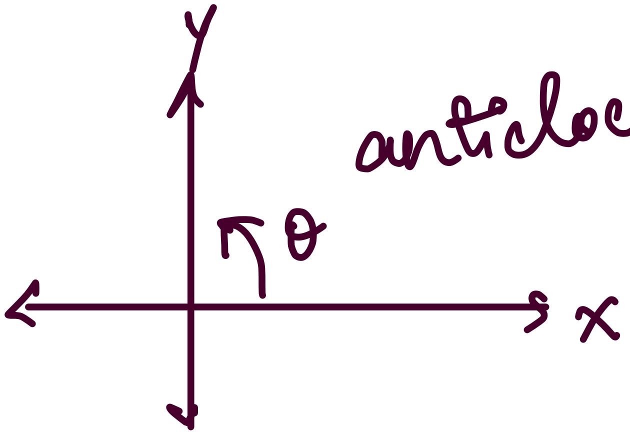
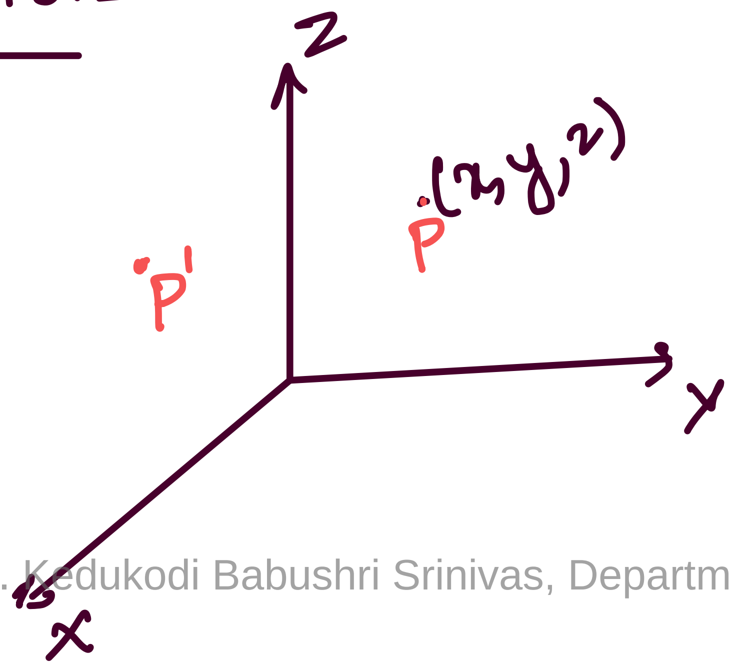
2 D Rotations.



anticlock wise

$$\longleftrightarrow \quad \begin{array}{cc} & \begin{array}{cc} x & \quad y \end{array} \\ \begin{array}{c} x \\ y \end{array} & \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \end{array}$$

3 D Rotations

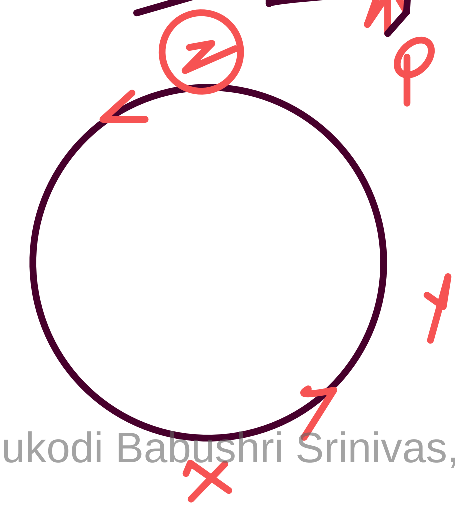


Rotation about z-axes or Rotation in XY-plane. (Yaw)

