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Soal UTS Aljabar Linier

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1. Matriks $A=\begin{bmatrix}1 & 5 & -3\\ 1 & 0 & 2\\ 3 & -1 & 2\end{bmatrix} B=\begin{bmatrix}1 & 5 & 0\\ 2 & 4 & -1\\ 0 & -2 & 0\end{bmatrix} C=\begin{bmatrix}2 & 1 & 0\\ -1 & 4 & -1\\ 0 & 3 & 0\end{bmatrix}$

Tunjukan sifat-sifat matrik:
a. Sifat asosiatif perkalian dan penjumlahan pada matriks
b. Komutatif penjumlahan pada matriks
c. Asosiatif perkalian dan pengurangan pada matriks

2. Pada matriks diatas:
a. Carilah determinan dan invers pada matriks A dengan cara sarrus
b. Carilah determinan dan invers pada matriks B dengan cara minor kofaktor

3. Tentukan penyelesaian SPL berikut dengan cara sarrus:

$$\begin{aligned} x+2y+z&=8\\ 2x-y+z&=5\\ x+y-z&=4 \end{aligned}$$

4. Tentukan penyelesaian SPL berikut dengan cara invers matriks:

$$\begin{aligned} x+2y+z&=8\\ 2x-y+z&=5\\ x+y-z&=4 \end{aligned}$$

5. Tentukan penyelesaian SPL berikut dengan cara:

$$\begin{aligned} 2w-x+3y-z&=11\\ -w-2x-y-z&=-4\\ 3w-x+y+3z&=-2\\ -2w+2x-2y+2z&=4 \end{aligned}$$

a. Eliminasi Gaus
b. Eliminasi Gaus-Jordan

jawab

1.a. Sifat asosiatif perkalian dan penjumlahan pada matriks

Asosiatif perkalian:

$(AB)C=A(BC)$

$$AB=\begin{bmatrix}1 & 5 & -3\\ 1 & 0 & 2\\ 3 & -1 & 2\end{bmatrix}\begin{bmatrix}1 & 5 & 0\\ 2 & 4 & -1\\ 0 & -2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1(1)+5(2)-3(0) & 1(5)+5(4)-3(-2) & 1(0)+5(-1)-3(0)\\ 1(1)+0(2)+2(0) & 1(5)+0(4)+2(-2) & 1(0)+0(-1)+2(0)\\ 3(1)-1(2)+2(0) & 3(5)-1(4)+2(-2) & 3(0)-1(-1)+2(0)\end{bmatrix}$$

$$= \begin{bmatrix} 1+10-0 & 5+20+6 & 0-5-0 \\ 1+0+0 & 5+0-4 & 0-0+0 \\ 3-2+0 & 15-4-4 & 0+1+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 11 & 31 & -5 \\ 1 & 1 & 0 \\ 1 & 7 & 1 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} 11 & 31 & -5 \\ 1 & 1 & 0 \\ 1 & 7 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ -1 & 4 & -1 \\ 0 & 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 11(2)+31(-1)-5(0) & 11(1)+31(4)-5(3) & 11(0)+31(-1)-5(0) \\ 1(2)+1(-1)+0(0) & 1(1)+1(4)+0(3) & 1(0)+1(-1)+0(0) \\ 1(2)+7(-1)+1(0) & 1(1)+7(4)+1(3) & 1(0)+7(-1)+1(0) \end{bmatrix}$$

$$= \begin{bmatrix} 22-31-0 & 11+124-15 & 0-31-0 \\ 2-1+0 & 1+4+0 & 0-1+0 \\ 2-7+0 & 1+28+3 & 0-7+0 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 120 & -31 \\ 1 & 5 & -1 \\ -5 & 32 & -7 \end{bmatrix}$$

$$BC = \begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ -1 & 4 & -1 \\ 0 & 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1(2)+5(-1)+0(0) & 1(1)+5(4)+0(3) & 1(0)+5(-1)+0(0) \\ 2(2)+4(-1)-1(0) & 2(1)+4(4)-1(3) & 2(0)+4(-1)-1(0) \\ 0(2)-2(-1)+0(0) & 0(1)-2(4)+0(3) & 0(0)-2(-1)+0(0) \end{bmatrix}$$

