

Time-Series trading strategy based on Factor stock selection

Yanzhong Huang, Qinqin Huang, Yongyi Tang, Haoyue Tang

Introduction

Most factor models used in trading strategies are cross-sectional, meaning they rely on the relative ranking of stocks at a specific time. It is assumed that this ranking will remain stable over time and that the factors used for ranking will predict future returns. However, this assumption may not hold true under all market conditions, and the predictability of these factors can change over time, as can market anomalies. This can result in a model that overfits historical data and fails to generalize to new data.

In this report, we will explore a rolling window approach to address this issue, we will selected significant factors in the rolling window and combine them with time-series components to create a more robust model. Then we will build a trading strategy accordingly. The goal is to create a trading strategy that is more robust to changes in market conditions and can adapt to new data.

To achieve this, we selected Fama-French 5 factors plus momentum as the base factors. Then we constructed anomalies based on 101 Formulaic Alphas [Zura Kakushadze, 2015]. Finally, we used a rolling window approach to select the most significant factors, anomalies (alphas) and combined them with time-series components to create a more robust model.

In this report, we will:

- Pre-modelling
 - Provide an overview of the data used in our analysis.
 - Analyze the time-series components of returns.
- Model construction
 - Discuss the trading strategy.
- Performance evaluation
 - Present the backtesting results.

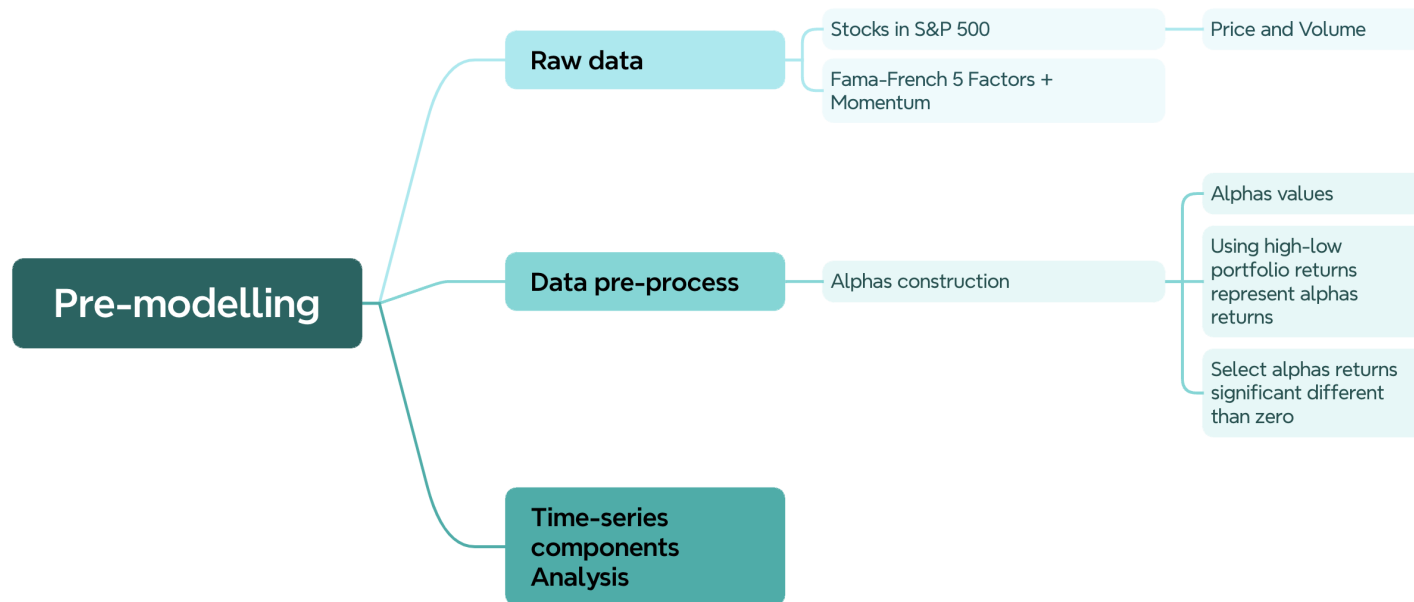


Figure 1: Pre-modeling

Data Overview

We selected the following data for our analysis:

- **Daily prices and volumes:** Daily prices and volumes for all stocks in the S&P 500 index.
- **Fama-French 5 factors plus momentum:** mktrf, smb, hml, rmw, cma, umd.
- **101 Formulaic Alphas:** We constructed these using daily prices and volumes.
- **Daily returns for 101 Formulaic Alphas:** We generated daily returns for the 101 Formulaic Alphas using high-low arbitrage portfolios.

The data spans from January 1, 2004, to December 31, 2024, covering 20 years. We believe this is a sufficient amount of data to capture market dynamics across various conditions.

Daily Prices and Volumes

The daily prices and volumes for all stocks in the S&P 500 index include:

- **Close, Open, High, Low:** Adjusted close, open, high, and low prices.
- **Vol:** Daily volume.
- **Pct_change:** Daily percentage change. For example, -0.83 indicates a -0.83% daily return.
- **VWAP:** Volume-weighted average price.

We will use this price data to construct the 101 Formulaic Alphas and conduct backtesting. Since we have a sufficient amount of data, we will not fill in any missing data. This approach utilizes a rolling window, which helps us avoid overfitting. For alpha construction, if a stock has missing data during the construction period, we will skip that stock.

