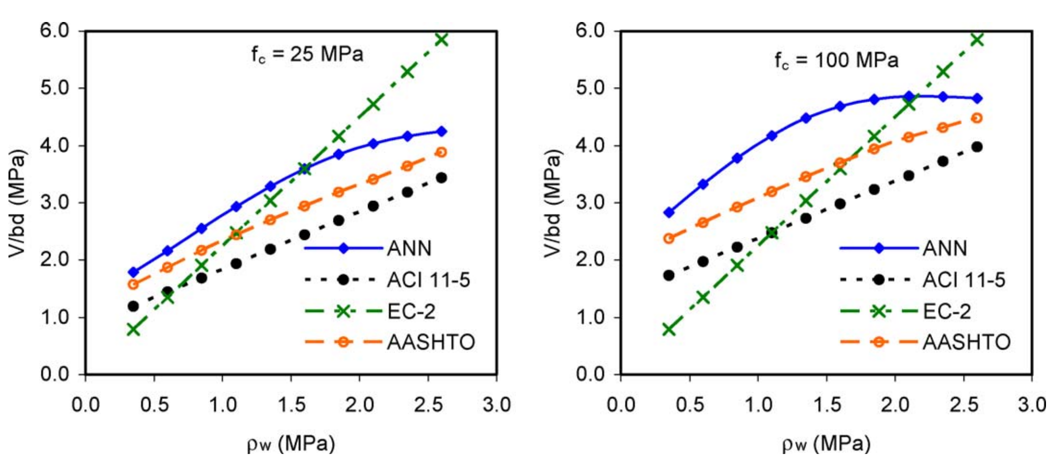

Fig. 1. ANN results compared to the ACI 11-5, EC-2 and AASHTO predictions for beams with web reinforcement. Influence of the amount of shear reinforcement in relation to the concrete compressive strength.

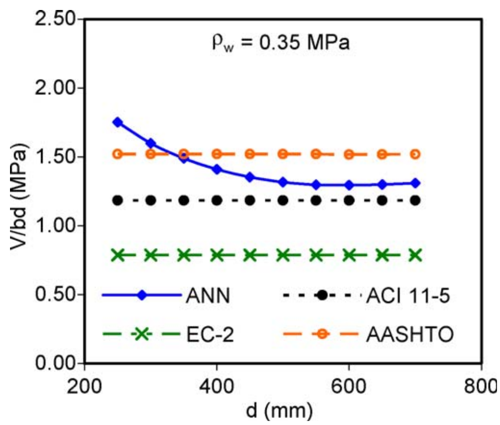

Fig. 2. ANN results compared to the ACI 11-5, EC-2 and AASHTO predictions for beams with web reinforcement. Size effect in relation to the amount of shear reinforcement.

Collins pointed out that for members with a low amount of web reinforcement a reduction in failure shear stress was detected [25]. This effect decreased when the amount of shear reinforcement increased.

The ANN analysis reveals that for lowly shear-reinforced members ($b_w = 300$ mm, $f_c = 25$ MPa, $\rho_w = 0.35$ MPa, $\rho_l = 2.5\%$, and $a/d = 3$) the size effect may reduce the failure shear stress about 25% as effective depth is increased from 250 to 700 mm, as is shown in Fig. 2. Members with twice this amount of shear reinforcement did not show any reduction in shear strength due to the size effect in the ANN analysis.

3.3. *Influence of the concrete compressive strength, f_c*

The influence of the concrete compressive strength depends on the beam size and the amount of transverse reinforcement. Fig. 3 plots the failure shear strength in relation to the concrete compressive strength. The beams presented the following characteristics: $b_w = 300$ mm, $a/d = 3$, $\rho_l = 3\%$, and $\rho_w = 0.50$ MPa. The effective depth and the concrete compressive strength varied as indicated in the graphs. The influence of the concrete compressive strength decreases for the $d = 700$ mm beam specimens in comparison to the $d = 350$ mm series. The EC-2 procedure [6] does not take into account this influence.

The relationship between the concrete compressive strength and the amount of transverse reinforcement is shown in Fig. 4 for two series of beams in which d is equal to 350 mm, $b_w = 300$ mm, $a/d = 3$, and $\rho_l = 3\%$. For normal strength concrete beams, the greater the amount of transversal reinforcement, the higher the influence of the concrete compressive strength. The AASHTO procedure [7] shows a satisfactory adaptation to this trend.

