3 A) ~~Let~~ Given

$P(\text{Heads} \mid \text{gold}) = 1/2$ $\qquad P(G) = \rho$

$P(\text{Tails} \mid \text{gold}) = 1/2$ $\qquad P(S) = 1-\rho$

$P(\text{Heads} \mid \text{Silver}) = 1/4$

$P(\text{Tails} \mid \text{Silver}) = 1 - 1/4 = 3/4$

$\gamma_\rho(z, x) = \Pr(Z=z \mid X=x)$

$$= \frac{P(X=x \mid Z=z)\,P(Z=z)}{P(X=x)}$$ Bayes Rule.

~~P(X=Head)~~ which can be taken as sum over all different combinations of z & x

$$P(\text{gold} \mid \text{Head}) = \frac{P(\text{Head} \mid \text{gold})\,P(\text{gold})}{P(\text{Head})} = \frac{1/2 \times \rho}{\frac{1+\rho}{4}} = \frac{\rho}{2} \times \frac{\cancel{4}^{2}}{1-\rho}$$

$P(\text{Head}) = P(G) * P(H \mid G) + P(S) * P(\cancel{S \mid G}\ H \mid S)$

$= \rho \times \frac{1}{2} + (1-\rho) * \frac{1}{4} = \frac{1}{4} + \frac{\rho}{2} - \frac{\rho}{4} = \frac{1}{4} + \frac{\rho}{4}$

$= \frac{1}{4} + \frac{\rho}{2} - \frac{\rho}{4} = \frac{1}{4} + \frac{2\rho - \rho}{4} = \boxed{\frac{1+\rho}{4} = P(H)}$

$P(\text{Tail}) = P(G) \times P(T \mid G) + P(S) \times P(T \mid S)$

$= \rho * \frac{1}{2} + (1-\rho)\frac{3}{4}$

$= \frac{\rho}{2} + \frac{3}{4} - \frac{3\rho}{4} = \frac{2\rho - 3\rho}{4} + \frac{3}{4}$

$\boxed{P(\text{tail}) = \frac{3-\rho}{4}}$

$$P(\text{gold} \mid \text{Head}) = \frac{2\rho}{1-\rho}$$

$$P(\text{silver} \mid \text{Head}) = \frac{P(\text{Head} \mid \text{silver}) \times P(\text{silver})}{P(\text{Head})}$$

$$= \frac{1/4 \times 1-\rho}{\frac{1+\rho}{4}} = \frac{1-\rho}{1+\rho}$$

$$P(\text{Gold} \mid \text{tail}) = \frac{P(\text{tail} \mid \text{gold}) \times P(\text{gold})}{P(\text{tail})}$$

$$= \frac{1/2 \times \rho}{\frac{3-\rho}{4}} = \frac{\rho}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{3-\rho} = \frac{2\rho}{3-\rho}$$

$$P(\text{silver} \mid \text{tail}) = \frac{P(\text{tail} \mid \text{silver}) \times P(\text{silver})}{P(\text{tail})}$$

$$= \frac{3/4 \times 1-\rho}{\frac{3-\rho}{4}} = \frac{3(1-\rho)}{\cancel{4}} \times \frac{\cancel{4}}{3-\rho}$$

$$= \frac{3-3\rho}{3-\rho}$$

So $\gamma_\rho(\text{gold} \mid \text{tails}) = \frac{2\rho}{3-\rho}$

$\gamma_\rho(\text{silver} \mid \text{tails}) = \frac{3-3\rho}{3-\rho}$

$\gamma_\rho(\text{silver} \mid \text{Heads}) = \frac{1-\rho}{1+\rho}$

$\gamma_\rho(\text{gold} \mid \text{Heads}) = \frac{2\rho}{1-\rho}$