Fama-French 5 Factors plus Momentum

The Fama-French 5 factors plus momentum daily factors returns were obtained from WRDS(Wharton Research Data Services). The factors include:

- **mktrf:** Market return minus risk-free rate.
- **smb:** Small minus big (size factor).
- **hml:** High minus low (value factor).
- **rmw:** Robust minus weak (profitability factor).
- **cma:** Conservative minus aggressive (investment factor).
- **umd:** Up minus down (momentum factor).

Alphas Construction

The 101 Formulaic Alphas paper [Zura Kakushadze, 2015] provides a comprehensive list of 101 real-life quantitative trading alphas. The original paper have a average holding period approximately ranges from 0.6 to 6.4 days. For our analysis, since we aim to construct a daily frequency trading strategy, we will hold the alphas for 1 day. However, the look back period for the alphas construction still varies based on the alphas construction method, details can be found in the paper.

We calculated alphas for all stocks in the S&P 500. A sample of the alpha values is shown below, including the alpha value for **AAPL** from January 3, 2005, to January 6, 2005:

trade_date	alpha001	alpha002	alpha003
2005-01-03	0.359342330355	-0.00060693612848	-0.22883143600105
2005-01-04	0.204808971678	-0.06933950066987	0.39975096612885
2005-01-05	0.065386808591	0.24332731404990	0.23698080798364
2005-01-06	0.817049990496	0.13108774439945	0.24530729110312

Then we sorted the alphas by their values on daily basis. Dividing the stocks into 5 groups, We used the arbitrage portfolio (long the top 20% and short the bottom 20%) to calculate the daily returns of each alpha. Sample of the daily returns for the alphas are calculated as follows:

trade_date	alpha001	alpha002	alpha003
2005-01-03	-0.001604	0.004097	0.003343
2005-01-04	-0.002053	-0.001746	0.003168
2005-01-05	0.000321	-0.001533	-0.001220
2005-01-06	-0.002266	0.003295	-0.001950

Overall Significant Alphas Selection In our backtesting period, 2004-2024, we first selected the alphas daily return is significantly different from 0. We used a t-test to determine the significance of the alphas. At the 5% significance level, 29 alphas were found to be significant. The t-test results are shown below:

Alpha	t-stat	Alpha	t-stat	Alpha	t-stat
alpha003	-3.989	alpha060	-4.593	alpha006	-4.558
alpha051	-9.596	alpha025	-11.888	alpha020	-5.886

Alpha	t-stat	Alpha	t-stat	Alpha	t-stat
alpha049	-10.508	alpha028	-4.890	alpha101	9.541
alpha047	-3.564	alpha018	-7.907	alpha040	-2.907
alpha046	-8.055	alpha017	-11.038	alpha005	-2.740
alpha042	-5.334	alpha014	-13.343	alpha016	-2.534
alpha038	-10.072	alpha013	-4.074	alpha053	2.530
alpha035	4.996	alpha012	-4.152	alpha008	-2.374
alpha034	-12.283	alpha010	-3.947	alpha045	1.983
alpha033	-14.349	alpha009	-5.562		

The absolute correlation between the significant alphas daily returns is relatively low, the average absolute correlation is 0.15. The maximum absolute correlation is 0.71, which is between alpha009 and alpha010 (Due to their similar construction method).

Time-Series Components Analysis

Data Preparation for Time-Series Analysis

We started by reading multiple .csv files from the raw data folder. Each file corresponds to a stock and contains columns such as `trade_date`, `open`, `high`, `low`, `close`, `vol`, `pct_change`, and `vwap`. The preprocessing included:

- Parsing `trade_date` as datetime and setting it as the index.
- Converting numerical columns to appropriate types and handling errors.
- Sorting by date and filling missing values using forward and backward fill.
- Computing daily returns using:

$$\text{return}_t = \frac{\text{pct_change}_t}{100}$$

To mitigate the influence of extreme values, we clipped the returns at the 1st and 99th percentiles:

$$r_t^{\text{clipped}} = \begin{cases} q_{0.01}, & \text{if } r_t < q_{0.01} \\ r_t, & \text{if } q_{0.01} \leq r_t \leq q_{0.99} \\ q_{0.99}, & \text{if } r_t > q_{0.99} \end{cases}$$

After transformation, the new plot shows that all the returns are now clipped between -13% and 17%. This transformation reduces the impact of outliers and enhances the robustness of subsequent statistical analysis.

Time Series Analysis

In this section, we conduct a diagnostic analysis of the return series.

The primary tests are: - **Stationarity Tests**: To check if the return series are stationary. - **Ljung-Box Test**: To check for autocorrelation in the return series. - **ARCH LM Test** (Lagrange Multiplier Test): To check for ARCH effects, indicating volatility clustering.

Stationarity Tests

Unit Root Test To assess the stationarity of stock return series, we performed the Augmented Dickey-Fuller (ADF) and KPSS tests across all 496 stocks.

Augmented Dickey-Fuller (ADF) Test

- Null hypothesis (H_0): The series has a unit root (non-stationary)
- If p-value < 0.05 , we reject (H_0), suggesting stationarity.

KPSS Test

- Null hypothesis H_0 : The series is stationary
- If p-value > 0.05 , we fail to reject H_0 , suggesting stationarity.

Decision Rule A series is considered stationary only if:

- ADF test rejects H_0 ($p < 0.05$), and
- KPSS test fails to reject H_0 ($p > 0.05$)

This dual-criteria approach ensures robustness in stationarity conclusions.

