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SOAL QUIS 1

1. Tentukan; a. (A+B).D b. (C-A)+B.A c. (A+C)-(D-B)

a. $A=\begin{bmatrix}-1 & 1\\ 5 & 3\end{bmatrix}$ b. $B=\begin{bmatrix}5 & -1\\ -4 & 0\end{bmatrix}$ c. $C=\begin{bmatrix}4 & 8\\ -3 & -6\end{bmatrix}$ d. $D=\begin{bmatrix}10 & -6\\ 8 & -5\end{bmatrix}$

2. Tentukan matriks X jika :

a. $X\begin{bmatrix}4 & 5\\ 2 & 0\end{bmatrix}=\begin{bmatrix}8 & 5\\ 14 & 15\end{bmatrix}$ b. $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}X=\begin{bmatrix}4 & 3\\ 2 & -1\end{bmatrix}$

c. $\begin{bmatrix}3 & -2\\ 1 & 4\end{bmatrix}X=\begin{bmatrix}28\\ -14\end{bmatrix}$ d. $X\begin{bmatrix}2 & -1\\ 4 & 1\end{bmatrix}=\begin{bmatrix}8 & 2\\ 14 & 5\\ 10 & -2\end{bmatrix}$

3. Carilah Determinan dan Invers matriks : a. $D=\begin{bmatrix}10 & -6\\ 8 & -5\end{bmatrix}$ b. $A=\begin{bmatrix}2 & 1 & -1\\ 1 & 1 & 1\\ 1 & -2 & 1\end{bmatrix}$

4. Tentukan determinan dan invers matriks berikut menggunakan metode Minor dan Kofaktor.

$$A=\begin{bmatrix}1 & 4 & 6 & 1\\ 1 & 0 & 0 & 1\\ 3 & 2 & 4 & 5\\ 5 & 8 & 4 & 2\end{bmatrix}$$

5. Selesaikan persamaan berikut dengan Eleminasi da Substitusi:

$$x+y+2z=9$$
$$2z+4y-3z=1$$
$$3z+6y-5z=0$$

6. Selesaikan sistem persamaan berikut dengan cara Matriks:

$$5x_1+2x_2+10x_3+16x_4=16$$
$$3x_1+x_2-2x_4=3x_1+x_2-9x_3-19x_4=-4x_1+x_2-3x_4=5$$

Jawab

1

1a. $(A+B).D$

$$A+B=\begin{bmatrix}-1 & 1\\ 5 & 3\end{bmatrix}+\begin{bmatrix}5 & -1\\ -4 & 0\end{bmatrix}=\begin{bmatrix}4 & 0\\ 1 & 3\end{bmatrix}$$

$$(A+B).D=\begin{bmatrix}4 & 0\\ 1 & 3\end{bmatrix}\begin{bmatrix}10 & -6\\ 8 & -5\end{bmatrix}$$

$$=\begin{bmatrix}4(10)+0(8) & 4(-6)+0(-5)\\ 1(10)+3(8) & 1(-6)+3(-5)\end{bmatrix}$$

$$=\begin{bmatrix}40 & -24\\ 34 & -21\end{bmatrix}$$

1b. $(C-A)+BA$

$$C-A=\begin{bmatrix}4 & 8\\ -3 & -6\end{bmatrix}\begin{bmatrix}-1 & 1\\ 5 & 3\end{bmatrix}=\begin{bmatrix}5 & 7\\ -8 & -9\end{bmatrix}$$

$$BA=\begin{bmatrix}5 & -1\\ -4 & 0\end{bmatrix}\begin{bmatrix}-1 & 1\\ 5 & 3\end{bmatrix}$$

$$=\begin{bmatrix}5(-1)+(-1(5)) & 5(1)+(-1(3))\\ -4(-1)+0(5) & -4(1)+0(3)\end{bmatrix}$$

