Assignment 6

CSCE 4323: Formal Languages and Computability
Fall 2018

Solutions to exercises 1 and 2 should be provided as code in a text document following the format for a TM on the web site `http://web.cs.ucdavis.edu/~doty/automata/`. A template file "CSCE4323-F18-HW-template.txt" can be found on the course site on Blackboard. Solutions to the remaining problems should be typed and contained in a PDF, but may include state diagrams or other figures neatly drawn by hand and included in the PDF.

All binary numbers should be represented so that their most significant bits are leftmost and their least significant bits are rightmost.

1. Give the formal definition of a Turing machine with $\Sigma = \{0, 1\}$ which decides the language $\{w \mid w$ contains twice as many 0s as 1s $\}$.

2. Give the formal definition of a Turing machine with $\Sigma = \{0, 1\}$ which decides the language $\{w \mid w$ represents $n \in \mathbb{N}$ in binary, $n > 0$, and $n \mod 4 = 3\}$.

3. A *Turing machine with a doubly infinite tape* has a tape which extends infinitely in both directions. It initially contains blanks everywhere other than the input string, and the head is assumed to begin at the leftmost symbol of the input. Clearly show that the class of languages recognized by this class of Turing machines is identical to the class of languages recognized by regular Turing machines (i.e. those with tapes which are infinite only to the right).

4. A *Turing machine with left reset* is similar to an ordinary Turing machine, but the transition function has the form
$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, \texttt{RESET}\}.$$
If $\delta(q, a) = (r, b, \texttt{RESET})$, when the machine is in state q reading an a, the machine's head jumps to the left-hand end of the tape after it writes b on the tape and enters state r. Note that these machines do not have the usual ability to move the head one symbol left. Clearly show that Turing machines with left reset recognize the class of Turing-recognizable languages (i.e. they recognize the same class of languages as regular Turing machines).

5. Show that the collection of decidable languages is closed under the operation of

   a. complementation

   b. intersection

   c. concatenation

6. Assume that the digital representation of a picture consists of a single number, representing a color value, for each pixel location. Let I be a picture and $\langle I\rangle$ be an encoding (to be specified by you) of I as a string over Σ, for $\Sigma = \{0, 1, ;, \#\}$. Give an implementation-level description of a Turing machine M which takes as input a string $w =$ "$x, y, c\#\#\langle I\rangle$" where x is the x-coordinate of a pixel location, y is the y-coordinate, c is a number representing a color, and $\langle I\rangle$ is an encoding of a picture. M should accept w if the pixel of I at coordinate (x, y) has color c, and should reject otherwise. (Note that the representation of $\langle I\rangle$ given as input must be generic and cannot have any special marking which distinguishes the pixel at location (x, y).)

   Before giving the description of M, clearly specify your encoding of I as $\langle I\rangle$, i.e. how the image is represented as a string over Σ.

   Your description of M should also specify how the numbers for x, y, and c are represented. (Hint: a long representation of numbers may make your implementation easier to define.)