3.4. *Influence of the amount of longitudinal reinforcement, ρ_l*

Based on experimental tests, the artificial neural network predicts that the flexural reinforcement will have an important influence on both beams with high and beams with low reinforcement, as can be seen in Fig. 5. The influence is greater than ACI318-02 equation 11-5 [8] considers, and it does not follow the trend given by the AASHTO LRFD Specifications [7].

3.5. *Influence of the a/d ratio*

As one can see in Fig. 6, the ANN predicts lower failure shear stress values for higher M/V relationships. The beams shown in the figure presented the following characteristics: $b_w = 300$ mm, $d = 500$ mm, $f_c = 35$ MPa, $\rho_l = 3\%$ and $\rho_w = 1$ MPa. Both ACI318-02 equation 11–5 [8] and the AASHTO shear procedure [7] consider satisfactorily its influence.

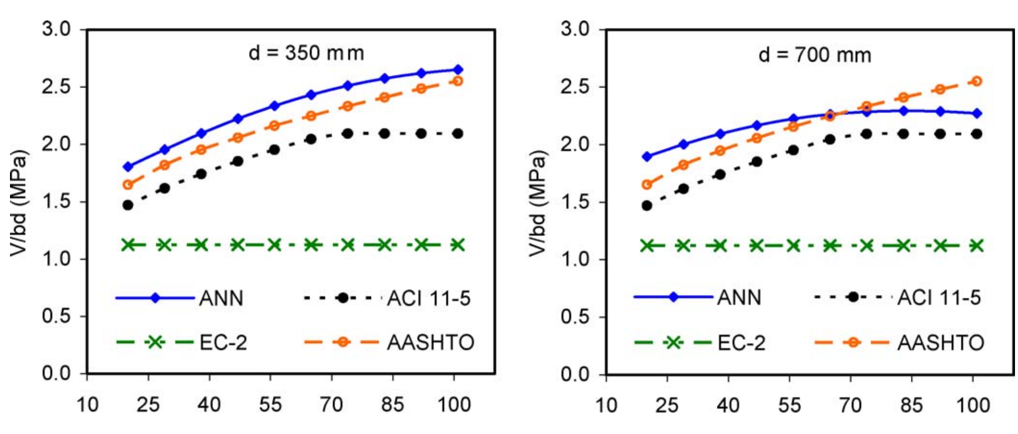

Fig. 3. ANN results as compared to the ACI 11-5, EC-2, and AASHTO predictions for beams with web reinforcement. Influence of the concrete compressive strength in relation to the beam size.

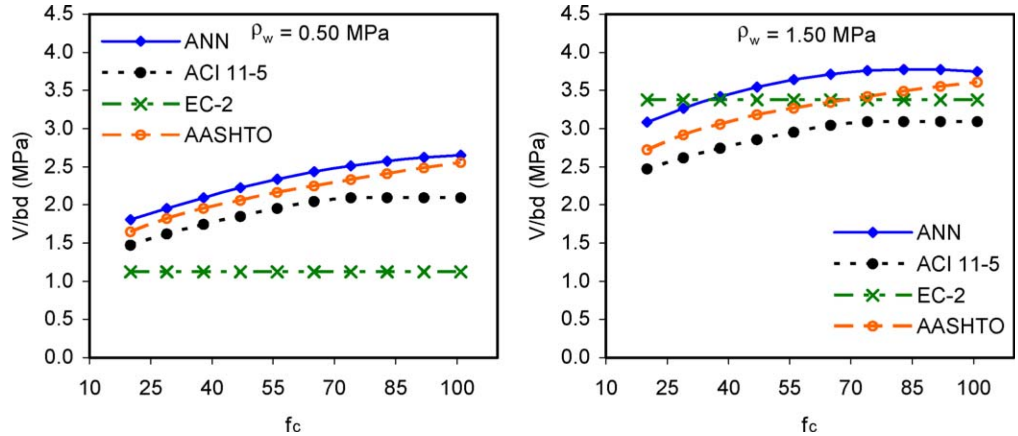

Fig. 4. ANN results as compared to the ACI 11-5, EC-2 and AASHTO predictions for beams with web reinforcement. Influence of the concrete compressive strength in relation to the amount of transverse reinforcement.

