ECEN 679

Computer Relays

Project 1

Fourier Transform Algorithm

March 23, 2021

Statement of originality

This work is done be me and me only.

Summary (contains major findings)

This project report consists of several parts.

First part of the project dealt with the theoretical foundation of Fourier analysis and the applications of Fourier analysis in the field of Digital Relaying. The basic approach was discovered: Fourier method is used to obtain phasors from the sampled data. One application is digital distance relaying, in that case the current and voltage samples are transformed into fundamental frequency phasors and then the apparent impedance is calculated.

The second part explored the properties of the Fourier algorithm, its shortcomings, and advantages. The effects of noise, high frequency components, DC offset, fundamental frequency change, window size, number of samples were researched and analyzed. Also, ability of the method to extract not only fundamental frequency phasor, but also higher harmonics phasor was discovered. Along with it, the frequency response of the half-cycle and full-cycle versions of algorithm was analyzed. The method was implemented in Python code and some of the features were tested and results are presented.

Finally, answers to the questions from project assignment were addressed.

The report is organized as instructed in “outline” document on eCampus.

Introduction (describe the assignment)

Protection of power systems is one of the most important tasks in industry [1]. With introduction of digital relaying and advent of digital processors [2] relay protection engineers were able to apply signal processing techniques for implementing power system protection functions. Distance protection relays are one of the most complex relays, which combined with modern circuit breakers, are capable of clearing faults at extremely fast clearing times. One of the widely used algorithms in digital relaying is Fourier Transform Algorithms [3].

The goal of this project is to study the Fourier Transform Algorithm (FTA) and analyze its properties from the sources in the literature. Fourier Algorithm allows rapid extraction of phasors of measured quantities for further use in protection logic. One example of the application is the extraction of fundamental frequency phasors from samples of current and voltage, which are then used to calculate apparent impedance. Apparent impedance is then used in mho plane to determine if there is a fault in protected segment. In case of detecting the fault, the trip signal is sent to the circuit breaker.

The following questions were covered in this project report:

- What the basic performance and implementation characteristics of the Fourier Transform algorithm are? (The underlying theory shall be explained)
- How the Fourier Transform Algorithm is used in distance relaying? (The related relaying function implementation shall be described)
- What the benefits and shortcomings of this algorithm when used for relaying are? (The implementation advantages and disadvantages shall be indicated)
- What the literature used is? (A list of references shall be provided)

Algorithm description (brief outline of major algorithm design steps and design parameters)

Theoretical overview

Fourier Analysis is a well-known tool among the engineers. However, the author of this report found himself in a confused state about some matters. A concise summary about theoretical basis is therefore given below.

First, the concept of Fourier Series would be discussed. The idea is that almost any function f(x) on some interval can be represented as a sum of sines and cosines of increasing frequency, with some coefficients. We say “almost”, because the function needs to satisfy Dirichlet conditions, which are almost always true for signals in electrical engineering. The mathematical expressions are as follows:

$$f(x) = \frac{A_0}{2} + \sum_{k=1}^{\infty} A_k cos(kx) + B_k sin(kx)$$

Where f(x) is a function on interval [-π, π], A_0, A_k, B_k are some coefficients, that are found by using following expressions:

$$A_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)cos(kx)dx;\ B_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)sin(kx)dx$$

The same can be rewritten using complex notations of sine and cosine, leading to a more concise expression:

$$f(x) = \sum_{k=-\infty}^{\infty} C_k\, e^{jkx}; \qquad \text{where } C_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-jkx}\, dx$$

C_k is a complex coefficient. The function may be determined not only in $[-\pi, \pi]$ interval, but also on arbitrary interval [-L, L]. Then the expressions are modified to make the interval be a multiple of sine and cosine periods.

There is a geometric meaning to Fourier Series. It can be thought of as projection of a function into orthogonal “directions” of sines and cosines in function space (as opposed to vector space) [4].

The Fourier Transform is a mathematical transform (one can think of transform as a mapping from one “environment” to another) that puts original signal into frequency domain. That is, using a well-defined mathematical expression, we can represent a function from time domain in frequency domain and back to time domain. The “frequency domain” is the aforementioned orthogonal function space. The need to transform into frequency domain is justified by a number of useful properties, that allow engineers to work with the signal in a more convenient way.

