$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$R_2 \to R_2 - 3R_1 \; ; \sim \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$$

$$R_2 \to R_2/-2 \; ; \sim \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$R_1 \to R_1 - 2R_2 \; ; \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$R_2 \to R_2 - 3R_1$ ; $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$E_1 = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

$R_2 \to R_2/-2$ ; $E_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1/2 \end{bmatrix}$

$R_1 \to R_1 - 2R_2$ ; $E_3 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

$$E_3E_2E_1A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1/2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 0 & -1/2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$E_3E_2E_1A = I$$

$$\Rightarrow \quad A = E_1^{-1} E_2^{-1} E_3^{-1} I.$$

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$B^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ c & 1 \end{bmatrix} \qquad B^{-1} = \begin{bmatrix} 1 & 0 \\ -c & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}, \quad B^{-1} = \frac{1}{ad}\begin{bmatrix} d & 0 \\ 0 & a \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & 1/d \end{bmatrix}$$

$$B = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} 1 & -b \\ 0 & 1 \end{bmatrix}$$

$$A = E_1^{-1} E_2^{-1} E_3^{-1} I \longrightarrow \text{Elementary decomposition of matrices.}$$

$$= \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\downarrow$ $(x, 3x+y)$

Shear about y-axis by an amount 3.

$\downarrow$ $(x, -2y)$

Reflection about x-axis and scaling by an amount $k=2$

$\downarrow$ $(x+2y, y)$

$\downarrow$

Shear about x-axis by an amount 2

Q. Find elementary decomposition of A,

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

Give a geometric interpretation.

Soln:

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

$R_1 \to R_1/4$; $\begin{bmatrix} 1 & 3/4 \\ 2 & 1 \end{bmatrix}$, $E_1 = \begin{bmatrix} 1/4 & 0 \\ 0 & 1 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$; $\begin{bmatrix} 1 & 3/4 \\ 0 & -1/2 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

$R_2 \to -2R_2$, $\begin{bmatrix} 1 & 3/4 \\ 0 & 1 \end{bmatrix}$, $E_3 = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$

$R_1 \to R_1 - 3/4 R_2$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$, $E_4 = \begin{bmatrix} 1 & -3/4 \\ 0 & 1 \end{bmatrix}$

$$E_4 E_3 E_2 E_1 A = I.$$

$$A = E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1/2 \end{bmatrix} \begin{bmatrix} 1 & 3/4 \\ 0 & 1 \end{bmatrix}$$

Reflection about Y. axis by an amount 4.

Shear about Y. axis by amt 2.

Reflection about X-axis and scaling about Y-axis by 1/2

Shear about X. axis by a factor of 3/4.

let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$R_2 \to R_2 - 3R_1$ ; $\sim \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} \longrightarrow U$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , $E_1 = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

$R_2 \to R_2 / -2$ ; $\sim$ ; $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$E_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1/2 \end{bmatrix}$

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}}_{\text{lower triangular matrix with diagonal no's as } 1} \underbrace{\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}}_{U} = A$$

lower triangular matrix with diagonal no's as $\underline{1}$

U – upper triangular matrix

# LU-Decomposition.

Q. Solve $x + 2y = -1$ by LU-Decomposition
$3x + 4y = -1$

**Soln:**

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$A\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$LU\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$L\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$ where $$\begin{bmatrix} a \\ b \end{bmatrix} = U\begin{bmatrix} x \\ y \end{bmatrix}$$ — (I)

$$L\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$a = -1$$

$$3a + b = -1$$

$$\Rightarrow b = 2$$

put in I

$$\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow \quad x + 2y = -1$$

$$-2y = 2$$

$$\Rightarrow \quad \boxed{y = -1}$$

and $\boxed{x = 1}$

Therefore $\underline{\underline{x = 1}}$, $\underline{\underline{y = -1}}$

Q. Find LU decomposition of

$$A = \begin{bmatrix} \boxed{2} & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 0 & 7 & -3 & 1 \end{bmatrix}_{4\times 5}$$

Soln:- $R_2 \to R_2 + 2R_1$; $R_4 \to R_4 + 3R_1$

$R_3 \to R_3 - R_1$;

$$\sim \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ 0 & \boxed{3} & 1 & 2 & -3 \\ 0 & -9 & -3 & -4 & 10 \\ 0 & 12 & 4 & 12 & -5 \end{bmatrix}$$

$R_3 \to R_3 + 3R_2$ ; $R_4 \to R_4 - 4R_2$ ;

$$\sim \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ 0 & 3 & 1 & 2 & -3 \\ 0 & 0 & 0 & \boxed{2} & 1 \\ 0 & 0 & 0 & 4 & 7 \end{bmatrix}$$

$R_4 \to R_4 - 2R_3$

$$\sim \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ 0 & 3 & 1 & 2 & -3 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & \boxed{5} \end{bmatrix} = U$$

$A = LU$

$$= \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -3 & 1 & 0 \\ -3 & 4 & 2 & 1 \end{bmatrix}}_{L}{}_{4\times 4} \underbrace{\begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ 0 & 3 & 1 & 2 & -3 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}_{4\times 5}}_{U}$$

$$= \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 0 & 7 & -3 & 1 \end{bmatrix}_{4 \times 5}$$