$$= \begin{bmatrix} 2-5+0 & 1+20+0 & 0-5+0 \\ 4-4-0 & 2+16-3 & 0-4-0 \\ 0+2+0 & 0-8+0 & 0+2+0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 21 & -5 \\ 0 & 15 & -4 \\ 2 & -8 & 2 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 1 & 5 & -3 \\ 1 & 0 & 2 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} -3 & 21 & -5 \\ 0 & 15 & -4 \\ 2 & -8 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-3)+5(0)-3(2) & 1(21)+5(15)-3(-8) & 1(-5)+5(-4)-3(2) \\ 1(-3)+0(0)+2(2) & 1(21)+0(15)+2(-8) & 1(-5)+0(-4)+2(2) \\ 3(-3)-1(0)+2(2) & 3(21)-1(15)+2(-8) & 3(-5)-1(-4)+2(2) \end{bmatrix}$$

$$= \begin{bmatrix} -3+0-6 & 21+75+24 & -5-20-6 \\ -3+0+4 & 21+0-16 & -5+0+4 \\ -9-0+4 & 63-15-16 & -15+4+4 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 120 & -31 \\ 1 & 5 & -1 \\ -5 & 32 & -7 \end{bmatrix}$$

$$(AB)C = A(BC)$$

$$\begin{bmatrix} -9 & 120 & -31 \\ 1 & 5 & -1 \\ -5 & 32 & -7 \end{bmatrix} = \begin{bmatrix} -9 & 120 & -31 \\ 1 & 5 & -1 \\ -5 & 32 & -7 \end{bmatrix}$$

Asosiatif penjumlahan:

$$A+(B+C)=(A+B)+C$$

$$B+C = \begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 0 \\ -1 & 4 & -1 \\ 0 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 0 \\ 1 & 8 & -2 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A+(B+C) = \begin{bmatrix} 1 & 5 & -3 \\ 1 & 0 & 2 \\ 3 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 6 & 0 \\ 1 & 8 & -2 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 11 & -3 \\ 2 & 8 & 0 \\ 3 & 0 & 2 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 1 & 5 & -3 \\ 1 & 0 & 2 \\ 3 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 10 & -3 \\ 3 & 4 & 1 \\ 3 & -3 & 2 \end{bmatrix}$$

$$(A+B)+C = \begin{bmatrix} 2 & 10 & -3 \\ 3 & 4 & 1 \\ 3 & -3 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 0 \\ -1 & 4 & -1 \\ 0 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 11 & -3 \\ 2 & 8 & 0 \\ 3 & 0 & 2 \end{bmatrix}$$

$$A+(B+C)=(A+B)+C$$

$$\begin{bmatrix} 4 & 11 & -3 \\ 2 & 8 & 0 \\ 3 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 11 & -3 \\ 2 & 8 & 0 \\ 3 & 0 & 2 \end{bmatrix}$$

1.b. Komutatif penjumlahan pada matriks

komutatif penjumlahan:

$$A+B+C=C+B+A$$

$$\begin{bmatrix} 1 & 5 & -3 \\ 1 & 0 & 2 \\ 3 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 0 \\ -1 & 4 & -1 \\ 0 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 4 & -1 \\ 0 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 5 & -3 \\ 1 & 0 & 2 \\ 3 & -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 11 & -3 \\ 2 & 8 & 0 \\ 3 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 11 & -3 \\ 2 & 8 & 0 \\ 3 & 0 & 2 \end{bmatrix}$$