The Discrete Fourier Transform (DFT) is the same concept but applied to the discrete samples of the signal. The DFT is widely used in many fields. It allows to determine frequency content of a sampled signal, that is, it represent the digital signal as a sum of periodic signals of increasing frequency, just like Fourier Series do with analog signal.

Signal model

Let us first present the signal model. The signal model of the measured quantity can be represented as follows [5]:

$$u(t) = Vcos(w_0 t + \gamma) + \sum_k C_k^u\, f_k^u(t) + n_u(t)$$

$$i(t) = I\, cos(w_0 t + \varphi) + \sum_k C_k^i\, f_k^i(t) + n_i(t)$$

Where $C_k^u, C_k^i, V, I, \gamma, \varphi$ are unknown coefficients (Signal Parameters), $f_k^u(t)$, $f_k^u(t)$ are known functions, representing higher harmonics (HH), $n_u(t), n_i(t)$ are noise terms. As can be seen from the equations above the signal is represented as a sum of fundamental frequency harmonic, higher harmonics, and a noise term. The goal of the algorithm is to extract the unknown coefficients from the signal. After coefficients are determined, one can calculate apparent impedance, which is used to determine if the fault has occurred:

$$z = R + jw_0L = \frac{V}{I}cos(\gamma - \varphi) + j\frac{V}{I}sin(\gamma - \varphi)$$

From the classification point of view, presented in [5, 6], the Fourier Algorithms belong to the class of lumped parameter model algorithms, as opposed to distributed parameter model. FTA is also a frequency domain algorithm, which does not require optimization. However, it is worth mentioning that under assumption of a constant variance error model and external removal of the offset, the Discrete Fourier Transform (DFT) is an optimal estimate of the fundamental and of all of the higher harmonics permitted by the sampling rate, as mentioned in [7].

Another way of representing the signal is of the following form [8]:

$$y(t) = Yc\ cos(w_0t) + Ys\ sin(w_0t) + \varepsilon(t)$$

where y(t) represents the measured signal, Yc and Ys are unknown coefficient, ε(t) is a noise term. The reader must not be confused, as the author of this report was in the beginning, since the latest equation is just another form of representation of the signal. It transforms the cosine function with an arbitrary phase argument into sum of sine and cosine with phase argument equal to zero. And the higher harmonics are included in the noise term.

Phasor estimation from sampled data

The introduced signal is being sampled at some sampling rate, which needs to be at least twice the highest frequency, that needs to be obtained from the signal, in order to avoid aliasing. The anti-aliasing filters are used before the signal is sampled to attenuate higher frequencies that may appear in the signal [9].

To represent the samples from the signal x(t), we would use the following notation. Let's assume that the signal is being sampled at the t = kT, k = 0, 1, 2, . . ., T – is a sampling interval, and x_k is the corresponding samples [10]:

$$x_k = x(kT), k = 0,1,2...$$

Then we can write phasor representation of the fundamental frequency (FF) component of the signal x(t) as a function of samples x_k:

$$X = f(x_k), k = 0,1,2...$$

On the multiples of one-half of the period the sine and cosine functions form an orthogonal basis. Therefore, the projection of the signal onto that basis gives us the accurate phasor estimation [11]. More information about window size for the algorithm would be given in the Performance assessment section. In a meantime, if the window size is a multiple of half-cycle FF period, and the number of samples per period is denotes as N, then the phasor in polar form for FF components could be written as [10]:

$$X = \frac{2}{N}\sum_{k=0}^{N-1} x_k B_k, \text{ where } B_k = exp\left(-j\frac{2\pi T}{T_0}k\right)$$

Where T_0 is the period of the FF. The equations above transform into following form for full-cycle (N = 12) and half-cycle (N = 6) data windows, we would further refer to these implementations as full-cycle and half-cycle:

$$X = \frac{1}{6}\sum_{k=0}^{11} x_k \cdot exp\,(-j\,\frac{k\pi}{6}), for\ full - cycle$$

$$X = \frac{1}{3}\sum_{k=0}^{5} x_k \cdot exp\,(-j\,\frac{k\pi}{6}), for\ half - cycle$$