4. Proposal for a new shear design method

A general shear design method is proposed in this section, as well as two simplified shear procedures based on the following observations for members with web reinforcement:

- The AASHTO LRFD shear procedure [7], based on the modified compression field theory, performs much better than the other current codes do, as it will be seen in the last section of this paper. In this procedure, the angle between the compression struts and the longitudinal axis of the beam, θ, is obtained by compatibility, and it depends on the shear stress and the longitudinal strain.
- The concrete contribution to the shear strength is the vertical component of the shear stress transferred across the crack and therefore depends on the crack width. The greater the amount of shear reinforcement, the lesser crack width, and the larger the concrete contribution will be.
- The influence of the amount of web reinforcement is not linearly proportional to the failure shear strength. Truss models, like EC-2 [6], could be unconservative for highly reinforced concrete members.
- The use of high-strength concrete tends to prevent shear-compression failure and to ensure a diagonal tension failure instead, thus increasing the effectiveness of the shear reinforcement.
- For members with low shear reinforcement, the size effect reduces the shear stress at failure, although most codes do not take it into account for members with stirrups.
- An increase in the amount of longitudinal reinforcement produces an increase in shear strength in members with stirrups.

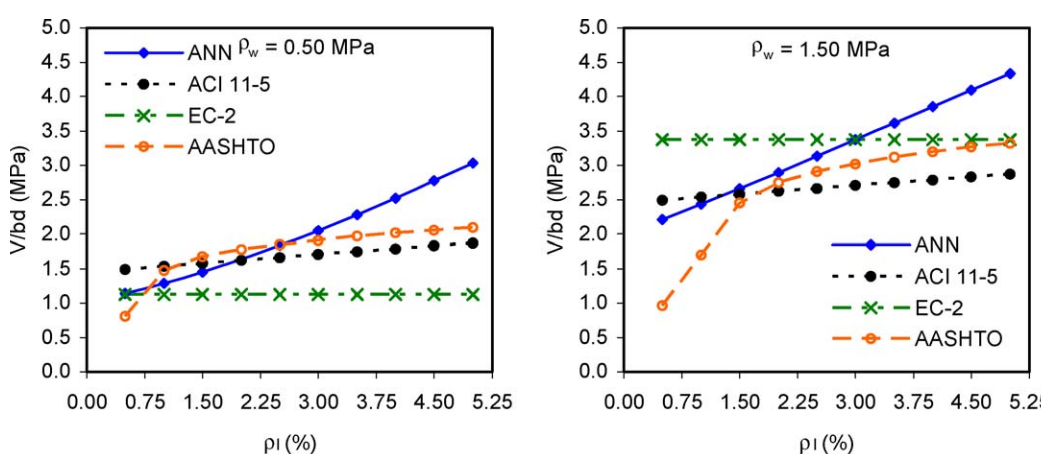

Fig. 5. ANN results as compared to the ACI 11-5, EC-2, and AASHTO predictions for beams with web reinforcement. Influence of the amount of longitudinal reinforcement.

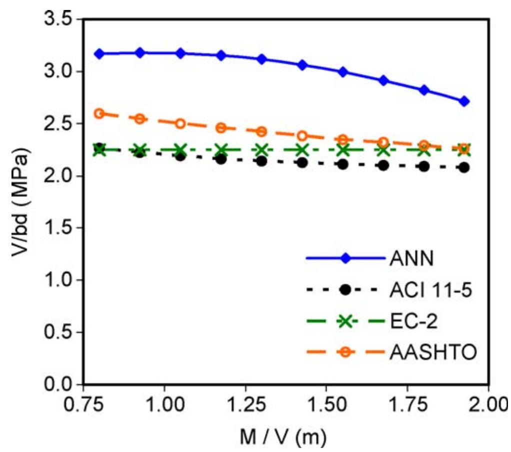

Fig. 6. ANN results as compared to the ACI 11-5, EC-2 and AASHTO predictions for beams with web reinforcement. Influence of the moment/shear (M/V) relationship.