The following summarizes the statistical properties of the test results:

Metric	ADF Statistic	ADF p-value	KPSS Statistic	KPSS p-value
Count	496	496	496	496
Mean	-26.32	2.29×10^{-10}	0.116	0.097
Min	-77.92	0.0	0.013	0.010
Max	-6.07	1.14×10^{-7}	1.012	0.100

Interpretation

- The **ADF test** statistic has a strongly negative mean of -26.32 and nearly all p-values are effectively 0, indicating strong evidence to reject the null hypothesis of a unit root. This suggests the return series are stationary under the ADF test.
- The **KPSS test** statistic has a mean of 0.116, with the maximum value slightly above 1. Most of the p-values are exactly 0.1 (the upper bound reported by the test), suggesting that the null hypothesis of stationarity is not rejected in most cases.
- About 96.8% of the return series are confirmed to be stationary. These results confirm that the majority of stock return series in our dataset can be treated as stationary, validating their suitability for further time series modeling and statistical analysis.

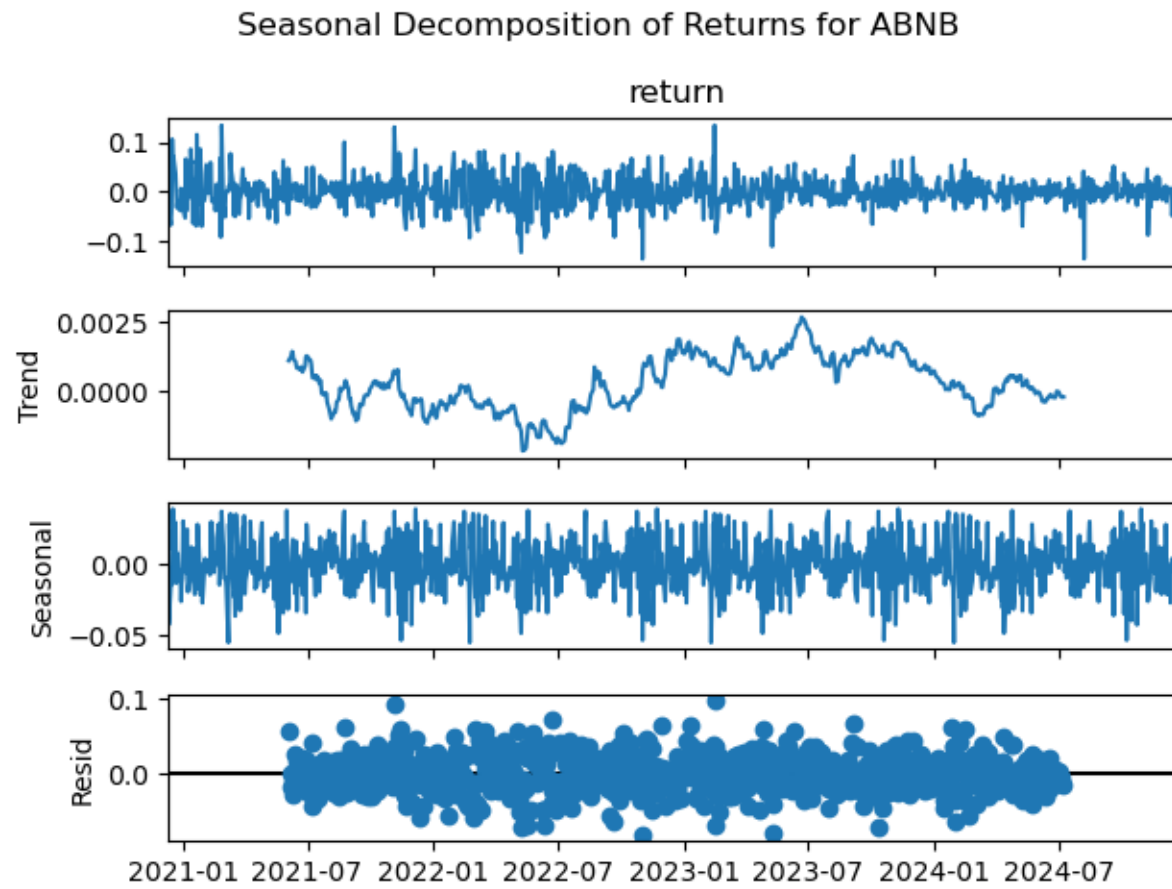


Figure 2: Seasonal Decomposition

Seasonal Decomposition

Our tests show that there is no seasonality and trend in stock return rates. Therefore, we do not consider seasonal decomposition in our models.

Autocorrelation and ARCH Effect Detection

The Ljung-Box test checks the null hypothesis:

$$H_0 : \text{The series is white noise (no autocorrelation up to lag } k \text{).}$$

The test statistic accumulates the squared autocorrelation coefficients up to lag (k). A significant p-value (typically $p < 0.05$) suggests that the series exhibits autocorrelation.

Since we conduct GARCH model on the residuals of the ARIMAX model, we should also check if there is ARCH effect in the residuals. We simply apply the LM test on the residuals of the AR(1) model to check if there is autocorrelation after removing the one lag autocorrelation.

The ARCH LM Test evaluates whether residual variance shows autocorrelation, suggesting the presence of volatility clustering. If ARCH effects are detected, models like GARCH are recommended to better capture the dynamic behavior of volatility. The LM test checks the null hypothesis:

$$H_0 : \text{No ARCH effect (the squared residuals are not autocorrelated, variance is constant over time).}$$

p-value	Conclusion
$p\text{-value} < 0.05$	Reject H_0 : ARCH effects detected (volatility clustering exists).
$p\text{-value} \leq 0.05$	Fail to reject H_0 : No significant ARCH effects, variance can be considered constant.