Using the second notation, presented earlier, we can write the FF phasor as its orthogonal components [8]:

$$Yc = \frac{2}{N}\sum_{k=0}^{N-1} x_k\, cos\left([k+1]\frac{2\pi}{N}\right)$$

$$Ys = \frac{2}{N}\sum_{k=0}^{N-1} x_k\, sin\left([k+1]\frac{2\pi}{N}\right)$$

The relation between polar and orthogonal form is well known:

$$X = \sqrt{Yc^2 + Ys^2}\ , arg(X) = tan^{-1}\left[\frac{Ys}{Yc}\right]$$

Recursive form

Since in relay during its operation the samples are coming at every sample instant, the relay needs to compute phasor at each of these instants. Considerable reduction in computation time can be achieved by using recursive form of DFT. There is a big number of suggested algorithms in the literature, that are using recursive form in some manner. Here we would just outline one of them. Let us write the phasor estimate from last L measurements as X^L (we assume here for simplicity that we are using half-cycle version with N = 6), that is the X^L is obtained from measurements x_{L-5}, x_{L-4}, x_{L-3}, x_{L-2}, x_{L-1}, x_L. Then as new measurement (x_{L+1}) comes in and the oldest (x_{L-5}) is discarded, the new phasor estimate can be represented as [10]:

$$X^{L+1} = X^L + \frac{1}{3}\exp\left[-j\frac{(L+1)\pi}{6}\right] \cdot \Delta x_{L+1}$$
$$\Delta x_{L+1} = x_{L+1} + x_{L-5}$$

The saving in computation time, however, comes at cost. The recursive form is known to have some drawbacks. One of them that it is prone to accumulating error. Reference [12] states that the usage of the recursive form is preferred from computational perspective. The authors note that adoption of integer arithmetic, truncation or round-off, that are imposed by finite wordlengths in recursive application of the method make recursive DFT less reliable that non-recursive version. It also recommended to use a combination of recursive and non-recursive versions to achieve optimal results. The issue with recursive method is also noted in [10]. Authors give an example of how integer arithmetic leads to cumulative error. The adverse effect can be reduced by using special programming techniques.

Performance assessment (discussion of the algorithm features affecting the accuracy and response time)

In that section we would cover major parameters of the algorithm as well as its performance in different situations. In general, the algorithm's accuracy is affected by all the factors listed in [13] and [6].

Window size

The algorithm requires even number of samples per period. As was mentioned earlier, conventional applications of FTA use the window size equal to a multiple of half-cycle. It is for the reason that cosine and sine forming an orthogonal basis under such circumstances, that is the inner product of sine and cosine in Hilbert space is equal to zero [4], which can be confirmed by taking a definite integral of multiplication of sin(x) and cos(x) on the interval of [-π/2, π/2]. However, in general, the FTA can be implemented on shorter windows [11]. It should be noted that for windows that are not integer multiple of half-cycle sine and cosine no longer form an orthogonal basis. For that reason, the corrective matrix should be used in order to compute FF phasor. For that reason, the most widely used window sizes are half-cycle window and full-cycle window. We would now explore other properties and point out the differences of two versions.

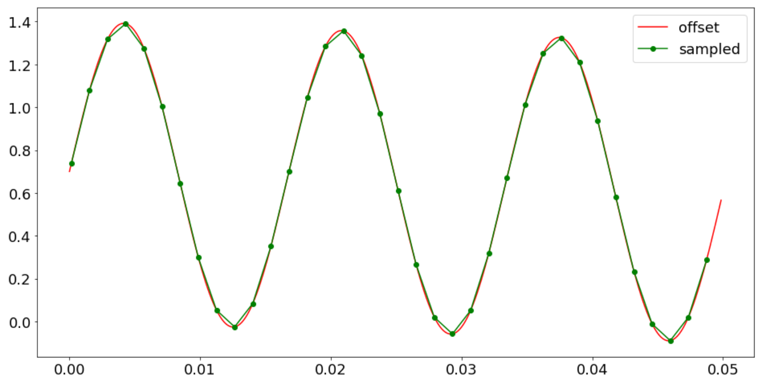
Effect of DC offset

The decaying DC offset that may be present at the sampled signal gives rise to errors in phasor estimation. Therefore, the distance relays have a tendency to overreach or underreach, when DC components are present in a signal. Typical EHV line has a time constant for decaying DC component of 30-50 ms. The initial amplitude of the DC offset can be as high as the peak of fault current [8]. In literature several approaches were suggested to reduce or partially remove the effect of DC offset: mimic filter, modified FTA with need one cycle and two samples of measurements, partial-sum based method, use of odd- and even-sample sets etc. [14].