4.1. General shear design method (GSDM)

The general shear design method is based on a truss model, where the angle θ was directly derived from the AASHTO LRFD procedure [7], in an attempt to find the simplest equation that still followed the general trend. The proposed value for θ is always conservative compared with the AASHTO predictions. Once the angle is obtained, the steel contribution is able to be determined.

The failure shear strength given by the simplified shear design method for beams without stirrups proposed by Cladera [4] was taken to be a good procedure for evaluating the concrete contribution for a beam with only longitudinal reinforcement. An extra term was added (Eq. (2)) to take into account the stirrups' influence on the shear friction which is the most important improvement of this shear design method. As the amount of transversal reinforcement is unknown during the design process, the new term is a function of the designing shear stress, τ, as the highest designing shear strength the more stirrups the beam will have . The value of the power $1/3$ and the constant 0.17, used to multiply V_c, were derived empirically to adjust the test beam results.

Thus, for members with web reinforcement, the failure tension shear strength is given by:

$$V = V_c + V_s \tag{1}$$

$$V_c = \left[0.17\xi(100\rho_l)^{1/2} f_c^{0.2}\ \tau^{1/3}\right]\ b_w d \tag{2}$$

where

$$\xi = 1 + \sqrt{\frac{200}{s_x}} \leq\ 2.75 \text{ is the size effect,}$$

and s_x is whichever is smaller, d_v or the vertical distance between longitudinal distributed reinforcement [4], d_v is the mechanical depth which can be taken as $0.9 \cdot d$, d is the effective depth in mm, $\rho_l = \frac{A_l}{b_w d} \leq 0.04$, is the amount of longitudinal reinforcement, $f_c \leq$ 100 MPa,

$$\tau = \frac{V_d}{b_w\ d_v} \leq 3 \text{ MPa}$$

, V_d is the designing (factored) shear strength, $V_d = V_k \cdot \gamma_f$, and b_w, the smallest width of the cross-section area in mm. And,

$$V_s = d_v \frac{A_w}{s} f_{ywd} \cot\theta \tag{3}$$

where A_w is the cross-sectional area of the shear reinforcement; s is the spacing of the stirrupsf_{ywd} is the design yielding strength of the shear reinforcement, and θ is the angle of the compression struts, derived as follows:

$$\theta = 20 + 15\ \varepsilon_x + 45\frac{\tau}{f_c} \leq 45^\circ \tag{4}$$

where ε_x is the longitudinal strain in the web (Fig. 7), expressed in $1/1000$, calculated by the following expression:

$$\varepsilon_x \approx 0.5\frac{\frac{M_d}{d_v} + V_d}{E_s A_l} \cdot 1000 \leq 1$$
$$\frac{\tau}{f_c} \geq 0.05 \tag{5}$$

The expression of the longitudinal strain in the web is a conservative simplification of the real strain. It

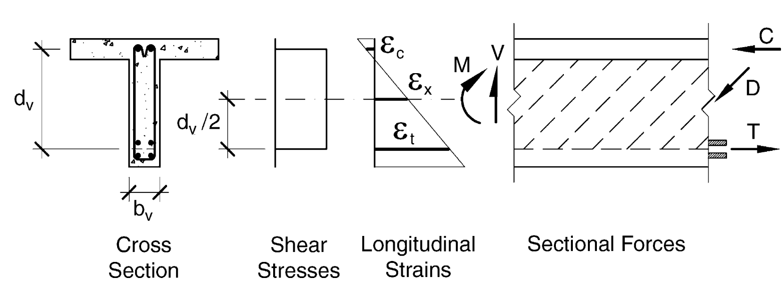

Fig. 7. Longitudinal strain in the web [26].

assumes that in the web the strain is equal to one half the strain in the tension reinforcement, and that the maximum longitudinal strain of the reinforcement is 0.002.

4.2. Simplified shear design method (SSDM)

To apply the general shear design procedure it is necessary to evaluate the shear strength in different sections of the beam, due to the interaction between the bending moment and the shear strength. The simplified shear design method assumes that the longitudinal strain in the web, ε_x, is equal to 0.1%, and therefore that the longitudinal reinforcement yields; this is the worst condition under which to calculate the shear strength. Hence:

$$\theta = 35 + 45\frac{\tau}{f_c} \leq 45^\circ \tag{6}$$

where

$$\frac{\tau}{f_c} \geq 0.05$$

With the value of the angle of the compression struts given by Eq. (6), the failure shear strength can be calculated using Eqs. (1), (2) and (3).

