

Mid Semester Model Question Paper

Sub: Computational Linear Algebra (MAT 2135)

BTech Mathematics & Computing

Type: MCQs

Q1. If $T(x, y) = (3x + y, x + 2y)$, then $T^{-1}(5, 10)$ is (0.5)

1. $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$
2. $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$
3. $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$
4. $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Q2. The dimension of the vector space of all 2×3 matrices over real numbers is (0.5)

1. 2.
2. 3.
3. 5.
4. 6.

Q3. The eigenvalues of the matrix of $T(x, y) = (x, kx + y)$ are (0.5)

1. $0, k$
2. $1, k$
3. $1, 1$
4. k, k

Q4. Let $T: V \rightarrow W$ be a linear transformation and suppose $\dim(V) = 8$ and $\dim(W) = 6$. If $\dim(\text{Image}(T)) = 6$, then (0.5)

1. T is one-one but not onto.
2. T is onto but not one-one.
3. T one-one and onto.
4. T neither one-one nor onto.

Q5. Which of the following matrices is not orthogonal?

(0.5)

1. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
2. $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$
3. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
4. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Q6. Suppose you are given data of the following points: $\{(1,0), (0, -1), (2, 1), (1, -1)\}$. The line of best fit which makes minimum least squares error with the data is

(0.5)

- (a) $y = 1.25 + 1.5x$
- (b) $y = -1.5 + 0.25x$
- (c) $y = -1.25 + x$
- (d) $y = 0.25 + x$

Q7. If A is a 5×6 matrix with left inverse as B , then the order of the matrix B is

(0.5)

1. 5×5
2. 5×6
3. 6×5
4. 6×6

Q.8 If $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}$, then the projection of $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ in the column space of A is

(0.5)

- (a) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$
- (b) $\begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$
- (c) $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$
- (d) $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Q.9 The least squares solution for the inconsistent system $Ax = b$ where $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$ is (0.5)

- (a) $X = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
- (b) $X = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
- (c) $X = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$
- (d) $X = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

Q10 Which of the following matrices **cannot** be a projection matrix? (0.5)

- (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- (b) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
- (c) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
- (d) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Type: Descriptive

Q11. Let W be a subspace of a vector space V . Show that W^\perp is a subspace of V . Then prove that $V = W \oplus W^\perp$. (4)

Q12. If $T(x, y) = (x + 3y, 3x + y)$, find $T^{100}(x, y)$. (3)

Q13. Fit a curve $y = a + b \frac{x}{\log_{10} x}$ given the data

x	10	10^2	10^3	10^4
y	4	25	168	1229

Hence find $y(10^5)$. (3)

Q14. Find the projection matrix corresponding to $\begin{bmatrix} -3 & -1 \\ 0 & 0 \\ 4 & 7 \end{bmatrix}$. Validate your answer using a different method. (3)

Q15. Find the least squares solution for the following system of inconsistent equations.

$$x = 0$$

$$y = 0$$

$$x + y = 1.$$

Also quantify the error involved in your solution.

(3)

Q16. Orthonormalize $\{1, x, x^2, x^3\}$ using the Gram-Schmidt process by taking the inner product as

$$\langle f(x), g(x) \rangle = \int_{-\pi}^{\pi} f(x)g(x)dx.$$

(3)

Q17. If $T(x, y) = (x + y, 2x + 3y)$, then

- (i) show that T is a linear transformation.
- (ii) find $T^{-1}(x, y)$.

(2)

Q18. Let $T: V \rightarrow W$ be a linear transformation. Show that T is onto if and only if $\text{Im}(T) = W$.

(2)

Q18. Verify the Rank-Nullity theorem for $T(x, y, z) = (x - y, x + y, z)$.

(2)