Both full-cycle and half-cycle algorithms are effected by DC component. However, the full-cycle is less effected, when compared to full-cycle. The author has modeled the full and half cycle algorithms and tested them on several signals. The implementation is available at the Github [15]. The below pictures demonstrate how the algorithm is effected by DC component. The modeled signal is represented as

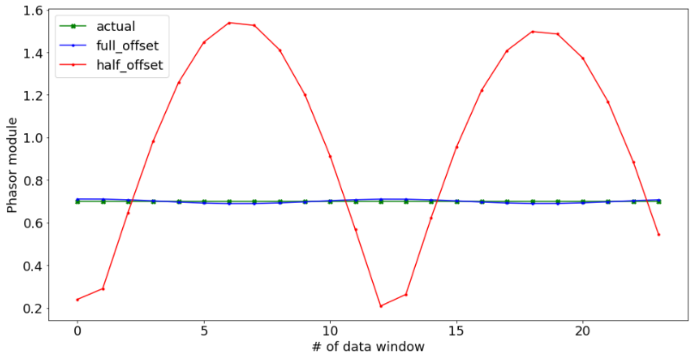
$$x_{offset}(t) = 0.7sin(2\pi 60 * t) + 0.7exp(-3 * t)$$

The original signal and the sample points are shown below:



Sampling rate is 12 samples per cycle. The full-cycle method therefore takes a data window of 12 samples and half-cycle works with window of 6 samples. The half-cycle starts 6 samples later than full-cycle, in that way the results are comparable at a given data window number.

The result of phasor module estimation of half and full cycle versions of FTA are presented below. Red line shows the results of half-cycle algorithm, blue line shows the full-cycle and green line shows the actual magnitude of first harmonic for reference:



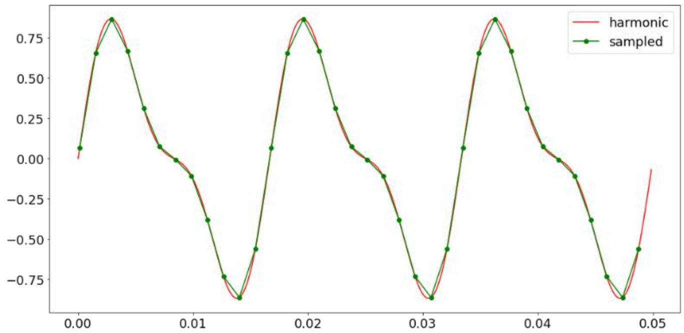
As can be seen, the half-cycle method is heavily affected by DC offset. The full-cycle method is affected as well, but to a lesser degree. For that reason, the half-cycle algorithm needs an external means of removing DC offset.

Higher Harmonics effect

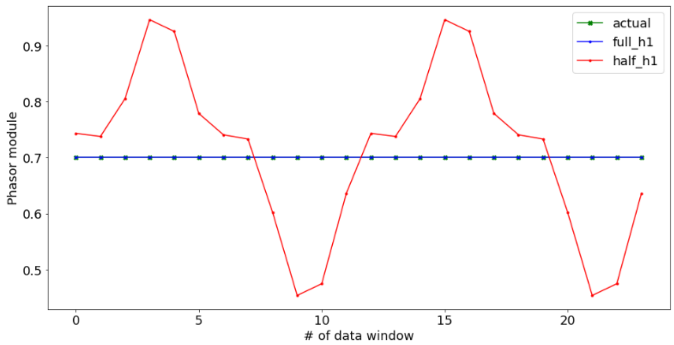
Fourier algorithm is cable of extracting not only fundamental frequency components, but also higher harmonics (HH) components. Even though the HH are not necessarily needed in line protection, they are very helpful in other applications, such as transformer protection. In latter case the HH are used to detect magnetizing inrush currents, which lead to maloperations of relays [16]. Actually, literature names DFT as the best choice for computing fundamental and harmonic components of input signals [12]. The decision whether to include HH in signal model is made by relay designer, the assumption of the content of the signal may affect the accuracy.