$$=\begin{bmatrix}-10 & 2\\ 4 & -4\end{bmatrix}$$

$$(C-A)+BA=\begin{bmatrix}5 & 7\\ -8 & -9\end{bmatrix}\begin{bmatrix}-10 & 2\\ 4 & -4\end{bmatrix}$$

$$=\begin{bmatrix}-5 & 9\\ -4 & -13\end{bmatrix}$$

1c. $(A+C)-(D-B)$

$$A+C=\begin{bmatrix}-1 & 1\\ 5 & 3\end{bmatrix}\begin{bmatrix}4 & 8\\ -3 & -6\end{bmatrix}$$

$$=\begin{bmatrix}3 & 9\\ 2 & -3\end{bmatrix}$$

$$D-B=\begin{bmatrix}10 & -6\\ 8 & -5\end{bmatrix}\begin{bmatrix}5 & -1\\ -4 & 0\end{bmatrix}$$

$$=\begin{bmatrix}5 & -5\\ 12 & -5\end{bmatrix}$$

$$(A+C)-(D-B)=\begin{bmatrix}3 & 9\\ 2 & -3\end{bmatrix}\begin{bmatrix}5 & -5\\ 12 & -5\end{bmatrix}$$

$$=\begin{bmatrix}-2 & 14\\ -10 & 2\end{bmatrix}$$

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2a. $X\begin{bmatrix}4 & 5\\ 2 & 0\end{bmatrix}=\begin{bmatrix}8 & 5\\ 14 & 15\end{bmatrix}$

$XA=B$

$X=BA^{-1}$

Menentukan $A^{-1}=\frac{1}{ad-bc}\begin{bmatrix}a & b\\ c & d\end{bmatrix}$

$$X=\begin{bmatrix}8 & 5\\ 14 & 15\end{bmatrix}\cdot\frac{1}{-10}\begin{bmatrix}4 & 5\\ 2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}8 & 5\\ 14 & 15\end{bmatrix}\begin{bmatrix}\frac{4}{-10} & \frac{5}{-10}\\ \frac{2}{-10} & \frac{0}{-10}\end{bmatrix}$$

$$= \begin{bmatrix} -\frac{32}{10} - \frac{10}{10} & -\frac{40}{10} - \frac{0}{10} \\ -\frac{56}{10} - \frac{30}{10} & -\frac{70}{10} - \frac{0}{10} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{21}{5} & -4 \\ -\frac{43}{5} & -7 \end{bmatrix}$$

2b. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} X = \begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix}$

$AX = B$
$X = A^{-1}B$

$$X = \frac{1}{-2}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2} & -\frac{2}{2} \\ -\frac{3}{2} & -\frac{4}{2} \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2} & -1 \\ -\frac{3}{2} & -2 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+(-2) & -\frac{3}{2}+1 \\ -6+(-4) & -\frac{9}{2}+2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -\frac{1}{2} \\ -10 & -\frac{5}{2} \end{bmatrix}$$

2c. $\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} X = \begin{bmatrix} 28 \\ -14 \end{bmatrix}$

$AX = B$
$X = A^{-1}B$

$$X = \frac{1}{14}\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 28 \\ -14 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{14} & -\frac{2}{14} \\ \frac{1}{14} & \frac{4}{14} \end{bmatrix}\begin{bmatrix} 28 \\ -14 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{84}{14} + \frac{28}{14} \\ \frac{28}{14} - \frac{56}{14} \end{bmatrix}$$

$$= \begin{bmatrix} 28 \\ -2 \end{bmatrix}$$

2d. $X \begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 14 & 5 \\ 10 & -2 \end{bmatrix}$

$$X = BA^{-1}$$
$$X = \begin{bmatrix} 8 & 2 \\ 14 & 5 \\ 10 & -2 \end{bmatrix} \frac{1}{6} \begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 2 \\ 14 & 5 \\ 10 & -2 \end{bmatrix} \begin{bmatrix} \frac{2}{6} & -\frac{1}{6} \\ \frac{4}{6} & \frac{1}{6} \end{bmatrix}$$

$$= \begin{bmatrix} 8\left(\frac{2}{6}\right) + 2\left(\frac{4}{6}\right) & 8\left(-\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) \\ 14\left(\frac{2}{6}\right) + 5\left(\frac{4}{6}\right) & 14\left(-\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) \\ 10\left(\frac{2}{6}\right) + -2\left(\frac{4}{6}\right) & 10\left(-\frac{1}{6}\right) + -2\left(\frac{1}{6}\right) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -1 \\ 8 & -\frac{3}{2} \\ 2 & -2 \end{bmatrix}$$

