Computational Linear Algebra

22/7/25

Subject code: MAT 2135

References :

1) S. Kumaresen, Linear Algebra Geometric Approach.

2) R. Rao and P. Bhimshankaran, Linear Algebra.

3). S. H. Friedberg, A. J. Insel, L. E. Spence. Linear Algebra.

4) D. C. Lay ; Linear Algebra & its applications

5) Introduction to Linear Algebra ; Gilbert Strang.

$H \rightarrow 1$

$T \rightarrow 0$

Toss a coin 8 times

$n = 8$: HTHHTTHTH

10100101

$P(H/T) = \frac{1}{2}$

1st : $\frac{n}{2}$; 2nd = $\frac{n}{2^2}$, ...

toss

Given n people.

Q. Find no. of rounds ?

Let x = no. of rounds.

$$\frac{n}{2^x} < 1$$

$$\Rightarrow n < 2^x$$

$$\Rightarrow \log_2 n < x$$

Eg: $n = 2^6$

$k = 6$ rounds.

Q. Given a matrix A, $A^{16} = ?$

$A^{16} = A * A * \cdots * A$

↳ 15 multiplications.

$A^{16} = A^8 * A^8$

↳ 7 multiplications involved to compute A^8.

In total, A^{16} can be computed using $7+1 = 8$ multiplications.

Q. Reducing no. of multiplications ?

$$A^8 = (\underbrace{A * A * A * A}_{B_1}) * B_1$$

$\hookrightarrow 3+1$ multiplications

$= 4$

In total, $4+1=5$ multiplications.

15
↓
8
↓
5

$B_1 = A^4 = \underbrace{(A*A)}_{B_2} * B_2$

$\hookrightarrow 1+1 = 2$ multiplications

To compute A^{16}.

Step 1 : Find $A^2 = A * A =: B$

↳ 1 multiplication (1 * used)

Step 2 : Find $B^2 = B * B = C$

(1 * used)

Step 3 : Find $C^2 = C * C = D$

(1 * used)

Step 4: Find $D^2 = D * D$ (1 * used)

precisely, 4 multiplications used.

Also, $4 = \log_2 16$

So, A^n can be computed in $\log_2 n$ steps.

↓

within 2 multiplications using linear algebra.

Kurt Baryon : Billion Dollar Eigen vectors

3 Blue 1 Brown : channel explaining linear algebra

Suppose $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

vector Y

vector X

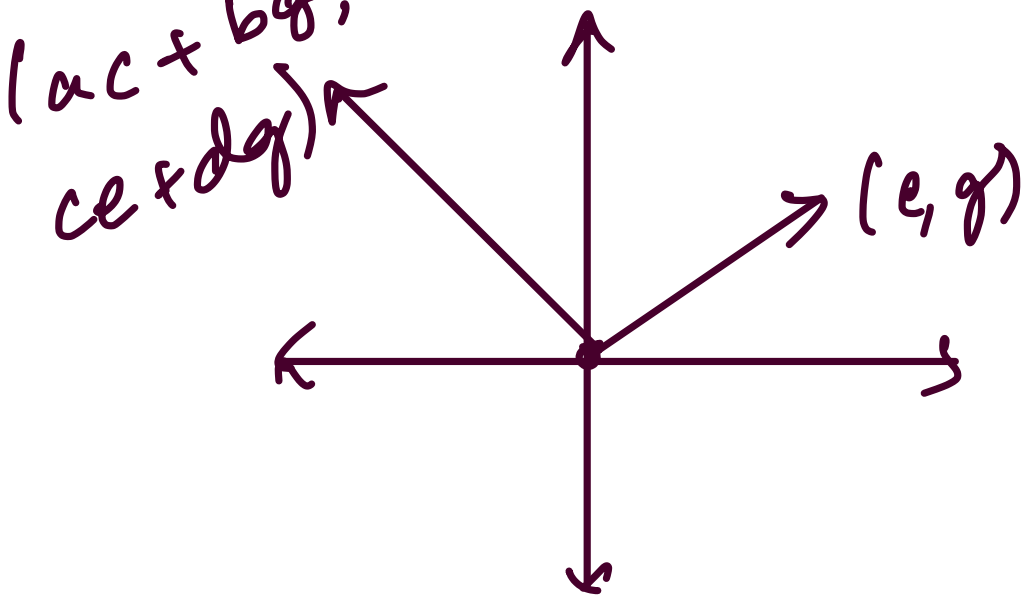
$$= \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

transformed vector.

$$AX = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e \\ g \end{bmatrix}$$

$$= \begin{bmatrix} ac + bg \\ ce + dg \end{bmatrix}$$

; rotation | scaling of $X = \begin{bmatrix} e \\ g \end{bmatrix}$



$$AX = \lambda X \quad ; X \neq 0$$

↳ only scaling of X (No rotation)

eigen → in German, stands for 'own'.

λ : eigen value

X : eigen vector of A corresponding to λ.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix} = A[x, y]$$

$$= [Ax, Ay]$$

$$= [\lambda_1 x, \lambda_2 y]$$

$$= \begin{bmatrix} \lambda_1 e & \lambda_2 f \\ \lambda_1 g & \lambda_2 h \end{bmatrix}$$

↳ Decompose this matrix as product of 2 matrices

$$A[X, Y] = \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$\uparrow$ P

$\uparrow$ Diagonal matrix D

$$A = PDP^{-1}$$

$$A^2 = A \cdot A$$

$$= (PDP^{-1})(PDP^{-1})$$

$$= PD^2P^{-1}$$, and so on.

$$A^n = P * D^n * P^{-1}.$$

↳ involves only 2 multiplications.