Exam Date & Time: 26-Sep-2024 (10:45 AM - 12:15 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

III Semester B.Tech. Mathematics and Computing
Mid Term Examination

COMPUTATIONAL LINEAR ALGEBRA [MAT 2135]

Marks: 30   **Duration: 90 mins.**

MCQs

Answer all the questions.   Section Duration: 20 mins

1)   Let $T : V \to W$ be a one-one linear transformation. Then $T^{-1}(0)$ is   (0.5)

1) 0	2) 1	3) V	4) W

2)   Let S denote the unit square with vertices (0,0), (1,0), (0,1) and (1,1) and let $T(x, y) = (3x+y, x+3y)$. Then the area of the parallelogram T(S) is   (0.5)

1) 4	2) 6	3) 8	4) 9

3)   Let Q be a 2×2 orthogonal matrix. Which of the following is always true?   (0.5)

1) Q is the identity matrix.	2) The determinant of Q is zero.	3) Q is either a rotation matrix or a reflection matrix.	4) The determinant of Q is 1.

4)   If $T: R^2 \to R^2$ is a linear transformation such that $T(1, 0) = 7$ and $T(0, 1) = 8$, then $T(7, 8) = ...$ (0.5)

1) 49	2) 64	3) 78	4) 113

5)   If Q is an orthogonal matrix, then Q^{-1} is   (0.5)

1) Q	2) Q^T	3) $(Q^T)^{-1}$	4) the identity matrix.

6)   The sum of the eigenvalues of the 2×2 identity matrix is   (0.5)

1) 0	2) 1	3) 2	4) 4

7)   If P is a projection matrix, then which of the following is always true?   (0.5)

1) $P^2 = P$	2) $P^2 = P^T$	3) $P^{-1} = P^T$	4) $(P^T)^{-1} = P$

8)   Let $T:V \to W$ be a linear transformation. Which of the following is NOT true?   (0.5)

1) If T is one-one and onto, then dim (V) =	2) If dim (V) = dim (W), then T is one-one	3) If dim (V) = dim (W) and T is one-one,	4) If dim (V) = dim (W) and T is onto, then T

	dim (W).			and onto.			then T is onto.			is one-one.	

9)   If A is a 3 × 4 matrix with right inverse as B, then the order of the matrix AB is

(0.5)

1)	3×3	2)	3×4	3)	4×3	4)	4×4

10)   Consider a system Ax = b such that b is in the column space of A. Then

1)	the system has an exact solution.		2)	the system has no solution.		3)	the system is inconsistent.		4)	the system has a solution with error greater than zero.		(0.5)

DES

Answer all the questions.

11)   Let T : V → W be a linear transformation.

(i) Show that Ker T is a subspace of V.

(ii) Show that T is one-one if and only if Ker T = {0}.

(4)

12)   If T(x, y) = (7x + 2y, 2x + 7y), then find the matrix of the linear transformation T and hence compute T^{25} (x, y).

(3)

13)   Fit a curve y (x) = ab^x given the data

X	0	1	2
Y	3	6	12

(3)

Hence find y(3).

14)   Orthonormalize { 1, x, x^2 } using the Gram-Schmidt process by taking the inner product as

$$\langle f(x), g(x) \rangle = \int_{-1}^{1} f(x)g(x)dx$$

(3)

15)   Find the left inverse of the matrix $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}$

(3)

Validate your answer.

16)   Find the least squares solution for the inconsistent system Ax = b where   (3)

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \quad \text{and } b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$$

17)   Verify the Rank-Nullity theorem for $T(x, y) = (x + y, x - y)$.

(2)

18)   Test whether the following matrices are orthogonal matrices.

(i) $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$

(2)

(ii) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

19)   Let P be a projection matrix corresponding to the orthogonal projection on the subspace W of V. Find the eigenvalues and the corresponding eigenvectors of P.

(2)

-----End-----