We randomly selected 5 stocks from the dataset to illustrate the results of the Ljung-Box test and LM test. In the table below, the last two columns indicate whether autocorrelation and ARCH effects exist based on the p-values (5% significance level). Four out of five stocks show evidence of autocorrelation and ARCH effects.

symbol	box_stat	box_pvalue	LM_stats	LM_pvalue	Exist_Arch	Exist_Autocorr
CSCO	67.4781	0.0005	321.0422	0.0000	True	True
UAL	103.5778	0.0000	674.0067	0.0000	True	True
TROW	97.5386	0.0000	107.2964	0.0000	True	True
ISRG	20.5516	0.4239	0.8469	1.0000	False	False
NVR	52.5150	0.0001	1092.4693	0.0000	True	True

Model Construction

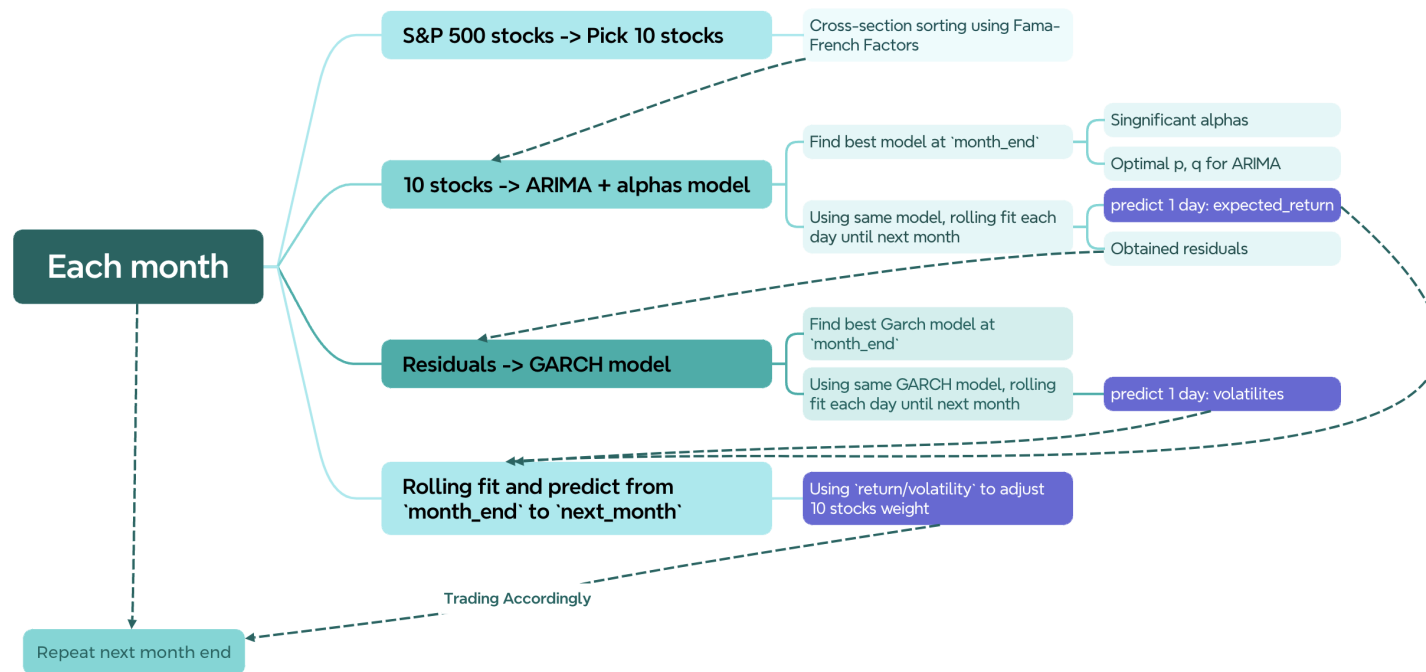


Figure 3: Model

The general methodology begins with using a cross-sectional factor model to select the top n stocks based on their exposure to the chosen factors. This selection will occur monthly, as we are employing a fundamental factor model.

Next, we will use the selected stocks to create a time-series model. This model will be updated monthly, but its fit will be refreshed daily. The time-series model will predict the daily returns and volatility of the selected stocks. Specifically, we will use an ARIMAX model to forecast daily returns and a GARCH model to estimate daily volatility based on the residuals. The general methodology is first using cross-sectional factor model to pick top n stocks based on the exposure of the selected factors. The cross-sectional picking will be done on a monthly basis, since we are using a fundamental factor model.

Factor Model Pick Top n Stocks Monthly

We used the Fama-French five factors plus momentum as our base factors. The model is as follows:

$$r_i - r_f = \beta_1 \cdot mktrf + \beta_2 \cdot smb + \beta_3 \cdot hml + \beta_4 \cdot rmw + \beta_5 \cdot cma + \beta_6 \cdot umd + \epsilon$$

Where:

- r_i is the daily return of stock i .
- r_f is the risk-free rate.
- $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6$ are the factor loadings for the Fama-French 5 factors plus momentum.
- ϵ is the error term.

Then we used the z-score approach to select the top n stocks. The z-score is calculated cross-sectionally for each stock based on its factor loadings.

$$z_i = \frac{\beta_i - \mu}{\sigma}$$

Where:

- z_i is the z-score for stock i .
- β_i is the factor loading for stock i .
- μ is the mean of the factor loadings for all stocks.
- σ is the standard deviation of the factor loadings for all stocks.