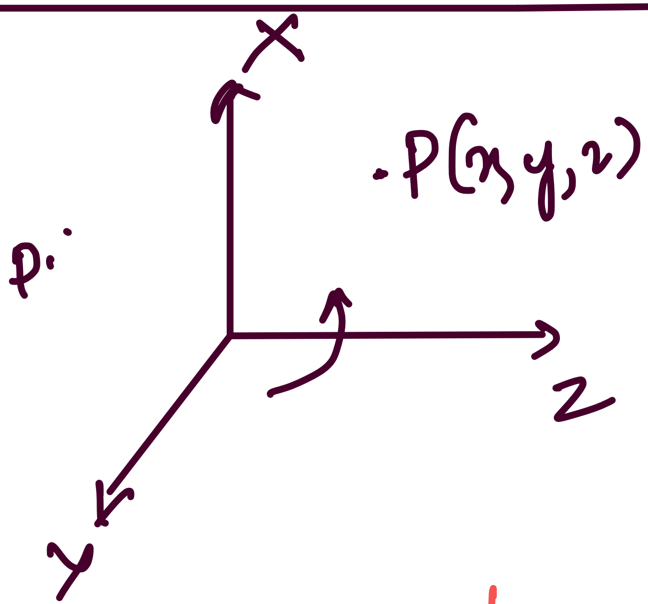
$$
\begin{array}{c|ccc}
 & x & y & z \\
\hline
x & & & \\
y & & \left[\begin{matrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{matrix}\right] & \\
z & & &
\end{array}
$$

$$
\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} x \\ y \\ z \end{bmatrix}
=
\begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \\ z \end{bmatrix}
\equiv P'
$$

P



Rotation about x-axis. (Roll)



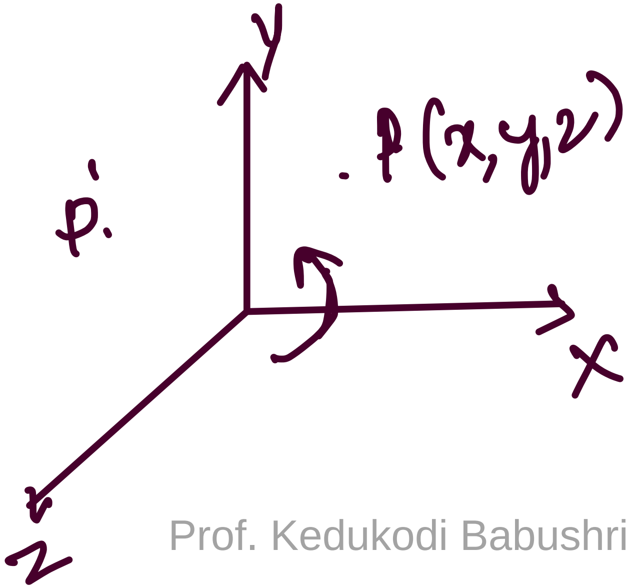
$$\longleftrightarrow \quad \begin{array}{c|ccc} & x & y & z \\ \hline x & & & \\ y & & & \\ z & & & \end{array} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

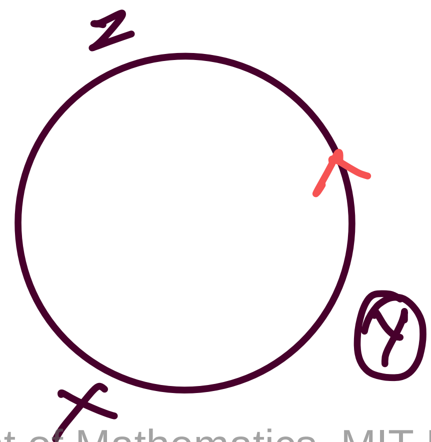
$\hookrightarrow$ Rotation about yz-plane.

$$\begin{array}{c} \\ y \\ z \end{array} \begin{array}{c} \begin{array}{cc} y & z \end{array} \\ \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \end{array}$$

$$\underbrace{\begin{bmatrix}1&0&0\\0&\cos\theta&-\sin\theta\\0&\sin\theta&\cos\theta\end{bmatrix}}\underbrace{\begin{bmatrix}x\\y\\z\end{bmatrix}}_{P}=\begin{bmatrix}x\\y\cos\theta-z\sin\theta\\y\sin\theta+z\cos\theta\end{bmatrix}=P'$$

Rotation about y-axis (Pitch)





Rotation about zx-plane.

$$\begin{array}{c|cc} & z & x \\ \hline z & \cos\theta & -\sin\theta \\ x & \sin\theta & \cos\theta \end{array} \quad ; \quad \begin{array}{c|ccc} & x & y & z \\ \hline x & \cos\theta & 0 & \sin\theta \\ y & 0 & 1 & 0 \\ z & -\sin\theta & 0 & \cos\theta \end{array}$$