4.3. Simplified shear verification method (SSVM)

To verify the ultimate shear strength of a given section it would be possible to use the expressions given in the general shear design method, although it would be necessary to iterate to find the solution, as V_d is an input to obtain both the concrete and the steel contributions. Moreover, the ultimate shear strength would depend not only on the cross-section of the beam, but also on the bending moment in that section.

The simplified shear verification method estimates τ as a function of the member depth, web width and the amount of shear reinforcement:

$$\tau_{est} = 3.5\sqrt{\frac{200}{s_x}\frac{f_{ywd}}{b_w}}\left(\frac{A_w}{s}\right)^{0.5} \tag{7}$$

The ultimate shear strength can be calculated from eqs. 1–3 assuming that $\tau = \tau_{\mathrm{est}}$ in Eq. (2) and ε_x= 1 in Eq. (3).

The above estimation (Eq. (7)) is equivalent to a truss model using a variable angle of inclination for the trusses. For small members, such as $s_x = 200$ mm, the estimated shear failure would be given by a truss model in which $\cot\theta = 3.5$. For a bigger beam, $s_x = 1000$ mm, the inclination of the truss would be given by $\cot\theta = 1.57$.

5. Verification of the proposed equations with the experimental database

The proposed equations are compared in Table 3 with the database's 123 test beams with shear reinforcement. The three proposed procedures correlate much better with empirical tests than do the EC-2, ACI318-02 11-5 or 11-3 procedures. For example, the $V_{\mathrm{test}}/V_{\mathrm{fail}}$ ratio for the ACI 318-02 (Eq. 11–5) is 1.36, with a CoV of 24.60, while it is 1.11, with a standard deviation of 18.77 for the general shear design method. Nevertheless, the AASHTO LRFD shear procedure performs very similarly to the proposed equations.

It can also be seen in Table 3, that the two simplified methods are slightly more conservative than the general design method, as they do not take into account the influence of the bending moment, and they assume that the longitudinal rebars yield.

To check the ability of the proposed methods to predict the shear strength for different types of beams, the results of the partial set of analyses are given in Table 4. The proposed procedures represent an improvement over the performance of the EC-2, ACI 11-5, and ACI 11-3 procedures for all the groups of beams studied.

For the biggest members, where $d \geq 750$ mm, most codes do not take into account the size effect when stirrups are provided. As it was shown in Fig. 2 for members with low shear reinforcement, the size effect causes a reduction in shear strength. The proposed equations correctly reproduce this behavior, as very little reduction in the overall safety factor ($V_{\mathrm{fail}}/V_{\mathrm{pred}}$) is observed.

Another set of beams that requires special attention is the group of seven beams with high shear reinforcement ($\rho_w > 2$ MPa). The EC-2 procedure is absolutely unconservative for this set of beams with an average

Table 3
Verification of proposed shear procedures for beams with web reinforcement using the entire database

Procedure	EC-2	AASHTO	ACI 11-5	ACI 11-3	GSDM	SSDM	SSVM
Average	1.83	1.18	1.36	1.41	1.11	1.17	1.18
Median	1.72	1.17	1.37	1.42	1.11	1.17	1.19
Standard deviation	0.74	0.23	0.34	0.38	0.21	0.23	0.22
COV (%)	40.29	19.23	24.60	26.70	18.77	19.56	18.71
Minimum	0.50	0.69	0.69	0.67	0.67	0.67	0.73
Maximum	4.85	1.96	2.66	2.83	2.01	2.20	2.14

Table 4
Verification of different code procedures using partial sets of the database for beams with web reinforcement