Finally use the average z-scores to select the top (n) stocks. The selection will be based on their average z-scores across all factors. In backtesting, we tested different lookback periods for the z-scores, including 3, 6, 9, 12, 18, and 24 months. The results indicate that the 9-month lookback period performs best, according to the information coefficient (IC):

$$IC = \text{corr}(z_{it}, Ri_{t+1})$$

Time-Series ARMAX Model

Each month, we employ a systematic approach to construct a time-series ARMA(p, q) model using our selected stocks. At the conclusion of every month, denoted as time t , we harness the daily returns of these stocks along with the alpha values from the preceding three months to identify the most suitable ARMAX(p, q) model.

In defining what constitutes an “appropriate” model, we adhere to the following criteria:

- The model incorporates only those alpha values that are statistically significant, specifically those with a p-value less than 0.05.
- It achieves the lowest Akaike Information Criterion (AIC) value, with the parameters p and q ranging from 1 to 4.

Once the model is established at time t , it does not remain static. Throughout the following month, we fit the model on a daily basis, continuously updating the parameters to reflect the latest data and maintain the model’s accuracy until the next stock selection period.

The ARIMAX model, once established at time t , serves several critical forecasting functions:

- It predicts the expected return $E(R_{t+1})$ for the subsequent trading day, providing a valuable forward-looking insight into potential market movements.
- By employing a rolling model approach, it generates a series of expected returns that extend up to the next stock selection period, offering a continuous stream of predictive insights.
- It also produces a residual series for the lookback period, which is instrumental in fitting the GARCH model. This residual series is crucial for estimating the volatility, thereby enhancing our ability to manage risk effectively.

Time-Series GARCH Model

Our trading strategy is primarily centered around leveraging the expected returns predicted by the ARIMAX model. While this model provides valuable insights into potential returns, it is crucial to also account for the inherent volatility in these returns to make informed trading decisions. To address this, we incorporate a GARCH model to estimate and manage the volatility of the residuals derived from the ARIMAX model.

The GARCH model operates on a similar foundational principle as the ARIMAX model. At the end of each month, we calibrate the GARCH model using the latest available data. This involves fitting the model daily with the newest ARIMAX residuals, allowing us to continuously update and refine the model parameters. This dynamic fitting process ensures that our volatility estimates remain responsive to the latest market conditions.

Once calibrated, the GARCH model forecasts the volatility for the subsequent trading day. This forecast is crucial, as it provides us with a forward-looking measure of risk, enabling us to adjust our trading positions accordingly. By employing a rolling estimation approach, the model generates a series of expected volatility measures. These measures extend until the next stock selection period, providing a continuous stream of volatility predictions that inform our trading strategy.

This dual-model approach—utilizing ARIMAX for expected returns and GARCH for volatility—allows us to construct a more comprehensive and robust trading strategy. By integrating these models, we can better manage risk and optimize our trading decisions, ultimately aiming to enhance our portfolio's performance while mitigating potential downsides associated with market fluctuations.

Prediction Sample

The following table shows the predicted returns and volatility for the selected stocks on 2024-10-31. The prediction period would be 2024-11-01 to 2024-11-29. The predicted returns are based on the ARIMAX model, while the predicted volatility is derived from the GARCH model.

Selected 10 stocks using Fama-French 5 factors plus momentum:

Symbol	Name	Sector	Industry
NVDA	NVIDIA Corporation	Information Technology	Semiconductors
AVGO	Broadcom Inc.	Information Technology	Semiconductors
IP	International Paper Company	Materials	Containers & Packaging
BLDR	Builders FirstSource, Inc.	Industrials	Building Products
QCOM	QUALCOMM Incorporated	Information Technology	Semiconductors
URI	United Rentals, Inc.	Industrials	Rental & Leasing Services
POOL	Pool Corporation	Industrials	Building Products
IVZ	Invesco Ltd.	Financials	Asset Management
KLAC	KLA Corporation	Information Technology	Semiconductors
PHM	PulteGroup, Inc.	Consumer Discretionary	Homebuilding

Expected returns prediction:

Date	NVDA	AVGO	IP	BLDR	QCOM	URI	POOL	IVZ	KLAC	PHM
2024-11-01	-0.0018	0.0037	0.0293	0.0171	0.0223	0.0006	-0.0088	-0.0042	0.0139	0.0012
2024-11-04	-0.0299	0.0022	-0.2409	-0.0132	0.0054	-0.0005	0.0004	-0.0100	0.0006	0.0003

Date	NVDA	AVGO	IP	BLDR	QCOM	URI	POOL	IVZ	KLAC	PHM
...
2024-11-27	-0.0122	0.0000	-0.0065	0.0108	-0.0025	0.0012	-0.0062	0.0073	0.0001	-0.0012
2024-11-29	0.0066	-0.0026	-0.0068	0.0097	-0.0046	0.0145	-0.0067	0.0136	-0.0059	-0.0001

Expected volatility prediction:

Date	NVDA	AVGO	IP	BLDR	QCOM	URI	POOL	IVZ	KLAC	PHM
2024-11-01	0.0268	0.0309	0.0685	0.0196	0.0235	0.0126	0.0147	0.0155	0.0369	0.0172
2024-11-04	0.0295	0.0281	0.1361	0.0194	0.0205	0.0148	0.0142	0.0162	0.0204	0.0169
...
2024-11-27	0.0281	0.0191	0.0132	0.0217	0.0204	0.0191	0.0217	0.0171	0.0318	0.0232
2024-11-29	0.0284	0.0306	0.0143	0.0215	0.0201	0.0191	0.0185	0.0165	0.0203	0.0231

