MANIPAL ACADEMY OF HIGHER EDUCATION, MANIPAL
MANIPAL INSTITUTE OF TECHNOLOGY
End Semester EXAMINATION NOVEMBER 2024
MAT 2136 – PROBABILITY AND STOCHASTIC PROCESS

Time: 3 hours **Marks: 50**

Q. No.	Description	Marks	COs	BL
1A	Four balls are randomly selected from an urn containing 2 white, 4 red and 5 black balls. The person who selects the ball wins Rs 5 for each white ball and lose 5 Rs for each red ball. Let X be the random variable denotes total winning from the experiment. Find E(X).	4	3	3
1B	In a bank, 70% of customers have a fixed deposit account, and 30% do not. The bank offers a special investment plan, and 20% of the customers with a fixed deposit account are interested in this investment plan, while 10% of the customers without a fixed deposit account are interested in it. What is the probability that a randomly selected customer from the bank is interested in the investment plan?	3	1	3
1C	Two percentage of the population have a certain blood disease in serious form, 10 percentage have it in a mild form and 88 percentage do not have it at all. A new blood test is developed. The probability that test is positive is 0.9 if the subject has the serious form, 0.6 if the subject has the mild form and 0.1 if the subject does not have the disease. A subject has tested positive. What is the probability that the subject has the serious form of the disease?	3	1	3
2A	Let X be a random variable which follows a Poisson distribution with parameter m. Then, find the moment generating function (m,g,f,) of X. Hence, find E(X) and V(X).	4	2	3
2B	A cricket player has a 40% chance of hitting a boundary (4 runs) each time they face a ball. Suppose the player faces 10 balls in an over. 1. What is the probability that the player hits exactly 4 boundaries in this over? 2. What is the probability that the player hits at least 1 boundary in this over?	3	2	3
2C	The income of a group of 100000 persons was found to be normally distributed with mean Rs.18000 and S.D Rs. 600. How many had incomes between Rs.18500 and Rs 21500. Also, find the lowest income among the richest 100?	3	2	3
3A	Let (X, Y) is a two-dimensional random variable with the joint p.d.f. $f(x,y) = \begin{cases} x+y & ; \ 0 \le x,y \le 1 \\ 0 & ; \ elsewhere \end{cases}$. Find the p.d.f. of Z=XY.	4	3	3

3B	If the joint p.d.f. of a continuous two dimensional random variable (X,Y) is $f(x,y) = \begin{cases} 8xy & 0 < x < y < 1 \\ 0 & elsewhere \end{cases}$ then, find the marginal p.d.f. of X and Y.	3	3	3
3C	If X is uniformly distributed in (0, 1) then find the p.d.f. of $Y = \frac{1}{2X+1}$.	3	3	3
4A	Let $\bar{X}$ be the mean of a random sample of size 16 from a population X with a p.d.f. $f(x) = \begin{cases} \frac{3}{8}x^2 & ; 0 < x < 2 \\ 0 & ; otherwise \end{cases}$. Using central limit theorem, find $P\left(\frac{8}{5} < \bar{X} < \frac{9}{5}\right)$?	4	4	4
4B	Let $(X_1, X_2, \ldots, X_n)$ be a random sample of size n from a population X with p.d.f. $f(x;\theta) = \frac{\theta^x e^{-\theta}}{x!}$ for $0 < x < 1$ and $0 < \theta < \infty$. Find the maximum likelihood estimate (MLE) for θ.	3	4	3
4C	Find the least sample size required, if the length of the 99% confidence interval for the mean of a normal population with S.D. 10 should be less than 12.	3	5	4
5A	A die was thrown 120 independent times, and the following data resulted Spots up: 1 | 2 | 3 | 4 | 5 | 6 Frequency: b | 20 | 20 | 20 | 20 | $40 - b$ If we use Chi-Square test, for what values of b would the die is unbiased be rejected at 0.025 significance level.	4	5	4
5B	In the Dark Ages, Harvard (H), Dartmouth (D), and Yale (Y) admitted only male students. Assume that, at that time, 80 percent of the sons of Harvard men went to Harvard and the rest went to Yale, 40 percent of the sons of Yale men went to Yale, and the rest split evenly between Harvard and Dartmouth; and of the sons of Dartmouth men, 70 percent went to Dartmouth, 20 percent to Harvard, and 10 percent to Yale. (i) Find the probability that the grandson of a man from Harvard went to Harvard. (ii) Modify the above by assuming that the son of a Harvard man always went to Harvard. Again, find the probability that the grandson of a man from Harvard went to Harvard. (iii) Draw the Stochastic graph in both scenarios.	3	6	3
5C	Let $\{X_n ; n \geq 1\}$ be a Markov chain with state space $S = \{0,1,2\}$ and the transition probability matrix (t.p.m.) is $P = \begin{pmatrix} 0.2 & 0.5 & 0.3 \\ 0.4 & 0.3 & 0.3 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$. Then, whether the chain is 1. irreducible or not (Justify). 2. ergodic or not (Justify).	3	6	4