Semester III BTech (Mathematics and Computing) Makeup Examination
Discrete Mathematics – MAT 2138

Time: 3 Hours | 07/01/2025 | Max. Marks: 50

1A. Prove that ∪ distributes over ∩ and vice-versa: (3)

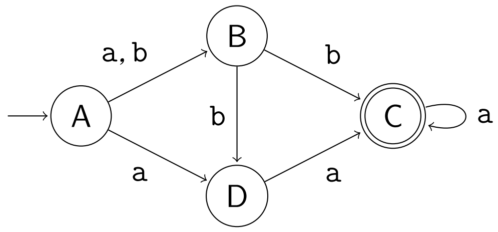
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

1B. Show that a every asymmetric relation is reflexive, and every transitive, irreflexive relation is asymmetric. (3)

1C. Prove that the set of rational numbers is countable. (4)

2A. Construct a DFA for the language of all binary strings that contain an even number of 0s and an even number of 1s. (5)

2B. Convert the NFA given below to a DFA using the subset construction: (5)

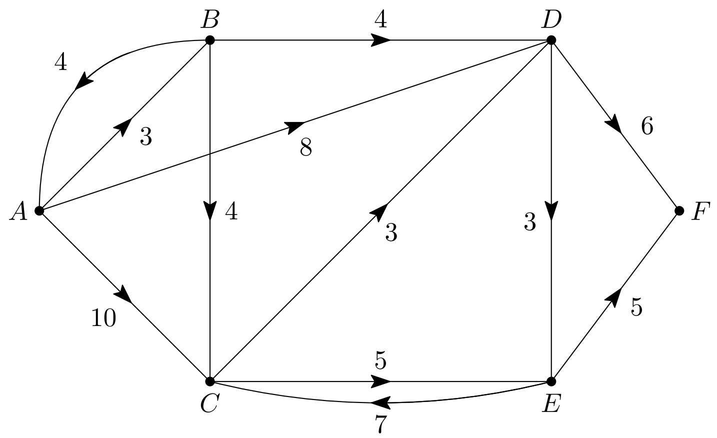


3A. Show that the sum of the degrees of all the vertices of a graph is twice its number of edges. (3)

3B. Prove that a graph T is a tree if and only if every two of its vertices have a unique path joining them. (3)

3C. Show that every self-complementary graph has order $4k$ or $4k+1$ for some integer k. Give an example of a self-complementary graph of order 5. (4)

4A. In the following network, find the shortest distances from A to all the other vertices using Dijkstra's algorithm. (5)



4B. Show that if A is the adjacency matrix of a graph G with vertices $v_1, \ldots, v_n$, then the (i,j)-entry of A^2 is the number of walks of length 2 from v_i to v_j. Hence show that $\text{tr}(A^2) = 2|E(G)|$. (5)

5A. Using generating functions, find the number of ways of distributing (5)
 (i) 30 identical objects into 3 distinct boxes such that each box is non-empty.
 (ii) 30 distinct objects into 3 distinct boxes such that each box is non-empty.

5B. Using generating functions, solve the following recurrence relation: (5)

$$a_n = 2a_{n-1} + 3a_{n-2}, \quad n \geq 2$$
$$a_0 = 0, \; a_1 = 1.$$