Exam Date & Time: 26-Dec-2024 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

END SEMESTER MAKEUP EXAMINATION, DECEMBER 2024

COMPUTATIONAL LINEAR ALGEBRA [MAT 2135]

Marks: 50

Duration: 180 mins.

Descriptive type questions

Answer all the questions.

Draw diagrams, and write equations wherever necessary.

1) If $T(x,y)=(3x+y,x+3y)$ then find $T^{-1}(10,20)$ and $T^{25}(x,y)$.

A) (3)

B) Express the following matrix A as product of elementary matrices and then describe the geometric effect of multiplication of a vector by A.

$$A=\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

(3)

C) Prove that any linearly independent set of a finite dimensional vector space V can be extended to a basis of V. Hence derive the rank-nullity theorem. Can a linearly dependent set in V be extended to a basis of V? Justify your answer. (4)

2) Find the least squares solution to the system of equations given by $AX=b$ given

A)

$$A=\begin{bmatrix} -1 & -1 \\ 2 & -2 \\ -1 & 1 \end{bmatrix}$$

$$X=\begin{bmatrix} x \\ y \end{bmatrix}$$

$$b=\begin{bmatrix} 10 \\ 5 \\ 20 \end{bmatrix}.$$

(3)

Compute the error in the solution.

B) Find QR decomposition of the following matrix:

(3)

$$\begin{bmatrix} 3 & 1 & 1 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

C) Give definition of a positive definite matrix. If A is a positive definite matrix then show that there exists a lower triangular matrix L such that $A = LL^T$. (4)

3)

A) Compute the basis for the four fundamental subspaces given the matrix:

$$A = \begin{bmatrix} 1 & -2 & 2 \\ -1 & 2 & -2 \end{bmatrix}$$ (3)

B) Fit $y = a + bx$ given the following data:

X	0	-1	1	2
Y	-1	0	4	9

Hence find $y(3)$. (3)

C) Let A be a symmetric matrix with all real entries. Then

(i) prove that all eigenvalues of A are real.

(ii) show that there is an orthogonal change of variable $X = PY$ that transforms the quadratic form $X^T AX$ into quadratic form $Y^T DY$ with no cross-product term. (4)

4)

A) Let $m \geq n$ and suppose $B_{n \times m}$ has n independent rows. Then show that

(i) BB^T is invertible. (3)

(ii) right inverse of B exists.

B) Find LU decomposition of the following matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 9 \end{bmatrix}$$ (3)

Mention how to solve system of equations by LU Decomposition.

C) Find the singular value decomposition (SVD) of the following matrix: (4)

$$\begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$$

Give the geometrical interpretation of the obtained SVD.

5) Using eigenvalues and eigenvectors, find the maximum and minimum values of the function

A) $$17x^2 - 30xy + 17y^2$$

subject to the constraint

$$x^2 + y^2 = 1.$$ (3)

Give a pictorial representation of your computations using standard axes and principal axes.

B) Using eigenvalues and eigenvectors, solve the following system of simultaneous differential equations:

$$\frac{dy_1}{dt} = y_1 + y_2$$

$$\frac{dy_2}{dt} = y_1$$ (3)

C) Carry out Principal Component Analysis (PCA) given the following data.

X	0	1	2	3	-1	-2	-3
Y	0	1	2	3	-1	-2	-3

Give the explicit geometric interpretation with a diagram. (4)

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