

**MANIPAL ACADEMY OF HIGHER EDUCATION, MANIPAL**  
**MANIPAL INSTITUTE OF TECHNOLOGY**

**End Semester MAKE-UP EXAMINATION January 2025**

**MAT 2136 – PROBABILITY AND STOCHASTIC PROCESS**

**Time: 3 hours**

**Marks: 50**

Q. No.	Description	Marks	COs	BL
1A	An urn contains 3 white, 6 red, and 7 black balls. Five balls are randomly selected from the urn. A person wins Rs. 4 for each white ball and loses Rs. 3 for each red ball. Let $X$ denote the total winnings from this experiment. Find $E(X)$ .	4	3	3
1B	In a gym, 60% of members use the gym in the morning, and 40% do not. Among those who use the gym in the morning, 50% are interested in personal training. Among those who do not use the gym in the morning, 20% are interested in personal training. What is the probability that a randomly selected gym member is interested in personal training?	3	1	3
1C	It is suspected that a patient has one of the 3 diseases $A_1, A_2, A_3$ . Suppose that the population percentages suffering from these illness are in the ratio 2:1:1. The patient is given a test which turns out to be positive in 25 percentage of cases of $A_1$ , 50 percentage cases of $A_2$ and 90 percentage cases of $A_3$ . Given that out of the 3 tests taken by the patient two are positive, and the probability that he has the illness $A_1$ .	3	1	3
2A	Let $X$ be a random variable which follows a Binomial distribution with parameters $n$ & $p$ . Then, find the moment generating function (m.g.f.) of $X$ . Hence, find $E(X)$ and $V(X)$ .	4	2	3
2B	A basketball player has a 60% chance of making a successful free throw each time they attempt a shot. Suppose the player attempts 10 free throws in a game. 1. What is the probability that the player makes exactly 6 successful free throws? 2. What is the probability that the player makes at least 1 successful free throw?	3	2	3
2C	The annual rainfall at a certain locality is known to be normally distributed random variable with mean 29.5 inches and standard deviation 2.5 inches. How many inches of rain (annually) is exceeded about 5 percent of the time.	3	2	3
3A	If $X$ and $Y$ are two independent random variables having standard normal distribution then, find the p.d.f. of $Z = \frac{X}{Y}$	4	3	3
3B	A two-dimensional random variable is uniformly distributed over the region bounded by the curve $y=2x^2$ and the line $y=x$ then find $E(XY)$ .	3	3	3
3C	If $X$ has uniform distribution over $(-\pi/2, \pi/2)$ . Find the p.d.f of $Y = \tan x$	3	3	3
4A	A distribution with an unknown mean $\mu$ has variance 2.25. How large a sample should be taken from the population so that the probability is at least 0.99 that the sample mean will be within 0.3 of the population mean?	4	4	4
4B	Let $(X_1, X_2, \dots, X_n)$ be a random sample of size $n$ from a population $X$ with p.d.f. $f(x; \theta) = \theta x^{\theta-1}$ for	3	4	3



	$0 < x < 1$ and $0 < \theta < \infty$ . Find the maximum likelihood estimate (MLE) for $\theta$ .			
4C	The mean of a sample of size 20 from a population follows normal distribution $N(\mu, 80)$ was found to be 81.2. Find a 90% confidence interval for $\mu$ .	3	5	4
5A	A spinner with 5 equal sectors numbered 1,2,3,4,5 was spun 200 independent times, and the following data were recorded: $\begin{array}{ c c c c c } \hline \text{Number} & 1 & 2 & 3 & 4 & 5 \\ \hline \text{Frequency} & k & 45 & 35 & 60 & 60-k \\ \hline \end{array}$ Using the Chi-Square test, determine for what values of $k$ the null hypothesis ( $H_0$ : the spinner is fair) would be rejected at the 0.05 significance level.	4	5	4
5B	In a small town, three schools — Greenfield (G), Blue Ridge (B), and Red Valley (R) each have alumni associations that encourage students to attend their alma mater. Over time, the probability of a child attending a specific school, given the school their parent attended, is as follows: 70% of the children of Greenfield alumni go to Greenfield, 20% go to Blue Ridge, and 10% go to Red Valley; 50% of the children of Blue Ridge alumni go to Blue Ridge, 30% go to Greenfield, and 20% go to Red Valley; 60% of the children of Red Valley alumni go to Red Valley, 25% go to Blue Ridge, and 15% go to Greenfield. 1. What is the probability that the grandchild of a Greenfield alumnus goes to Greenfield?  What is the steady-state probability that a randomly selected student from this town attends Greenfield, Blue Ridge, or Red Valley over many generations?	3	6	3
5C	Let $\{X_n; n \geq 1\}$ be a Markov chain with state space $S = \{0, 1, 2, 3\}$ and the transition probability matrix (t.p.m.) is $P = \begin{bmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ 0.2 & 0.5 & 0.1 & 0.2 \\ 0.1 & 0.2 & 0.6 & 0.1 \\ 0.3 & 0.3 & 0.2 & 0.2 \end{bmatrix}$ Then, 1. Draw the Stochastic graph of the chain. 2. Find the equivalence classes and check its irreducibility (Justify). Whether the chain is ergodic or not (Justify).	3	6	4