Universitatea “Alexandru Ioan Cuza” din Iaşi
Facultatea de Informatică

LUCRARE DE LICENŢĂ

Multilinear Maps over Ideal Lattices

propusă de

Student: Ciprian Băetu
Coordonator ştiinţific: Prof. Dr. Ferucio Laurenţiu Ţiplea

Sesiunea: iunie - iulie
2017

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Contents

Chapter 1

Introduction

Usually, cryptographic primitives are constructed under the assumption that several problems are intractable, i.e. there exist no polynomial-running time algorithm to solve them. Vercauteren [11] realized an extensive research regarding the intractable problems that are most used in cryptography, such as integer factoring, discrete logarithm, computational Diffie-Hellman, shortest vector problem and many others.

In the last ten years, bilinear maps proved to be very useful in cryptography. Making use of the interesting properties of such maps, cryptographers managed to construct schemes for one-round three-party key exchange [9], identity based encryption [2] and many other applications. After the moment that bilinear maps proved to be undoubtedly useful, researchers have tried to generalize the concept. Thus, multilinear maps were defined and the search for their applications has begun. Boneh and Silverberg [3] showed that symmetric multilinear maps can be used to realize a one-round multi-party key exchange scheme but, after their attempts to construct such maps failed, they drew the conclusion that "such maps might have to either come from outside the realm of algebraic geometry, or occur as 'unnatural' computable maps arising from geometry."

Garg, Gentry and Halevi [6] proposed a construction based on lattices that approximate the multilinear maps in hard-discrete-logarithm groups. Using this candidate, they could construct an application to multipartite Diffie-Hellman key exchange scheme and also the first construction of Attribute-Based Encryption for general circuits [7]. A short period after the aforementioned candidate was proposed, Coron, Lepoint and Tibouchi [5] created a similar construction, based on integers instead of lattices.

However, these construction proved to be susceptible to attacks, and a devastating zeroizing attack for the integer construction is presented thoroughly in [4]. Numerous fixing tentatives of these schemes were designed, but for each of them there was found at least another attack. Therefore currently, new methods of constructing multilinear maps and Graded Encoding Schemes still constitute an open field, very interesting for cryptographers.

1.1 Contribution

This work represents a survey over the bilinear and multilinear maps and their applications. Furthermore, in this paper is presented the concept of Graded Encoding Scheme, a modality of approximating multilinear maps. The core of the paper is represented by the review of the lattice-based construction, designed by Garg, Gentry and Halevi in [6].

1.2 Organization

The paper is divised into 5 sections. First one - introduction bla-bla. In the second section, bla bla bla ... etc etc.

Chapter 2

Multilinear Maps and Graded Encoding Systems

In this chapter, multilinear maps are defined and also, the particular case of bilinear maps is discussed, along with results concerning bilinear maps over groups of composite order. Thereafter, *Graded Encoding Schemes* are defined, as an approximate to multilinear maps.

Observation: Regarding the multilinear maps and Graded Encoding Systems schemes, and also for the lattice-based candidate designed in [6], the paper encompasses one subsection of efficient procedures, and another one of hardness assumptions. The reader should be aware of this detail and realize the analogy and differences of the mentioned schemes.

2.1 Bilinear Maps

As stated before, bilinear maps are a specific case of multilinear maps. They proved to be a highly useful tool in cryptography, with many applications, such as: tripartite protocol [9], identity based encryption [2] and Attribute-based encryption scheme for monotone boolean formulas [8]. In this section, bilinear maps are only defined, while next section presents a relationship between self-bilinear maps and multilinear maps.

Definition 1. (Bilinear Map [1]). *Given the cyclic groups G and G_t (written additively) of the same order p, a (symmetric) map $e : G \times G \to G_t$ is said to be bilinear if the following properties hold:*

1. ***(Bi-linearity)*** $e({g_1}^{x_1}, {g_2}^{x_2}) = e(g_1, g_2)^{x_1 x_2}$, *for any $x_1, x_2 \in \mathbb{Z}_p$ and any $g_1, g_2 \in G$;*
2. ***(Non-degeneracy)*** *There exists an element $g \in G$ such that $e(g, g) \neq 1_{G_t}$;*
3. ***(Efficient computability)*** *There exists a polynomially-bounded algorithm to compute $e(g_1, g_2)$, for any $g_1, g_2 \in G$.*

2.2 Cryptographic Multilinear Maps

Definition 2. (Multilinear Maps [10]). *Let $k \geq 1$ be an integer number and $G_1, G_2, ..., G_k, G_T$ be $k + 1$ cyclic groups (written additively), of same order p. Then, a $k-$multilinear map is a mapping $e : G_1 \times ... \times G_k \to G_T$, with the following properties:*

1. ***(Linearity)*** *For every $g_1 \in G_1, ..., g_k \in G_k$, every $i \in \{1, 2, .., k\}$ and every $\alpha \in \mathbb{Z}_p$, it holds that:*

$$e(g_1, ..., \alpha \cdot g_i, .., g_k) = \alpha \cdot e(g_1, ..., g_k))$$

2. ***(Non-degeneracy)*** *If $g_1 \in G_1, ..., g_k \in G_k$ are generators of their respective groups, then $e(g_1, ..., g_k)$ is a generator of G_T.*

2.2.1 From Bilinear to Multilinear Maps

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Chapter 3

Mathematical Background

Chapter 4

Proposed Encoding Scheme

Chapter 5

Security

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