Backtesting Results

Strategy Explanation

Our strategy proceeds as follows:

1. Monthly Stock Selection and Prediction

For each month τ , our model selects 10 stocks and forecasts for each trading day t within that month:

- Predicted return: $\hat{\mu}_{i,t}$
- Predicted volatility: $\hat{\sigma}_{i,t}$

2. Mean–Variance Ratio Calculation

On each day t , compute the mean–variance ratio for stock i :

$$\text{MVR}_{i,t} = \frac{\hat{\mu}_{i,t}}{\hat{\sigma}_{i,t}}.$$

3. Average MVR and Ranking

At the start of month τ , calculate the average daily MVR for each stock:

$$\overline{\text{MVR}}_{i,\tau} = \frac{1}{N_\tau} \sum_{t=1}^{N_\tau} \text{MVR}_{i,t},$$

where N_τ is the number of trading days in month τ .

Sort the 10 stocks in ascending order of $\overline{\text{MVR}}_{i,\tau}$, assign ranks $r_{i,\tau} \in \{1, \dots, 10\}$, then set weights by

$$w_{i,\tau} = \frac{r_{i,\tau}}{\sum_{j=1}^{10} r_{j,\tau}} = \frac{r_{i,\tau}}{55}.$$

4. Daily Portfolio Return

For each trading day t in month τ , the portfolio return is

$$R_{p,t} = \sum_{i=1}^{10} w_{i,\tau} R_{i,t},$$

where $R_{i,t}$ is the actual return of stock i on day t .

We repeat steps 1–4 for every month in the backtesting period to generate the full series of daily portfolio returns.

Backtesting and Benchmark Comparison

Our back-testing procedure runs the strategy side-by-side with a given benchmark index. For each month τ :

- We compute daily portfolio returns $R_{p,t}$ as described above.
- In parallel, we record the benchmark's daily return $R_{b,t}$. Our benchmark is the equal-weighted portfolio of the S&P 500 stocks (excluded stocks with missing data)
- Both series start on the same first trading day and proceed through the last trading day of the backtest.
- We assume daily rebalancing the model portfolio and no rebalancing for the benchmark (i.e., a buy-and-hold of the index).

At the end, we aggregate the daily returns into performance metrics for both:

- Annualized Return:

$$\text{AnnRet} = (1 + \overline{R})^{252} - 1,$$

where \overline{R} is the average daily return.

- Annualized Volatility:

$$\text{AnnVol} = \sqrt{252} \sigma_R,$$

where σ_R is the standard deviation of daily returns.

- Sharpe Ratio:

$$\text{SR} = \frac{\text{AnnRet} - r_f}{\text{AnnVol}},$$

assuming risk-free rate r_f is zero.

- Maximum Drawdown: the largest peak-to-trough decline in cumulative return.

These metrics are computed separately for our strategy and the benchmark to facilitate a clear comparison.

Assumptions:

1. The transaction cost is 20 basis point.
2. Trades executed at daily closing prices.
3. Stocks are picked on the first trading day of each month and rebalanced daily.
4. Benchmark is buy-and-hold with no dividends reinvested.
5. Unlimited liquidity and zero market impact.

Backtesting Performance Summary

Performance without Transaction Cost When transaction costs are ignored, the strategy outperforms the benchmark.

The cumulative returns plot shows the strategy's growth to about 20 times the initial capital by the end of the period, compared to about 4.5 times for the benchmark. Without trading costs, the Sharpe ratio approaches 1.0, indicating almost unit risk-adjusted return, while volatility remains similar to the with-cost scenario. However, the deep maximum drawdown of nearly 60% signals significant downside during stress periods.

Performance with Transaction Cost (20 basis point)