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3a. $D = \begin{bmatrix} 10 & -6 \\ 8 & -5 \end{bmatrix}$

$$Det\ D = \begin{bmatrix} 10 & -6 \\ 8 & -5 \end{bmatrix}$$
$$= 10(-5) - (-6(8))$$
$$= -50 + 48$$
$$= -2$$

$$D^{-1} = -\frac{1}{2} \begin{bmatrix} 10 & -6 \\ 8 & -5 \end{bmatrix}$$
$$= \begin{bmatrix} -5 & 3 \\ -4 & -\frac{5}{2} \end{bmatrix}$$

3b. $A=\begin{bmatrix}2 & 1 & -1\\ 1 & 1 & 1\\ 1 & -2 & 1\end{bmatrix}$

Metode minor kofaktor

$$\begin{aligned}Det\ A &= a_{11}.C_{11}+a_{12}.C_{12}+a_{13}.C_{13}\\ &=2.(-1)^{1+1}\begin{bmatrix}1 & 1\\ -2 & 1\end{bmatrix}+1.(-1)^{1+2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}+(-1).-1^{1+3}\begin{bmatrix}1 & 1\\ 1 & -2\end{bmatrix}\\ &=2.(1-(-2))-1.(1-1)+(-1(-2-1))\\ &=2(3)-1(0)+(-1(-3))\\ &=6-0+3\\ &=9\end{aligned}$$

$$A^{-1}=\frac{1}{9}\begin{bmatrix}2 & 1 & -1\\ 1 & 1 & 1\\ 1 & -2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\frac{2}{9} & \frac{1}{9} & -\frac{1}{9}\\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9}\\ \frac{1}{9} & -\frac{2}{9} & \frac{1}{9}\end{bmatrix}$$

4. $A=\begin{bmatrix}1 & 4 & 6 & 1\\ 1 & 0 & 0 & 1\\ 3 & 2 & 4 & 5\\ 5 & 8 & 4 & 2\end{bmatrix}$

Metode Minor kofaktor

$$M_{11}=\begin{bmatrix}0 & 0 & 1\\ 2 & 4 & 5\\ 8 & 4 & 2\end{bmatrix}\begin{bmatrix}0 & 0\\ 2 & 4\\ 8 & 4\end{bmatrix}=(0\times4\times2)+(0\times5\times8)+(1\times2\times4)-(1\times4\times8)-(0\times5\times4)-(0\times2\times2)$$

$$=0+0+8-32-0-0$$

$$=-24$$

$$C_{11}=(-1)^{1+1}\times M_{11}$$

$$=-24$$

$$M_{21}=\begin{bmatrix}4 & 6 & 1\\ 2 & 4 & 5\\ 8 & 4 & 2\end{bmatrix}\begin{bmatrix}4 & 6\\ 2 & 4\\ 8 & 4\end{bmatrix}=(4\times4\times2)+(6\times5\times8)+(1\times2\times4)-(1\times4\times8)-(4\times5\times4)-(6\times2\times2)$$

$$=32+240+8-32-80-24$$

$$=144$$

$$C_{21}=(-1)^{1+2}\times M_{21}$$

$$=-144$$

$$M_{31}=\begin{bmatrix}4 & 6 & 1\\ 0 & 0 & 1\\ 8 & 4 & 2\end{bmatrix}\begin{bmatrix}4 & 6\\ 0 & 0\\ 8 & 4\end{bmatrix}=(4\times0\times2)+(6\times1\times8)+(1\times0\times4)-(1\times0\times8)-(4\times1\times4)-(6\times0\times2)$$