Unfortunately, presence of higher harmonics may have adverse effect on the calculation of fundamental frequency phasor. It is worth noting that full-cycle method is “immune”[11] to higher harmonics, but is effected by intraharmonics. The half-cycle is negatively effected by even harmonics. To demonstrate this effect the signal with even-harmonic was modeled:

$$x_{harmonic}\ (t) = 0.7sin(2\pi 60 * t) + 0.3sin(4\pi 60 * t)$$



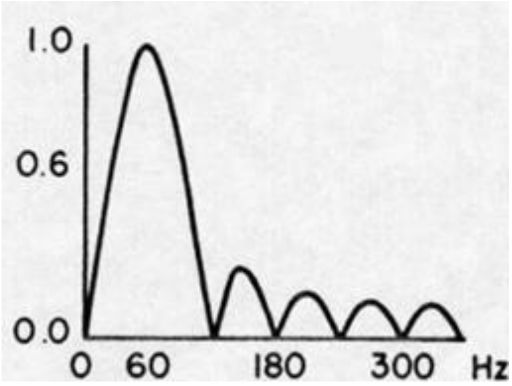
And the results of full and half methods are as follows:



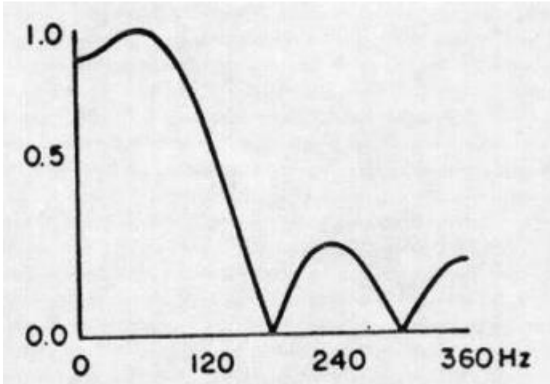
The full-cycle rejects the harmonic, whilst half-cycle produces oscillating result.

Frequency response

The digital algorithm can be thought of as digital filter, thus it should posses the frequency response. For that reason, it is convenient to summarize the effect of DC offset and HH as frequency response of the algorithm. The corresponding plots for 12 samples per period are presented below [7]:



Full-cycle



Half-cycle

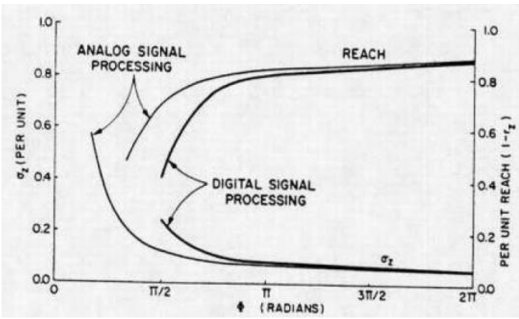
Effect of noise

Another source of error is the presence of noise in the signal. It can be of different nature ranging from inaccuracies of instrument transformer and harmonic signals from arc faults to noise from power system itself, which depend on fault location and nature of the system feeding the fault [8]. It is common to treat the noise term ε(t) in the signal as random process.

In general, there is inherent relationship between accuracy and speed of the algorithm. The authors of [17] show that an attempt to make impedance relay faster inevitably leads to less accurate measurement of the impedance, because of transients that occur during the fault. The variance of the impedance calculation can be expressed as a function of a data window. We denote a reach as

$$reach: 1 - 2.5\sigma_z$$

Where $\sigma_z$is variance of impedance calculation, the term $2.5\sigma_z$ represents the 99% confidence interval of impedance calculation, described by various distributions [17]. Then the reach and variance of an impedance relay as a function of data window is shown below:

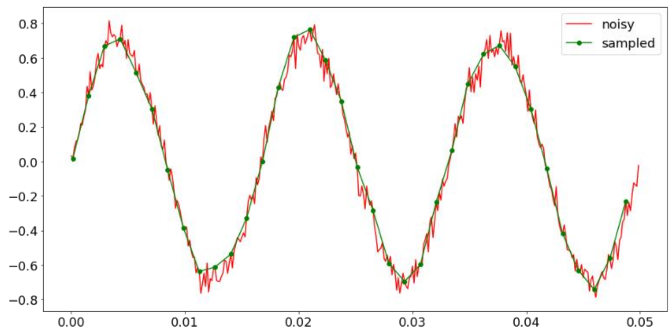


We should also note that as the data window moves further from disturbance moment, the noise, DC offset and harmonics in the signal are reduced, so the accuracy of the algorithm improves in that case. However, that comes at the price of time: you need to wait longer to get more accurate results.

