

A farmer purchased 100 head of livestock for a total cost of \$4000. Prices were as follow:
calves, \$120 each; lambs, \$50 each; piglets, \$25 each. If the farmer obtained at least one animal of each type, how many of each did he buy?

$$x + y + z = 100, \quad x, y, z \geq 1$$

$$120x + 50y + 25z = 4000$$

$$24x + 10y + 5z = 800$$

$$z = 100 - x - y$$

$$24x + 10y + 5(100 - x - y) = 800$$

$$19x + 5y = 300$$

$$19x(-1) + 5 \times 4 = 1 \quad \left(\because \begin{pmatrix} 19 & 5 \\ 1 & 1 \end{pmatrix} = 1 \right)$$

$$19x \underset{x_0}{(-300)} + 5 \underset{z_0}{\times (1200)} = 300$$

$$x = -300 + 5t$$

$$y = 1200 - 19t$$

$$x \geq 1 \Rightarrow -300 + 5x \geq 1$$

$$\Rightarrow 5x \geq 301$$

$$x \geq \frac{301}{5} = 60.2$$

$$1200 - 19x \geq 1 \Rightarrow$$

$$19x \leq 1199$$

$$\Rightarrow x \leq 63.10$$

$$\Rightarrow x = 61, 62, 63$$

$$\Rightarrow x = 5, 10, 15$$

$$y = 41, 22, 3$$

$$z = 54, 68, 82$$

When Mr. Smith cashed a check at his bank, the teller mistook the number of cents for the number of dollars and vice versa. Unaware of this, Mr. Smith spent 68 cents and then noticed to his surprise that he had twice the amount of the original check. Determine the smallest value for which the check could have been written. [Hint: If x denotes the number of dollars and y the number of cents in the check, then

$$100y + x - 68 = 2(100x + y)$$