1.c. Asosiatif perkalian dan pengurangan pada matriks

Asosiatif pengurangan:

$$A-(B-C)=(A-B)-C$$

$$B-C=\begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{bmatrix}-\begin{bmatrix} 2 & 1 & 0 \\ -1 & 4 & -1 \\ 0 & 3 & 0 \end{bmatrix}=\begin{bmatrix} -1 & 4 & 0 \\ 3 & 0 & 0 \\ 0 & -5 & 0 \end{bmatrix}$$

$$A-(B-C)=\begin{bmatrix} 1 & 5 & -3 \\ 1 & 0 & 2 \\ 3 & -1 & 2 \end{bmatrix}-\begin{bmatrix} -1 & 4 & 0 \\ 3 & 0 & 0 \\ 0 & -5 & 0 \end{bmatrix}=\begin{bmatrix} 2 & 1 & -3 \\ -2 & 0 & 2 \\ 3 & 4 & 2 \end{bmatrix}$$

2.a. Carilah determinan dan invers pada matriks A dengan cara sarrus

$$A=\begin{bmatrix} 1 & 5 & -3 \\ 1 & 0 & 2 \\ 3 & -1 & 2 \end{bmatrix}$$

$$\begin{aligned} Det\ A &= \begin{bmatrix} 1 & 5 & -3 \\ 1 & 0 & 2 \\ 3 & -1 & 2 \end{bmatrix} \begin{matrix} 1 & 5 \\ 1 & 0 \\ 3 & -1 \end{matrix} \\ &= (1\times 0\times 2)+(5\times 2\times 3)+(-3\times 1\times -1)-(-3\times 0\times 3)-(1\times 2\times -1)-(5\times 1\times 2) \\ &= 0+30+3+0+2-10 \\ &= 25 \end{aligned}$$

$$Kof\ A=\begin{bmatrix} +\begin{vmatrix} 0 & 2 \\ -1 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 3 & -1 \end{vmatrix} \\ -\begin{vmatrix} 5 & -3 \\ -1 & 2 \end{vmatrix} & +\begin{vmatrix} 1 & -3 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 5 \\ 3 & -1 \end{vmatrix} \\ +\begin{vmatrix} 5 & -3 \\ 0 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & -3 \\ 1 & 2 \end{vmatrix} & +\begin{vmatrix} 1 & 5 \\ 1 & 0 \end{vmatrix} \end{bmatrix}=\begin{bmatrix} 2 & 4 & -1 \\ -7 & 11 & 16 \\ 10 & -5 & -5 \end{bmatrix}$$

$$Adj\ A=\begin{bmatrix} 2 & -7 & 10 \\ 4 & 11 & -5 \\ -1 & 16 & -5 \end{bmatrix}$$

$$A^{-1}=\frac{1}{25}\begin{bmatrix} \frac{2}{25} & -\frac{7}{25} & \frac{10}{25} \\ \frac{4}{25} & \frac{11}{25} & -\frac{5}{25} \\ -\frac{1}{25} & \frac{16}{25} & -\frac{5}{25} \end{bmatrix}=\begin{bmatrix} \frac{2}{25} & -\frac{7}{25} & \frac{2}{5} \\ \frac{4}{25} & \frac{11}{25} & -\frac{1}{5} \\ -\frac{1}{25} & \frac{16}{25} & -\frac{1}{5} \end{bmatrix}$$

2.b. Carilah determinan dan invers pada matriks B dengan cara minor kofaktor

$$B=\begin{bmatrix}1 & 5 & 0\\ 2 & 4 & -1\\ 0 & -2 & 0\end{bmatrix}$$

$$\begin{aligned}Det\,B &= 1(-1)^2\begin{vmatrix}4 & -1\\ -2 & 0\end{vmatrix}+5(-1)^3\begin{vmatrix}2 & -1\\ 0 & 0\end{vmatrix}+0(-1)^4\begin{vmatrix}2 & 4\\ 0 & -2\end{vmatrix}\\ &= -2+0+0\\ &= -2\end{aligned}$$

