Q. Fit $z = a + bx + cy$; given

x	y	z
0	0	7
0	1	17
1	0	5
1	1	15.

Find a, b, c.

normal equations : $a + bx + cy = z$

$$a\sum 1 + b\sum x_i + c\sum y_i = \sum z_i$$

$$a\sum x_i + b\sum x_i^2 + c\sum x_i y_i = \sum x_i z_i$$

$$a\sum y_i + b\sum x_i y_i + c\sum y_i^2 = \sum y_i z_i$$

$$\begin{bmatrix} \sum 1 & \sum x_i & \sum y_i \\ \sum x_i & \sum x_i^2 & \sum x_i y_i \\ \sum y_i & \sum x_i y_i & \sum y_i^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum z_i \\ \sum x_i z_i \\ \sum y_i z_i \end{bmatrix}$$

$\sum 1 = n = 4$ data points.

x	y	z	x^2	y^2	xz	yz	xy
0	0	7	0	0	0	0	0
0	1	17	0	0	0	17	0
1	0	5	1	1	5	0	0
1	1	15	1	1	15	15	1
$\Sigma x = 2$	$\Sigma y = 2$	$\Sigma z = 44$	$\Sigma x^2 = 2$	$\Sigma y^2 = 2$	$\Sigma xz = 20$	$\Sigma yz = 32$	$\Sigma xy = 1$

$$\Rightarrow \begin{bmatrix} 4 & 2 & 2 \\ 2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 44 \\ 20 \\ 32 \end{bmatrix}.$$

$$\Rightarrow a = 7 \; ;$$

$$b = -2 \; ; \quad c = 10 .$$

Q. Fit $y = a.b^{x}$. $\longrightarrow$ exponential

$$\underbrace{\log_e y}_{Y} = \underbrace{\log_e a}_{A} + x \underbrace{\log_e b}_{B} .$$

$$\Rightarrow Y = A + Bx .$$

<u>To find</u>: $a = e^{A}$; $b = e^{B}$.

Eg : Fit $y = a \cdot b^x$; given

x	y
0	3
1	21
2	147
3	1029.

Find a, b.

Soln :

$$y = ab^x$$

$$\underbrace{\log_e y}_{Y} = \underbrace{\log_e a}_{A} + x\underbrace{\log_e b}_{B}$$

So, Fit $Y = A + Bx$.

normal equations: $y = A + Bx$

$$\sum y_i = A\sum 1 + B\sum x_i$$

$$\sum x_i y_i = A\sum x_i + B\sum x_i^2$$

x	y	$Y = \log_e y$	xY	x^2
0	3	1.0986	0	0
1	21	3.0445	3.0445	1
2	147	4.9904	9.9808	4
3	1029	6.9366	20.8089	9
$\sum x = 6$	$\sum y = 1200$	$\sum Y = 16.0698$	$\sum xY = 33.8342$	$\sum x^2 = 14$

$$\sum 1 = n = 4.$$

$$16.0698 = A(4) + B(6) \quad \text{--- (1)}$$

$$33.8342 = A(6) + B(14) \quad \text{--- (2)}$$

Solving (1) & (2),

$$A = 1.0986$$

$$B = 1.9459.$$

$$\therefore \quad a = e^{A} = e^{1.0986} = 2.9999$$

$$b = e^{B} = e^{1.9459} = 6.9999.$$

$$\therefore \quad y = ab^x$$

$$= (2.9999)(6.9999)^x .$$

Q: Find $\pi(10^5)$ given

x	$y = \pi(x)$
10	4
10^2	25
10^3	168
10^4	1229

prime counting function.

$$f(x) = \frac{x}{\log_e x}$$

$$f(10) = \frac{10}{\log_e 10} = 0.4342 \times 10 = 4.342$$

$$f(10^2) = f(100) = \frac{100}{\log_e 100} = 21.714$$

$$f(10^3) = 144.7648\ldots$$

$$f(10^4) = 1085.73\ldots$$

$$f(10^5) = 8685.88$$ (predicted value by the formula $\frac{x}{\log_e x}$).

Q. Fit $y = a + b\dfrac{x}{\log_e x}$; given

x	$y(x)$
10	4
10^2	25
10^3	168
10^4	1229

Hence find $y(10^5)$.

Soln:

$$y = a + b\underbrace{\frac{x}{\log_e x}}_{X}$$

$$y = a + bX \; ; \text{ where } \quad X = \frac{x}{\log_e x}$$

normal equations :

$$\Sigma y = a\Sigma 1 + b\Sigma X$$

$$\Sigma Xy = a\Sigma X + b\Sigma X^2$$

x	y	$X = x/\log_e x$	X^2	Xy
10	4	4.3429		
10^2	25	21.7147		
10^3	168	144.7648		
10^4	1229	1085.7362		
	$\Sigma y = 1426$	$\Sigma X = 1256.5586$	$\Sigma X^2 = 1250270.212$	$\Sigma Xy = 1359250.516$

$$\begin{bmatrix} \Sigma 1 & \Sigma x \\ \Sigma x & \Sigma x^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \Sigma y \\ \Sigma xy \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 1256.5586 \\ 1256.5586 & 1200270.212 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1426 \\ 1359250.516 \end{bmatrix}$$

$$\Rightarrow \quad a(4) + b(1256.5586) = 1426 \quad \text{---} (1)$$

$$a(1256.5586) + b(1200270.212) = 1359250.516 \quad \text{---} (2)$$

Solving (1) & (2),

$$a = 1.1196$$

$$b = 1.1312.$$

$$\therefore y(x) = a + bx$$

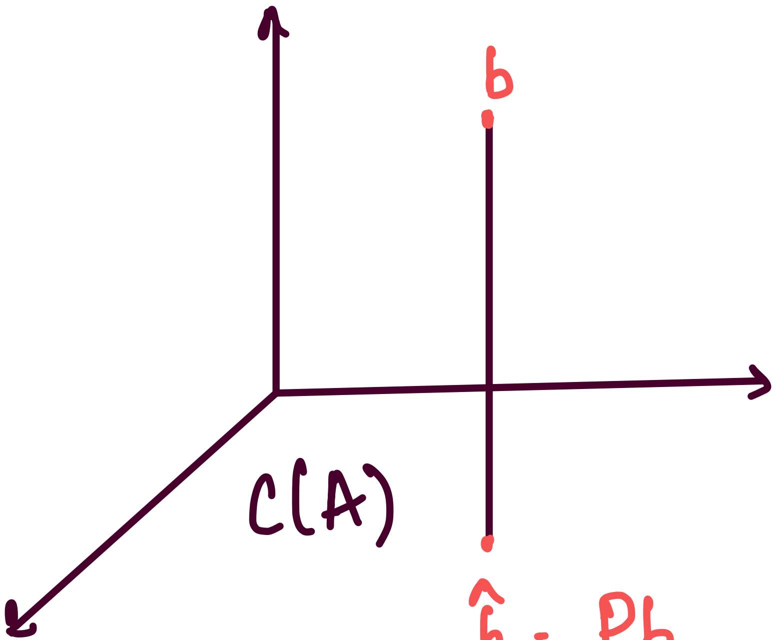
$$= 1.1196 + (1.1312) \frac{x}{\log_e x}$$

Now, $$y(10^5) = 1.1196 + (1.1312) \frac{10^5}{\log_e 10^5}$$

$$= 9826.6$$ (predicted value by regression)

Actual correct value of $\pi(10^5) = 9592$.
(exact answer)

Q: Given a matrix A;



$\hat{b} = Pb$; P?

$\hookrightarrow$ projection matrix
(or Hat matrix)