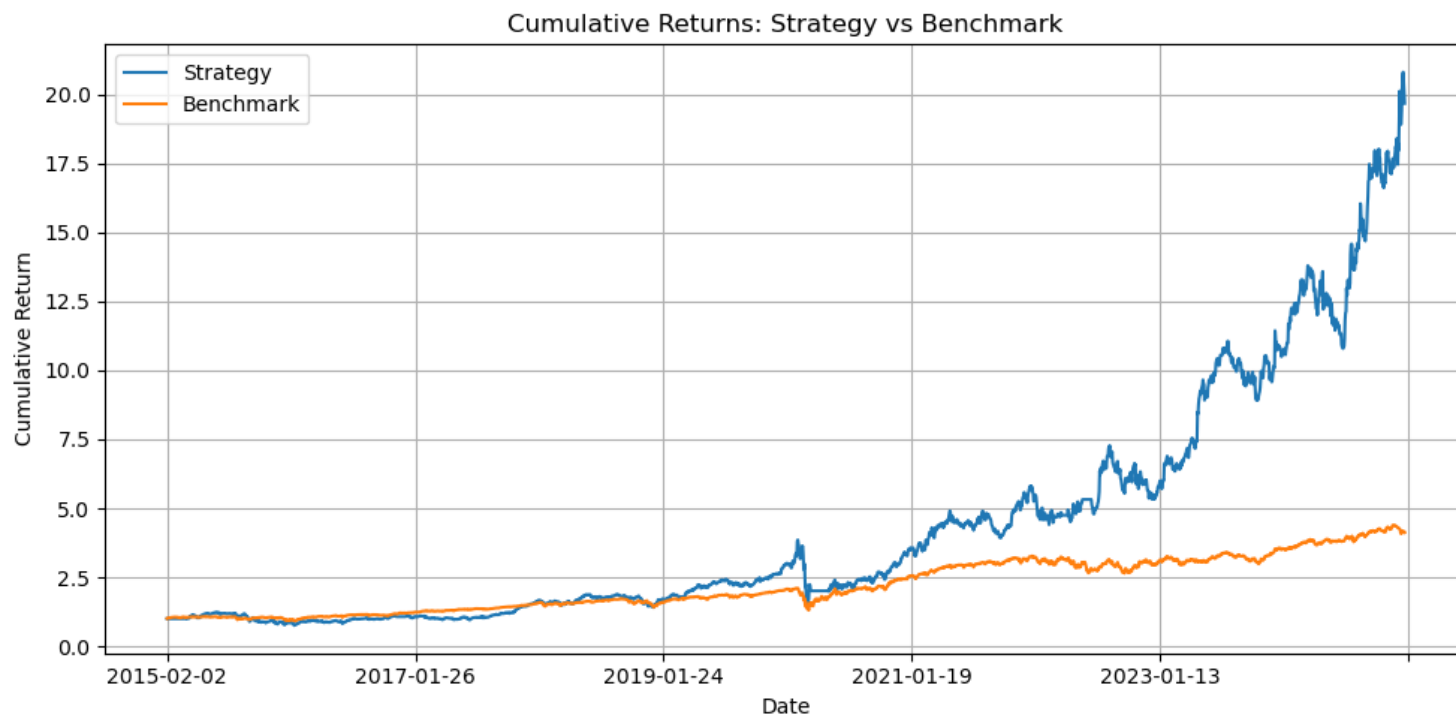


Figure 4: backtest_performance_no_cost

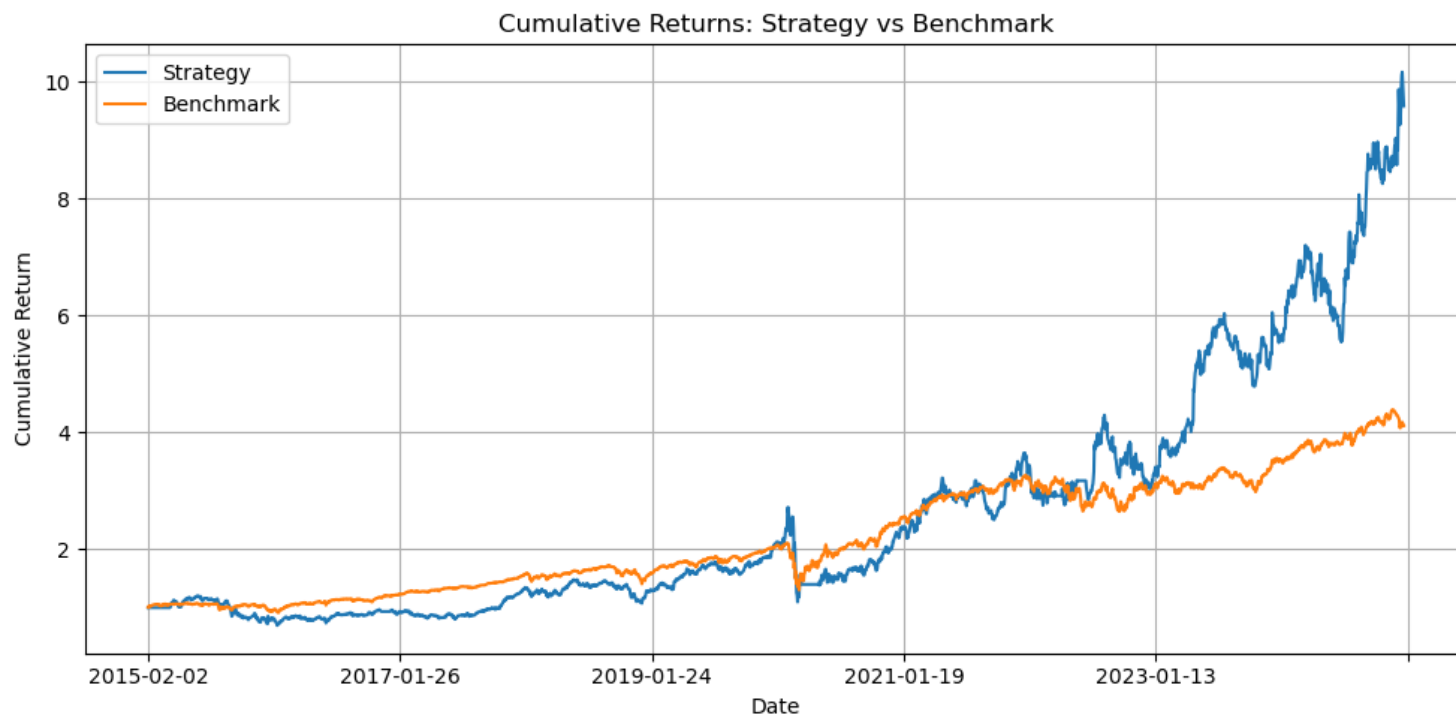


Figure 5: backtest_performance

Metric	Strategy	Benchmark
Annualized Return	25.67%	15.36%
Annualized Volatility	35.23%	18.22%
Sharpe Ratio	0.73	0.84
Maximum Drawdown	−59.64%	−38.15%

The cumulative returns chart shows that our strategy, after an early drawdown, generated a final cumulative return of over 10 times of the initial capital versus about 2.3 times for the benchmark. On an annualized basis, the strategy delivered 25.67% return compared to 15.36% for the benchmark, however with higher volatility (35.23% vs. 18.22%). This higher risk translated into a Sharpe ratio of 0.73—slightly below the benchmark’s 0.84—reflecting our model’s more aggressive positioning. The strategy also experienced a deeper maximum drawdown of −59.64% versus −38.15% for the benchmark, highlighting greater downside during stressed periods.