$$Kof\,B=\begin{bmatrix}+\begin{vmatrix}4 & -1\\ -2 & 0\end{vmatrix} & -\begin{vmatrix}2 & -1\\ 0 & 0\end{vmatrix} & +\begin{vmatrix}2 & 4\\ 0 & -2\end{vmatrix}\\ -\begin{vmatrix}5 & 0\\ -2 & 0\end{vmatrix} & +\begin{vmatrix}1 & 0\\ 0 & 0\end{vmatrix} & -\begin{vmatrix}1 & 5\\ 0 & -2\end{vmatrix}\\ +\begin{vmatrix}5 & 0\\ 4 & -1\end{vmatrix} & -\begin{vmatrix}1 & 0\\ 2 & -1\end{vmatrix} & +\begin{vmatrix}1 & 5\\ 2 & 4\end{vmatrix}\end{bmatrix}=\begin{bmatrix}-2 & 0 & -4\\ 0 & 0 & 2\\ -5 & 1 & -6\end{bmatrix}$$

$$Adj\,B=\begin{bmatrix}-2 & 0 & -5\\ 0 & 0 & 1\\ -4 & 2 & -6\end{bmatrix}$$

$$B^{-1}=-\frac{1}{2}\begin{bmatrix}-2 & 0 & -5\\ 0 & 0 & 1\\ -4 & 2 & -6\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & \frac{5}{2}\\ 0 & 0 & -\frac{1}{2}\\ 2 & -1 & 3\end{bmatrix}$$

3. Tentukan penyelesaian SPL berikut dengan cara sarrus:

$$\begin{aligned}x+2y+z&=8\\ 2x-y+z&=5\\ x+y-z&=4\end{aligned}$$

$$D=\begin{bmatrix}1 & 2 & 1\\ 2 & -1 & 1\\ 1 & 1 & -1\end{bmatrix}\begin{matrix}1 & 2\\ 2 & -1\\ 1 & 1\end{matrix}$$

$$\begin{aligned}&=(1\times-1\times-1)+(2\times1\times1)+(1\times2\times1)-(1\times-1\times1)-(1\times1\times1)-(2\times2\times-1)\\ &=1+2+2+1-1+4\\ &=9\end{aligned}$$

$$Dx = \begin{bmatrix} 8 & 2 & 1 \\ 5 & -1 & 1 \\ 4 & 1 & -1 \end{bmatrix} \begin{vmatrix} 8 & 2 \\ 5 & -1 \\ 4 & 1 \end{vmatrix}$$
$$= (8 \times -1 \times -1) + (2 \times 1 \times 4) + (1 \times 5 \times 1) - (1 \times -1 \times 4) - (8 \times 1 \times 1) - (2 \times 5 \times -1)$$
$$= 8 + 8 + 5 + 4 - 8 + 10$$
$$= 27$$

$$Dy = \begin{bmatrix} 1 & 8 & 1 \\ 2 & 5 & 1 \\ 1 & 4 & -1 \end{bmatrix} \begin{vmatrix} 1 & 8 \\ 2 & 5 \\ 1 & 4 \end{vmatrix}$$
$$= (1 \times 5 \times -1) + (8 \times 1 \times 1) + (1 \times 2 \times 4) - (1 \times 5 \times 1) - (1 \times 1 \times 4) - (8 \times 2 \times -1)$$
$$= -5 + 8 + 8 - 5 - 4 + 16$$
$$= 18$$

$$Dz = \begin{bmatrix} 1 & 2 & 8 \\ 2 & -1 & 5 \\ 1 & 1 & 4 \end{bmatrix} \begin{vmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{vmatrix}$$
$$= (1 \times -1 \times 4) + (2 \times 5 \times 1) + (8 \times 2 \times 1) - (8 \times -1 \times 1) - (1 \times 5 \times 1) - (2 \times 2 \times 4)$$
$$= -4 + 10 + 16 + 8 - 5 - 16$$
$$= 9$$

$$\frac{Dx}{D} = \frac{27}{9} = 3$$

$$\frac{Dy}{D} = \frac{18}{9} = 2$$

$$\frac{Dz}{D} = \frac{9}{9} = -1$$

4. Tentukan penyelesaian SPL berikut dengan cara invers matriks:

$$x + 2y + z = 8$$
$$2x - y + z = 5$$
$$x + y - z = 4$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$Det = 9$$