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x\cos\theta + z\sin\theta \\ y \\ -x\sin\theta + z\cos\theta \end{bmatrix} \equiv P'$$

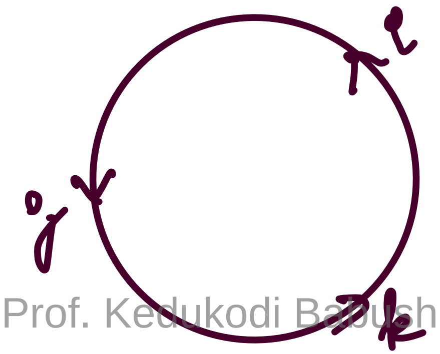
$\alpha \xrightarrow{\text{x-axis}}$ Roll

$\beta \xrightarrow{\text{y-axis}}$ Pitch

$\gamma \xrightarrow{\text{z-axis}}$ Yaw

Euler angles.

* $e^{i\theta}$, Euler angles are two standard ways to handle 3D rotations.



" Quarternions "

Suppose $T : V_F \longrightarrow W_F$

↳ vector spaces over a field F (scalars)

$\alpha A \in V$ → closed under scalar multiplication (scaling)

$A + B \in V$ → closed under vector addition (parallelogram law holds)

* $U \subseteq V$ is subspace iff it is closed under scaling & vector addition.

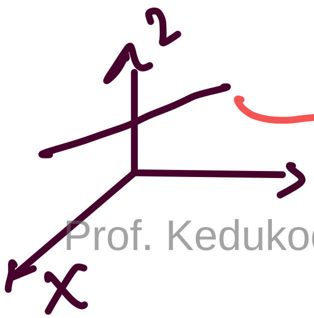
xy-plane is a subspace of $\mathbb{R}^3$.

$$xy\text{-plane} = \{(x, y, 0) \mid x, y \in \mathbb{R}\} = U.$$

$$\alpha(x, y, 0) = (\alpha x, \alpha y, 0) \in U$$

$$(x_1, y_1, 0) + (x_2, y_2, 0) = (x_1 + x_2, y_1 + y_2, 0) \in U$$

$\therefore$ U is a subspace of $\mathbb{R}^3$



$\longrightarrow$ not a subspace as it does not pass through origin.

$T : V \longrightarrow W$ be a L.T.

$\mathrm{Ker}(T) = \{x \in V \mid T(x) = 0\} \subseteq V$ (domain)

↳ Kernel of T or Nullity of T.

$= \mathrm{Nullity}(T)$

$\mathrm{Im}(T) = \{T(x) \mid x \in V\} \subseteq W$ (codomain)

↳ Image of T

Result : $T : V \to W$ be a linear transformation.

① $Ker(T)$ is a subspace of domain V.

② $Im(T)$ is a subspace of codomain W.

Proof : ① Let $x, y \in Ker(T) = \{a \in V \mid T(a) = 0\}$

$$\Rightarrow T(x) = 0 \; ; \; T(y) = 0 \quad \text{--- } (1)$$

$$T(x+y) = T(x) + T(y) \quad \text{(because } T \text{ is L.T.)}$$

$$= 0 + 0 = 0 \quad \text{(by (1).)}$$

$$\Rightarrow x + y \in Ker(T).$$

(ie Ker(T) is closed under + or parallelogram law holds)

Let $\alpha \in F$ (scalar)