Beam specimens	No. beams	Average V_{test}/V_{pred}							COV V_{test}/V_{pred}						
		EC-2	LRFD	ACI 11-5	ACI 11-3	GS DM	SS DM	SS VM	EC-2	LRFD	ACI 11-5	ACI 11-3	GS DM	SS DM	SS VM
All	123	1.83	1.18	1.36	1.41	1.11	1.17	1.18	40.29	19.23	24.60	26.70	18.77	19.56	18.71
$d \geq 750$ mm	12	1.34	1.00	0.85	0.88	1.08	1.12	1.14	24.66	20.38	19.33	20.97	16.83	16.26	15.05
$\rho_w \leq 1$ MPa	93	2.05	1.18	1.29	1.42	1.12	1.19	1.20	34.28	19.84	26.26	28.42	18.63	19.17	18.54
$\rho_w > 1$ MPa, $\rho_w \leq 2$ MPa	23	1.28	1.22	1.32	1.42	1.10	1.14	1.17	22.76	15.89	17.56	18.66	15.68	16.38	15.51
$\rho_w > 2$ MPa	7	0.78	1.07	1.14	1.23	0.99	1.02	1.06	19.63	20.91	22.84	23.84	29.50	31.08	29.62
$f_c \leq 70$ MPa	38	1.66	1.17	1.31	1.35	1.09	1.14	1.15	37.62	18.35	21.03	22.83	15.77	15.75	15.63
$f_c > 70$ MPa	85	2.05	1.20	1.43	1.48	1.14	1.21	1.23	39.74	20.22	27.30	29.61	21.49	22.87	21.19
$\rho_l \leq 2\%$	19	1.33	0.99	0.92	0.96	1.05	1.08	1.08	32.24	15.54	22.17	23.37	17.62	17.66	16.02

V_{fail}/V_{pred} ratio of 0.78, while the average ratio for the complete database is 1.83. The general shear design method presents a V_{fail}/V_{pred} ratio of 0.99, but the decrease in the safety factor is only by about 11% with respect to the whole database. The simplified shear design methods are not unconservative for this set of beams, although the best performance is achieved by the AASHTO procedure.

For beams with a low amount of longitudinal reinforcement, $\rho_l \leq 2\%$, the proposed methods perform satisfactorily, while other codes present slightly unconservative results.

6. Equivalence between the simplified shear design method and the shear verification method

The proposed simplified shear procedures are specially intended for design and verification. Both procedures assume the longitudinal strain in the web to be equal to 0.1%, and therefore, that the longitudinal reinforcement yields. The results obtained by the two methods are practically identical, the verification procedure being slightly (1%) more conservative than the design method.

The average V_{SSDM}/V_{SSVM} ratio for the database's 123 test beams with web reinforcement, where V_{SSDM} is the shear strength predicted by the simplified shear design method and V_{SSVM} is the shear strength predicted by the simplified shear verification method, is equal to 1.01, and its coefficient of variation is 3.85%. For 82% of the test beams, the value of the V_{SSDM}/V_{SSVM} ratio falls in the 0.95–1.05 band.

7. Design example

To illustrate the use of the proposed methods, beam H2-N tested by Yoon et al. [24] is calculated by the simplified shear verification method and by the general shear design method. The beam presented the following characteristics: $b_w = 375$ mm, $d = 655$ mm, $f_c = 87$ MPa, $f_y = 430$ MPa, $A_l = 7000$ mm^2 ($\rho_l = 0.028$), $A_w = 2 \cdot 71$ mm^2, $s = 160$ mm ($\rho_w = 0.99$ MPa), and it was tested with a shear span of 2150 mm. It failed under a shear of 721 kN.

7.1. Simplified shear verification method (SSVM)

The failure shear strength is given by the sum of the concrete and the steel contribution. Firstly, to calculate the concrete contribution is necessary to estimate the shear stress, τ, from Eq. (7):

$$\tau = 3.5\sqrt{\frac{200}{0.9 \cdot 655}\frac{430}{375}}\left(\frac{2 \cdot 71}{160}\right)^{0.5} = 2.20 \text{ MPa}$$

thus, the concrete contribution from Eq. (2) is as follows:

$$V_c = 0.17\left(1 + \sqrt{\frac{200}{0.9 \cdot 655}}\right)\left(100\frac{7000}{655 \cdot 375}\right)^{0.5} \times 87^{0.2}\, 2.20^{1/3}\, 375 \cdot 655 = 354 \text{ kN}$$

The angle of the compression struts is given by Eq. (6), taking into account that $\tau/f_c = 2.20/87 = 0.02$ but it can not be taken lesser than 0.05:

$$\theta = 35 + 45 \cdot 0.05 = 37.25^{\circ}$$

and the steel contribution from Eq. (3):

$$V_s = 0.9 \cdot 655\frac{2 \cdot 71}{160}430 \cdot \cot 37.25^{\circ} = 296 \text{ kN}$$

Therefore, the shear strength is equal to:

$$V = V_c + V_s = 354 + 296 = 650 \text{ kN}$$

which compared to the failure shear force gives a $V_{fail}/V_{SSVM} = 1.11$.