$$Kof\ A = \begin{bmatrix} + \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} & - \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} & + \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} \\ - \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} & + \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} & - \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} \\ + \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} & - \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} & + \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 0 & 3 & 3 \\ 3 & -2 & 1 \\ 3 & 1 & -5 \end{bmatrix}$$

$$Adj\ A=\begin{bmatrix}0 & 3 & 3\\ 3 & -2 & 1\\ 3 & 1 & -5\end{bmatrix}$$

$$A^{-1}=\frac{1}{9}\begin{bmatrix}0 & 3 & 3\\ 3 & -2 & 1\\ 3 & 1 & -5\end{bmatrix}$$

$$=\begin{bmatrix}0 & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{3} & -\frac{2}{9} & \frac{1}{9}\\ \frac{1}{3} & \frac{1}{9} & -\frac{5}{9}\end{bmatrix}$$

$x=3, y=2, x=1$

5. Tentukan penyelesaian SPL berikut dengan cara:

$$\begin{aligned}2w-x+3y-z&=11\\ -w-2x-y-z&=-4\\ 3w-x+y+3z&=-2\\ -2w+2x-2y+2z&=4\end{aligned}$$

a. Eliminasi Gaus

$$\begin{bmatrix}2 & -1 & 3 & -1 & 11\\ -1 & -2 & -1 & -1 & -4\\ 3 & -1 & 1 & 3 & -2\\ -2 & 2 & -2 & 2 & 4\end{bmatrix}$$

kalikan baris pertama dengan $\frac{1}{2}$, *kemudian tambahkan di baris* 2

$$\begin{bmatrix}2 & -1 & 3 & -1 & 11\\ 0 & -\frac{5}{2} & \frac{1}{2} & -\frac{3}{2} & \frac{3}{2}\\ 3 & -1 & 1 & 3 & -2\\ -2 & 2 & -2 & 2 & 4\end{bmatrix}$$

kalikan baris pertama dengan $\frac{3}{2}$, *kemudian kurangkan ke baris tiga*

$$\begin{bmatrix}2 & -1 & 3 & -1 & 11\\ 0 & -\frac{5}{2} & \frac{1}{2} & -\frac{3}{2} & \frac{3}{2}\\ 0 & \frac{1}{2} & -\frac{7}{2} & \frac{9}{2} & -\frac{37}{2}\\ -2 & 2 & -2 & 2 & 4\end{bmatrix}$$

kalikan baris pertama dengan 1 *, kemudian jumlahkan ke baris empat*

$$\begin{bmatrix} 2 & -1 & 3 & -1 & 11 \\ 0 & -\frac{5}{2} & \frac{1}{2} & -\frac{3}{2} & \frac{3}{2} \\ 0 & \frac{1}{2} & -\frac{7}{2} & \frac{9}{2} & -\frac{37}{2} \\ 0 & 1 & 1 & 1 & 15 \end{bmatrix}$$

kalikan baris ke dua dengan $-\frac{1}{5}$ *, lalu kurangkan dibaris tiga*

$$\begin{bmatrix} 2 & -1 & 3 & -1 & 11 \\ 0 & -\frac{5}{2} & \frac{1}{2} & -\frac{3}{2} & \frac{3}{2} \\ 0 & 0 & -\frac{17}{5} & \frac{21}{5} & -\frac{91}{5} \\ 0 & 1 & 1 & 1 & 15 \end{bmatrix}$$

kalikan baris ke dua dengan $\frac{2}{5}$ *, lalu tambahkan ke baris empat*

$$\begin{bmatrix} 2 & -1 & 3 & -1 & 11 \\ 0 & -\frac{5}{2} & \frac{1}{2} & -\frac{3}{2} & \frac{3}{2} \\ 0 & 0 & -\frac{17}{5} & \frac{21}{5} & -\frac{91}{5} \\ 0 & 0 & \frac{6}{5} & \frac{2}{5} & \frac{78}{5} \end{bmatrix}$$