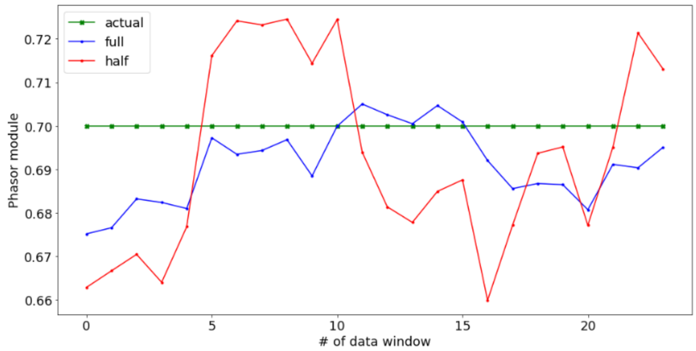
To demonstrate the effect of noise on the algorithm white noise was added to the signal and then the performance of full and half cycle versions were tested:

$$x_{noisy}(t) = 0.7sin(2\pi 60 * t) + 0.05N(0,1)$$

where N(0,1) is sampled from standard normal distribution with mean 0 and standard deviation 1.



The performance of full and half cycle algorithms is presented below:



As one can see, the full-cycle algorithm is more resistant to noise than half-cycle, which is in accordance with the accuracy-speed principle. In addition, the variances of both algorithms were calculated for a study case above:

std(full) = 0. 008567322241713478

std(half) = 0. 021724315265761164

One can see that full-cycle output has lesser variance, which again demonstrates the same principle.

Effect of Fundamental Frequency Change. Leakage

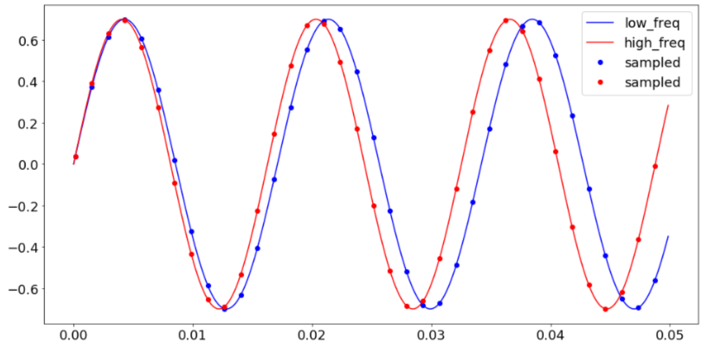
A common phenomenon in DFT analysis is called "leakage". Leakage error is characterized by the representation of false components at frequencies near the actual signal frequency. Leakage is a result of signal sampling in which an integer number of signal cycles are not captured by the data window [18].

Change of FF affects the results of FTA, since the method correlates sines and cosines with predefined frequencies, that are multiples of fundamental frequency. In practical applications the signal's frequency is not always equal to FF. In such case, the ripple in output occurs, which would be of frequency equal to the sum of system frequency and notch filter, therefore, it would be close to 120 Hz [19].

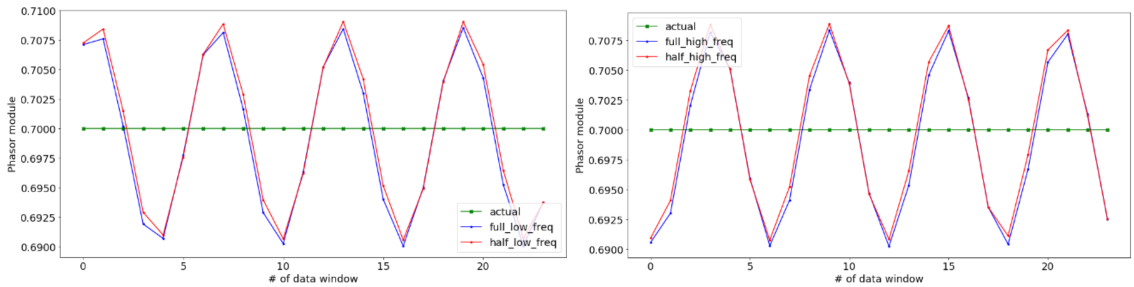
The same tests were performed for signal with frequency lower and higher than nominal. The results are shown below.

