MAT 2137 - Elementary Number Theory

Answer ALL questions.

1. If *a* divides *c* and *b* divides *c*, with *g.c.d.(a, b) = 1*, then prove that *ab* divides *c.* **(2 Marks)**
2. Find inverse of 143 mod 294. **(2 Marks)**
3. Solve $27x \equiv 72\ (mod\ 900)$. **(2 Marks)**
4. Given integers a and b, not both of which are zero, there exist integers x and y such that gcd(a, b) = ax + by **(3 Marks)**
5. Prove that 7 divides $5^{2n} + 3.2^{5n-2}$ **(3 Marks)**
6. Prove that Euler phi function is multiplicative. **(3 Marks)**
7. Solve

   $11x + 5y \equiv 7\ (mod\ 20)$.

   $6x + 3y \equiv 8\ (mod\ 20)$. **(3 Marks)**
8. Solve

   $x \equiv 5\ (mod\ 6)$.

   $x \equiv 4\ (mod\ 11)$.

   $x \equiv 3\ (mod\ 17)$. **(3 Marks)**
9. Express the prime 3877 which divides $15^6 + 1$, as a sum of two squares. **(4 Marks)**

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