kalikan baris ke tiga dengan $\frac{6}{17}$ *, lalu tambahkan ke baris empat*

$$\begin{bmatrix} 2 & -1 & 3 & -1 & 11 \\ 0 & -\frac{5}{2} & \frac{1}{2} & -\frac{3}{2} & \frac{3}{2} \\ 0 & 0 & -\frac{17}{5} & \frac{21}{5} & -\frac{91}{5} \\ 0 & 0 & 0 & \frac{32}{17} & \frac{156}{17} \end{bmatrix}$$

$$\frac{32}{17}z = \frac{156}{17}$$

$$z = \frac{39}{8}$$

$$-\frac{17}{5}y+\frac{21}{5}\left(\frac{39}{8}\right)=-\frac{91}{5}$$
$$\frac{-17}{5}y=-\frac{91}{5}-\frac{819}{40}$$
$$-\frac{17}{5}y=-\frac{1547}{40}$$
$$y=\frac{91}{8}$$

$$-\frac{5}{2}x+\frac{1}{2}\left(\frac{91}{8}\right)-\frac{3}{2}\left(\frac{39}{8}\right)=\frac{3}{2}$$
$$-\frac{5}{2}x+\frac{91}{16}-\frac{117}{16}=\frac{3}{2}$$
$$-\frac{5}{2}x=\frac{3}{2}-\frac{91}{16}+\frac{117}{16}$$
$$-\frac{5}{2}x=\frac{25}{8}$$
$$x=-\frac{5}{4}$$

$$2w-1\left(-\frac{5}{4}\right)+3\left(\frac{91}{8}\right)-1\left(\frac{39}{8}\right)=11$$
$$2w+\frac{5}{4}+\frac{273}{8}-\frac{39}{8}=11$$
$$2w=11-\frac{5}{4}-\frac{273}{8}+\frac{39}{8}$$
$$2w=-\frac{39}{2}$$
$$w=-\frac{39}{4}$$

Jadi, $w=-\frac{39}{4}, x=-\frac{5}{4}, y=\frac{91}{8}, z=\frac{39}{8}$

b.Eliminasi gaus jordan

$$\begin{bmatrix} 2 & -1 & 3 & -1 & 11 \\ -1 & -2 & -1 & -1 & -4 \\ 3 & -1 & 1 & 3 & -2 \\ -2 & 2 & -2 & 2 & 4 \end{bmatrix}$$

bagi baris pertama dengan 2

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{11}{2} \\ -1 & -2 & -1 & -1 & -4 \\ 3 & -1 & 1 & 3 & -2 \\ -2 & 2 & -2 & 2 & 4 \end{bmatrix}$$

kalikan baris pertama dengan 1 *, lalu tambahkan ke baris dua*

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{11}{2} \\ 0 & -\frac{5}{2} & -1 & \frac{1}{2} & \frac{3}{2} \\ 3 & -1 & 1 & 3 & -2 \\ -2 & 2 & -2 & 2 & 4 \end{bmatrix}$$

kalikan baris pertama dengan 3 *, lalu kurangkan ke baris tiga*

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{11}{2} \\ 0 & -\frac{5}{2} & -1 & \frac{1}{2} & \frac{3}{2} \\ 0 & \frac{1}{2} & -\frac{7}{2} & \frac{9}{2} & -\frac{37}{2} \\ -2 & 2 & -2 & 2 & 4 \end{bmatrix}$$

kalikan baris pertama dengan 2 *, lalu jumlahkan dibaris empat*

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{11}{2} \\ 0 & -\frac{5}{2} & -1 & \frac{1}{2} & \frac{3}{2} \\ 0 & \frac{1}{2} & -\frac{7}{2} & \frac{9}{2} & -\frac{37}{2} \\ 0 & 1 & 1 & 1 & 15 \end{bmatrix}$$

baris dua dibagi $-\frac{5}{2}$

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{11}{2} \\ 0 & 1 & -\frac{1}{5} & \frac{3}{5} & -\frac{3}{5} \\ 0 & \frac{1}{2} & -\frac{7}{2} & \frac{9}{2} & -\frac{37}{2} \\ 0 & 1 & 1 & 1 & 15 \end{bmatrix}$$