$$= 0+48+0-0-16-0$$
$$= 32$$

$$C_{31} = (-1)^{1+3} \times M_{31}$$
$$= 32$$

$$M_{41} = \begin{bmatrix} 4 & 6 & 1 \\ 0 & 0 & 1 \\ 2 & 4 & 5 \end{bmatrix} \begin{bmatrix} 4 & 6 \\ 0 & 0 \\ 2 & 4 \end{bmatrix} = (4\times0\times5)+(6\times1\times2)+(1\times0\times4)-(1\times0\times2)-(4\times1\times4)-(6\times0\times5)$$
$$= 0+12+0-0-16-0$$
$$= -4$$

$$C_{41} = (-1)^{1+4} \times M_{41}$$
$$= 4$$

$$Det\ A = (a_{11}\times C_{11})+(a_{21}\times C_{21})+(a_{31}\times C_{31})+(a_{41}\times C_{41})$$
$$= (1\times-24)+(1\times-144)+(3\times32)+(5\times4)$$
$$= -24-144+96+20$$
$$= -52$$

Invers Matriks

$$A^{-1} = \frac{1}{-52}\begin{bmatrix} 1 & 4 & 6 & 1 \\ 1 & 0 & 0 & 1 \\ 3 & 2 & 4 & 5 \\ 5 & 8 & 4 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} -\frac{1}{52} & -\frac{4}{52} & -\frac{6}{52} & -\frac{1}{52} \\ -\frac{1}{52} & 0 & 0 & -\frac{1}{52} \\ -\frac{3}{52} & -\frac{2}{52} & -\frac{4}{52} & -\frac{5}{52} \\ -\frac{5}{52} & -\frac{8}{52} & -\frac{4}{52} & -\frac{2}{52} \end{bmatrix}$$

5. $x+y+2z=9$ persamaan (1)
$2x+4y-3z=1$ persamaan (2)
$3x+6y-5z=0$ persamaan (3)

Hitung persamaan (1) dan (2), metode eliminasi

$$\begin{array}{rl} x+y+2z=9 & [\times2] \\ 2x+4y-3z=1 & [\times1] \end{array} \begin{array}{l} 2x+2y+4z=18 \\ \underline{2x+4y-3z=1} \\ -2y+7z=17 \end{array} \quad - \qquad (4)$$

Hitung persamaan (1) dan (3), metode eliminasi

$$\begin{array}{rl} x+y+2x=9 & [\times3] \\ 3x+6y-5z=0 & [\times1] \end{array} \begin{array}{l} 3x+3y+6z=27 \\ \underline{3x+6y-5z=0} \\ -3y+11z=27 \end{array} \quad - \qquad (5)$$

Hitung persamaan (4) *dan* (5), *metode eliminasi*

$$\begin{array}{rl} -2y+7z=17 & [\times 3] \\ -3y+11z=27 & [\times 2] \end{array} \quad \begin{array}{r} -6y+21z=51 \\ -6y+22z=54 \\ \hline z=3 \end{array} \; -$$

(6)

Substitusi persamaan (6) *ke* (4)

$$\begin{aligned} -2y+7(3)&=17 \\ -2y+21&=17 \\ -2y&=4 \\ y&=-2 \end{aligned}$$

(7)

substitusikan persamaan (6) *dan* (7) *ke persamaan* (1)

$$\begin{aligned} x+y+2z&=9 \\ x+(-2)+2(3)&=9 \\ x-2+6&=9 \\ x+4&=9 \\ x&=9-4 \\ x&=5 \end{aligned}$$

6. $5x_1+2x_2+10x_3+16x_4=16$

$3x_1+x_2-2x_4=3x_1+x_2-9x_3-19x_4=-4x_1+x_2-3x_4=5$