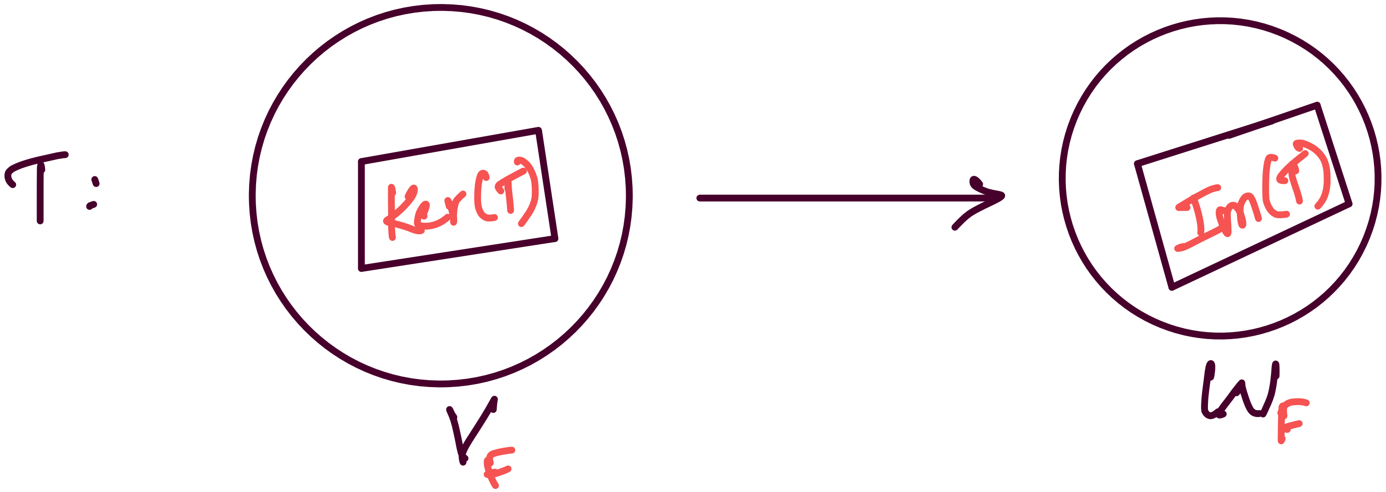
$T(\alpha x) = \alpha T(x)$ (because T is L.T.)

$= \alpha \cdot 0 = 0$
$(x \in Ker(T))$

$\Rightarrow \alpha x \in Ker(T)$

$\Rightarrow$ Ker(T) is closed under scalar multiplication (scaling holds)

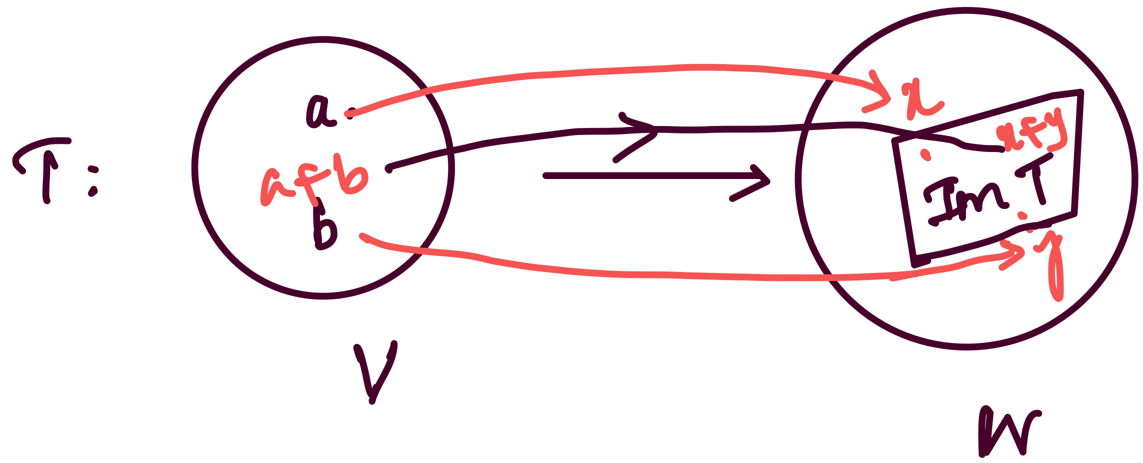
$\therefore$ Ker(T) is a subspace of V.



② Let $x, y \in Im(T) = \{T(a) \mid a \in V\}$

$\Rightarrow x = T(a)$

$y = T(b)$ for some $a, b \in V$



$$x + y = T(a) + T(b)$$

$$= T(\underbrace{a+b}_{\text{preimage}})$$

(because T is a L.T. & $a+b \in V$)

$\Rightarrow$ Im(T) is closed under $+$.

Let $\alpha \in F$ (scalar)

$\alpha x = \alpha T(a)$

image $= T(\alpha a)$ (because T is a L.T &

preimage

$\alpha a \in V$)

$\Rightarrow \alpha x \in Im(T)$

$\Rightarrow Im(T)$ is closed under scaling.

$\therefore Im(T)$ is a subspace of W.