Formulae
MAT 2135 - Computational Linear Algebra

Matrix of rotation by an angle of θ radians in the anticlockwise direction about the origin in $\mathbb{R}^2$:

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Matrix of reflection about the line making an angle of θ radians with the positive x-axis in $\mathbb{R}^2$:

$$\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$$

Matrix of scaling by k units in $\mathbb{R}^2$:

$$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

Matrix of shear parallel to x-axis with a shear factor k in $\mathbb{R}^2$:

$$\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$$

Matrix of shear parallel to y-axis with a shear factor k in $\mathbb{R}^2$:

$$\begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$$

Matrix of rotation by an angle of θ radians in the anticlockwise direction about x-axis in $\mathbb{R}^3$:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Matrix of rotation by an angle of θ radians in the anticlockwise direction about y-axis in $\mathbb{R}^3$:

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Matrix of rotation by an angle of θ radians in the anticlockwise direction about z-axis in $\mathbb{R}^3$:

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Least squares solution to the linear system of equations $Ax = b$:

$$\hat{x} = (A^T A)^{-1} A^T b.$$

Matrix of projection onto the column space of a matrix A:

$$P = A(A^T A)^{-1} A^T.$$

Gram-Schmidt Orthogonalization:

Let $\{w_1, w_2, \ldots, w_n\}$ be a linearly independent set of vectors. The vectors given by Gram-Schmidt orthogonalization are the following:

$$
\begin{aligned}
v_1 &= w_1 \\
v_2 &= w_2 - \frac{\langle w_2, v_1 \rangle}{\langle v_1, v_1 \rangle} v_1 \\
v_3 &= w_3 - \frac{\langle w_3, v_1 \rangle}{\langle v_1, v_1 \rangle} v_1 - \frac{\langle w_3, v_2 \rangle}{\langle v_2, v_2 \rangle} v_2 \\
v_4 &= w_4 - \frac{\langle w_4, v_1 \rangle}{\langle v_1, v_1 \rangle} v_1 - \frac{\langle w_4, v_2 \rangle}{\langle v_2, v_2 \rangle} v_2 - \frac{\langle w_4, v_3 \rangle}{\langle v_3, v_3 \rangle} v_3 \\
&\vdots \\
v_n &= w_n - \sum_{i=1}^{n-1} \frac{\langle w_n, v_i \rangle}{\langle v_i, v_i \rangle} v_i
\end{aligned}
$$

The set $\{v_1, v_2, \ldots, v_n\}$ is a set of orthogonal vectors.

Let $u_1 = \frac{v_1}{\|v_1\|}$, $u_2 = \frac{v_2}{\|v_2\|}$, $\ldots$, $u_n = \frac{v_n}{\|v_n\|}$.

Then $\{u_1, u_2, \ldots, u_n\}$ is an orthonormal set.