7.2. General shear design method (GSDM)

In this case the amount of shear reinforcement necessary to obtain a shear strength of 721 kN is going to be calculated using the proposed general shear design method.

The shear strength is again the sum of the concrete and steel contributions. From Eq. (2), and considering $\tau = 721 \cdot 10^3/(375 \cdot 0.9 \cdot 655) = 2.94 < 3$ MPa:

$$V_c = 0.17\left(1+\sqrt{\frac{200}{0.9\cdot 655}}\right)\left(100\frac{7000}{655\cdot 375}\right)^{0.5} \times 87^{0.2}\ 2.94^{1/3}\ 375\cdot 655 = 387\ \text{kN}$$

Prior to calculating the angle of the compression struts (Eq. (4)), is necessary to evaluate the longitudinal strain in the web from Eq. (5). The bending moment at the critical section (situated d_v away from the border of the application load) is 1071 kN:

$$\varepsilon_x = 0.5\frac{\frac{1071\cdot 10^6}{0.9\cdot 655}+721\cdot 10^3}{200000\cdot 7000}1000 = 0.91$$

$$\theta = 20 + 15\cdot 0.91 + 45\cdot 0.05 = 35.9^\circ$$

It is now possible to calculate from the steel contribution (Eq. (3)) the spacing of the stirrups necessary:

$$V_s = V_d - V_c = 721 - 387 = 334\ \text{kN}$$

$$V_s = 0.9\cdot 655\frac{2\cdot 71}{s}430\cdot \cot 35.9^\circ = 334\ \text{kN}$$

obtaining $s = 149$ mm, less than a 10% higher than the actual amount provided.

8. Conclusions

A study related to the shear strength of normal and high-strength concrete beams with stirrups has been carried out. An Artificial Neural Network has been developed to predict the shear strength of RC beams using a large database of experimental results and to perform parametrical analyses to determine the influence of each parameter affecting the shear strength. Finally design formulae have been developed. The following conclusions can be drawn for the present study:

- Artificial neural networks are a powerful tool for predicting the shear strength of concrete beams provided that a sufficient and representative number of test results are used for training and validating it.
- The influence of the amount of web reinforcement is not linearly proportional to the failure shear strength. The more stirrups, the less effective they are.
- For members with low shear reinforcement, the size effect reduces the failure shear stress by about 25% as effective depth is increased from 250 to 700 mm.
- The influence of the concrete compressive strength varies with the amount of transversal reinforcement and the size of the beam. Generally speaking, however, the higher the concrete compressive strength the higher the failure shear strength.
- The influence of the amount of longitudinal reinforcement is higher than that predicted by the codes of practice. No limitation is necessary up to an amount of about 5%.
- The influence of the a/d ratio, or the M/V relationship, has been also studied. The AASHTO LRFD shear procedure duplicates the behaviour observed in the Artificial Neural Network results.
- The proposed general shear design method correlates well with the empirical results, with a CoV of the $V_{\text{fail}}/V_{\text{pred}}$ ratio of 18.77%. This contrasts with the correlations made by the ACI 318-02 (Eq. 11–5) or Eurocode 2 (final draft 2002) procedures whose CoV are 24.68% and 40.29%, respectively.
- The general shear design method takes into account the interaction between bending moment and shear, and therefore, the shear strength is dependent upon the bending moment of each given section and not only the geometrical and reinforcing properties of that section. A simplified shear design method and a simplified shear verification method are proposed for obtaining conservative results independently of the bending moment.
- The simplified shear design method considers the designing shear force V_d to be an input, while the verification method considers V_d to be an output. The results obtained by these two methods are practically identical, with a coefficient of variation around 19%.

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