$$x_{f\ low}\ (t) = 0.7sin(2\pi[60 - 1.5] * t)$$

$$x_{f\ high}\ (t) = 0.7sin(2\pi[60 + 1.5] * t)$$



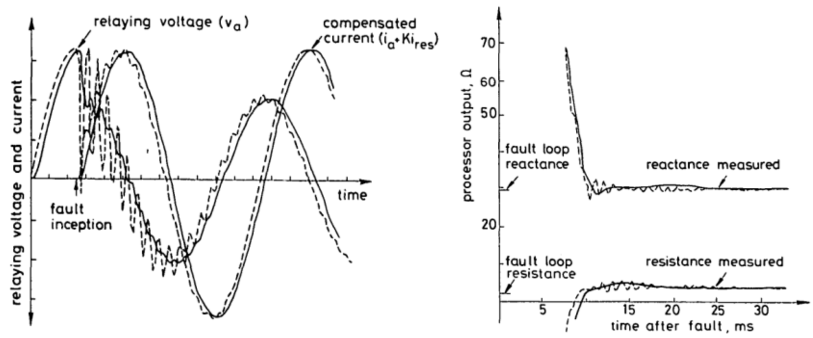
The resulting phasor magnitude estimates:



As can be seen from the plots, both algorithms are susceptible to FF change. The FF change causes the phasor magnitude to swing around true value.

Effect of sampling rate

As was mentioned earlier, the sampling rate should be at least twice the highest frequency present in the analogue signal. Otherwise, aliasing occurs, which may lead to maloperation of relay. Reference [20] gives an example of relay performance at different sampling rates. First signal is obtained by using anti-aliasing filter with cut-off frequency of 400 Hz and then sampling it at the rate of 800 Hz. The second signal is obtained by using anti-aliasing filter with cut-off frequency of 2000 Hz and then sampling it at the rate of 4000 Hz. The studies are performed on 400 kV line with solid a-earth fault applied approximately at the 80% length of the line. The pictures below use dashed line to represent high sampling rate and cut-off frequency and solid line for low sampling rate.



As can be seen, the lower sampling frequency with appropriate filtering outputs much smoother and rapidly converging output. An interested reader can access [15] and see for himself the effect of higher sampling rate on a noisy signal.

Answers to the questions from the project assignment

What the basic performance and implementation characteristics of the Fourier Transform algorithm are? (The underlying theory shall be explained)

The question was covered in first part of the report.

How the Fourier Transform Algorithm is used in distance relaying? (The related relaying function implementation shall be described)

The question was covered in the Signal Model section. The main aspects are summarized below.

FTA is used to extract phasor values of voltage and current from sampled signals. The phasor encodes information about signal's amplitude and phase angle. Using the FF phasors of voltage and current the apparent impedance can be calculated. The obtained apparent impedance is then compared to preset parameters in a relay, putting impedance in one of the zones in mho plane. If the impedance falls into tripping zone, then the trip signal is issued by the relay to the circuit breaker. The circuit breaker opens a primary circuit and fault is being deenergized.

What the benefits and shortcomings of this algorithm when used for relaying are? (The implementation advantages and disadvantages shall be indicated)

The question is covered performance assessment section. The summary is presented below.

Advantages:

- Method is easy to implement.
- Method is capable of extracting FF and HF components.
- Full-cycle implementation rejects DC offset and HF components. Half-cycle rejects even harmonics.
- Half-cycle is fast and needs only samples from half of the cycle.
- Method is resistant to noise.
- Recursive form is available for reduced computation load.

Disadvantages:

- Half-cycle is heavily affected by DC offset and requires external removal of it.
- Method is adversely affected by intraharmonics. Half-cycle is affected by odd harmonics.
- For some application that require ultra-fast operation method may not be suitable.
- Method had higher computational burden when compared to some other methods (for example Walsh methods)
- Method is adversely affected by FF change.
- Method is adversely affected by noise components.

What the literature used is? (A list of references shall be provided)

The literature list is given at the end of the report.

Conclusions (elaborate on major findings regarding design features affecting the accuracy and response time)

The response time of the method depends on the data window selected. The general dilemma of speed-accuracy applies - shorter window has poorer speed performance and is also more affected by higher harmonics, noise and DC offset, the longer window delays the output, but is more robust. Recursive form is less computationally demanding but may accumulate errors.

All in all, the method is fast and accurate. It is widely used in relay applications and is easily implemented. A number of variations of the method is proposed in the literature to enhance the performance and address the weak points.

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