kalikan baris kedua dengan $\frac{1}{2}$ *, lalu kurangkan ke baris tiga*

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{11}{2} \\ 0 & 1 & -\frac{1}{5} & \frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & -\frac{17}{5} & \frac{21}{5} & -\frac{91}{5} \\ 0 & 1 & 1 & 1 & 15 \end{bmatrix}$$

baris kedua dikali 1 *, lalu baris empat dikurangkan*

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{11}{2} \\ 0 & 1 & -\frac{1}{5} & \frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & -\frac{17}{5} & \frac{21}{5} & -\frac{91}{5} \\ 0 & 0 & \frac{6}{5} & \frac{2}{5} & \frac{78}{5} \end{bmatrix}$$

baris tiga dibagi $-\frac{17}{5}$

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{11}{2} \\ 0 & 1 & -\frac{1}{5} & \frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & 1 & -\frac{21}{17} & \frac{91}{17} \\ 0 & 0 & \frac{6}{5} & \frac{2}{5} & \frac{78}{5} \end{bmatrix}$$

baris empat dikurang $\left(\text{baris tiga dikali } \frac{6}{5} \right)$

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{11}{2} \\ 0 & 1 & -\frac{1}{5} & \frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & 1 & -\frac{21}{17} & \frac{91}{17} \\ 0 & 0 & 0 & \frac{32}{17} & \frac{156}{17} \end{bmatrix}$$

baris empat dibagi $\frac{32}{17}$

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{11}{2} \\ 0 & 1 & -\frac{1}{5} & \frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & 1 & -\frac{21}{17} & \frac{91}{17} \\ 0 & 0 & 0 & 1 & \frac{39}{8} \end{bmatrix}$$

baris tiga di tambah $\left(\text{ baris empat dikali } -\frac{21}{17}\right)$

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{11}{2} \\ 0 & 1 & -\frac{1}{5} & \frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & 1 & 0 & \frac{91}{8} \\ 0 & 0 & 0 & 1 & \frac{39}{8} \end{bmatrix}$$

baris dua dikurangi $\left(\text{ baris empat dikali } \frac{3}{5}\right)$

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{11}{2} \\ 0 & 1 & -\frac{1}{5} & 0 & -\frac{141}{40} \\ 0 & 0 & 1 & 0 & \frac{91}{8} \\ 0 & 0 & 0 & 1 & \frac{39}{8} \end{bmatrix}$$

baris pertama ditambah $\left(\text{ baris empat dikali } \frac{1}{2}\right)$

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & 0 & \frac{127}{16} \\ 0 & 1 & -\frac{1}{5} & 0 & -\frac{141}{40} \\ 0 & 0 & 1 & 0 & \frac{91}{8} \\ 0 & 0 & 0 & 1 & \frac{39}{8} \end{bmatrix}$$

baris dua ditambah $\left(\text{ baris ketiga kali } \frac{1}{5}\right)$

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & 0 & \frac{127}{16} \\ 0 & 1 & 0 & 0 & -\frac{5}{4} \\ 0 & 0 & 1 & 0 & \frac{91}{8} \\ 0 & 0 & 0 & 1 & \frac{39}{8} \end{bmatrix}$$

baris pertama di kurang $\left(\textit{baris tiga dikali } \frac{3}{2}\right)$

$$\begin{bmatrix} 1 & -\frac{1}{2} & 0 & 0 & -\frac{73}{8} \\ 0 & 1 & 0 & 0 & -\frac{5}{4} \\ 0 & 0 & 1 & 0 & \frac{91}{8} \\ 0 & 0 & 0 & 1 & \frac{39}{8} \end{bmatrix}$$

baris pertama ditambah $\left(\textit{baris kedua dikali } \frac{1}{2}\right)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -\frac{39}{4} \\ 0 & 1 & 0 & 0 & -\frac{5}{4} \\ 0 & 0 & 1 & 0 & \frac{91}{8} \\ 0 & 0 & 0 & 1 & \frac{39}{8} \end{bmatrix}$$

Jadi, $w=-\frac{39}{4}, x=-\frac{5}{4}, y=\frac{91}{8}, z=\frac{39}{8}$