1) ~~A~~ Flip Prob $= P(F) = \delta$

recieved correctly $= 1 - P(F) = 1 - \delta$

$P(\text{selecting } 0) = P(0) = p$

$P(\text{Selected bit being } 1) = 1 - P(0) = P(1) = 1 - p$

A) ~~P(0)?~~ There are 4 possibilities

Sent	Recieved	
0	0	— T_1
0	1	
1	0	— T_2
1	1	

Here T_1 & T_2 are of interest

$P(0 \text{ at } K^{th} \text{ instance}) = P(\text{sent } 0 \text{ recieved } 0) + P(\text{sent } 1 \text{ recieved } 0)$

$= P(0) \times (1 - P(F)) + (1 - P(0))\, P(F)$

$= p(1 - \delta) + (1 - p)\delta$

B) here we need to find out.

$P(\text{decoding } 0 \mid \text{selected } 0)$

$$= \frac{P(\text{decoding } 0 \cap \text{selecting } 0)}{P(\text{selecting } 0)}$$

~~For Numerator~~ : There are finite no of ways to get 0 after decoding

~~A~~ For Numerator

1) all bit zeros $= P(\text{all zeros}) = {}^3C_3 (1-\delta)^3 = (1-\delta)^3$

2) 2 bit zeros, 1 bit one $= \delta(1-\delta)^2 \times {}^3C_2$

$= 3\delta(1-\delta)^2$

$$0 \xrightarrow{1} 000 \quad \begin{cases} 000 \rightarrow (1-\delta)^3 \\ 001 \rightarrow \delta(1-\delta)^2 \\ 010 \rightarrow \delta(1-\delta)^2 \\ 100 \rightarrow \delta(1-\delta)^2 \end{cases}$$

p → 0

$1-p$ → 1

So $P(\text{selecting } 0 \cap \text{decoding } 0) = p\left((1-\delta)^3 + 3\delta(1-\delta)^2\right)$

$P(\text{Selecting } 0) = p$

$$P(\text{decoding } 0 \mid \text{selected } 0) = \frac{\cancel{p}\left((1-\delta)^3 + 3\delta(1-\delta)^2\right)}{\cancel{p}}$$

$$= (1-\delta)^3 + 3\delta(1-\delta)^2$$

c) Probability (sending 0 and receiving 0)

$$= p \times 7/8$$

$P(\text{send}=0 \,\&\, \text{recieve}=1) = p \times 1/8$

$P(\text{send } 1 \,\&\, \text{recieve}=0) = (1-p) \times 1/8$

$P(\text{send } 1 \,\&\, \text{recive}=1) = (1-p) \times 7/8$

now Assume that Probability of getting a 0 from decoder is ~~ρ_0~~ ρ_0, getting 1 from decoder ρ_1 and getting $\perp$ from decoder $\rho_\perp$

now ~~Expectation~~ $(\rho_0 + \rho_1 + \rho_\perp = 1)$ as the decoder o/p only 1 of value

$P(\text{send } 0 \,\&\, \text{receive } 0) \times (\rho_0 \times 0 + \rho_1 \times 2 + \rho_\perp \times 1)$

1)

$$E(P_1) = \frac{7p}{8} \times (2\rho_1 + \rho_\perp)$$

2) $P(\text{send } 0, \text{receive } 1) * P(\rho_1 * 2 + \rho_0 \times 0 + \rho_\perp \times 1)$

$$E(P_2) = \frac{P}{8}(2\rho_1 + \rho_\perp)$$

3) $P(\text{send } 1, \text{receive } 0) * (\rho_1 \times 0 + \rho_0 \times 2 + \rho_\perp \times 1)$

$$E(P_3) = \frac{1-P}{8} \times (2\rho_0 + \rho_\perp)$$

4) $P(\text{send } 1, \text{receive } 1) * (\rho_1 \times 0 + \rho_0 \times 2 + \rho_\perp \times 1)$

$$E(P_4) = \frac{(1-P)7}{8} * (2\rho_0 + \rho_\perp)$$

$$E(\text{Penalty}) = \sum_{i=1}^{4} E(P_i) :$$

$$= \frac{7P}{8}(2\rho_1 + \rho_\perp) + \frac{P}{8}(2\rho_1 + \rho_\perp) + \frac{1-P}{8}(2\rho_0 + \rho_\perp) + \frac{7-7P}{8}(2\rho_0 + \rho_\perp)$$

$$= \frac{7P(\rho_1)}{4} + \frac{7P(\rho_\perp)}{8} + \frac{P\rho_1}{4} + \frac{P\rho_\perp}{8} + \frac{(1-P)(\rho_0)}{4} + \frac{1-P}{8}(\rho_\perp) + \frac{7-7P}{4}(\rho_0) + \frac{7-7P}{8}\rho_\perp$$

$$= \rho_1\left(\frac{2P}{\cancel{8}}\right) + \rho_0\left(\cancel{2P}\right) + \rho_\perp(1)$$

$2(1-P)$ $\quad 1.5 + \frac{1}{2}\rho_0 \frac{1}{2}$

$$E(\text{Penalty}) = 2\rho_1 P + 2\rho_0(1-P) + \rho_\perp(1)$$

now as we have the expression we need to minimize Expectation and between 2 events we can have 3 cases

$P > \frac{1}{2}$ $\quad P = \frac{1}{2}$ $\quad P < \frac{1}{2}$

1) $\left(P = \frac{1}{2}\right) \Rightarrow E(P) = \rho_1 + \rho_0 + \rho_\perp$

we cannot minimize any one to reduce penalty so all outcomes are likely.

2) $P > \frac{1}{2} \Rightarrow$ here ρ_1 will have a larger coefficient which decoder will minimize

In order to do that it will need to decrease f_1 which will be done by outputting 0 . .

3) Similarly for $p < \frac{1}{2}$ the opposite case will occur so decoder should output 1