In summary, while the strategy outperformed in total and annualized return, it did so with substantially higher risk and drawdown, indicating the importance of further refinement in risk control.

Conclusion and Future Work

In this report, we presented a comprehensive analysis of the 101 Formulaic Alphas and their performance in a backtesting framework. We constructed alphas based on daily prices and volumes of S&P 500 stocks, and selected significant alphas using t-tests. The time-series analysis revealed that most stock return series are stationary, and we employed ARIMAX and GARCH models for predicting returns and volatility.

Major findings include:

- Time-series components did outperform the equal-weighted benchmark
- However, it incurred higher volatility and maximum drawdown, indicating a need for improved risk management.

We believe that the overall research structure is solid, we could further enhance the model by:

- Incorporating additional factors or alternative models to improve predictive power.
- Exploring different portfolio construction techniques to optimize risk-return profiles.
- Implementing advanced risk management strategies to mitigate drawdowns.
- Conducting further research on the impact of transaction costs and market conditions on strategy performance.
- Investigating the potential for machine learning techniques to enhance alpha generation and risk management.

Appendix A: Formulaic Expressions for Alphas

From the 101 Formulaic Alphas paper [Zura Kakushadze, 2015], we have the following formulaic expressions for the alphas:

Alpha#1: $\text{rank}(\text{Ts_ArgMax}(\text{SignedPower}(((\text{returns} < 0) ? \text{stddev}(\text{returns}, 20) : \text{close}), 2.), 5)) - 0.5)$

Alpha#2: $(-1 * \text{correlation}(\text{rank}(\text{delta}(\log(\text{volume}), 2)), \text{rank}(((\text{close} - \text{open}) / \text{open})), 6))$

Alpha#3: $(-1 * \text{correlation}(\text{rank}(\text{open}), \text{rank}(\text{volume}), 10))$

Alpha#4: $(-1 * \text{Ts_Rank}(\text{rank}(\text{low}), 9))$

Alpha#5: $(\text{rank}((\text{open} - (\text{sum}(\text{vwap}, 10) / 10))) * (-1 * \text{abs}(\text{rank}((\text{close} - \text{vwap}))))))$

Alpha#6: $(-1 * \text{correlation}(\text{open}, \text{volume}, 10))$

Alpha#7: $((\text{adv20} < \text{volume}) ? ((-1 * \text{ts_rank}(\text{abs}(\text{delta}(\text{close}, 7)), 60)) * \text{sign}(\text{delta}(\text{close}, 7))) : (-1 * 1))$

Alpha#8: $(-1 * \text{rank}(((\text{sum}(\text{open}, 5) * \text{sum}(\text{returns}, 5)) - \text{delay}((\text{sum}(\text{open}, 5) * \text{sum}(\text{returns}, 5)), 10))))$

Alpha#9: $((0 < \text{ts_min}(\text{delta}(\text{close}, 1), 5)) ? \text{delta}(\text{close}, 1) : ((\text{ts_max}(\text{delta}(\text{close}, 1), 5) < 0) ? \text{delta}(\text{close}, 1) : (-1 * \text{delta}(\text{close}, 1))))$

Alpha#10: $\text{rank}(((0 < \text{ts_min}(\text{delta}(\text{close}, 1), 4)) ? \text{delta}(\text{close}, 1) : ((\text{ts_max}(\text{delta}(\text{close}, 1), 4) < 0) ? \text{delta}(\text{close}, 1) : (-1 * \text{delta}(\text{close}, 1))))))$

Alpha#11: $((\text{rank}(\text{ts_max}((\text{vwap} - \text{close}), 3)) + \text{rank}(\text{ts_min}((\text{vwap} - \text{close}), 3))) * \text{rank}(\text{delta}(\text{volume}, 3)))$

Alpha#12: $(\text{sign}(\text{delta}(\text{volume}, 1)) * (-1 * \text{delta}(\text{close}, 1)))$

Alpha#13: $(-1 * \text{rank}(\text{covariance}(\text{rank}(\text{close}), \text{rank}(\text{volume}), 5)))$

Alpha#14: $((-1 * \text{rank}(\text{delta}(\text{returns}, 3))) * \text{correlation}(\text{open}, \text{volume}, 10))$

Alpha#15: $(-1 * \text{sum}(\text{rank}(\text{correlation}(\text{rank}(\text{high}), \text{rank}(\text{volume}), 3)), 3))$

Alpha#16: $(-1 * \text{rank}(\text{covariance}(\text{rank}(\text{high}), \text{rank}(\text{volume}), 5)))$

Alpha#17: $(((-1 * \text{rank}(\text{ts_rank}(\text{close}, 10))) * \text{rank}(\text{delta}(\text{delta}(\text{close}, 1), 1))) * \text{rank}(\text{ts_rank}((\text{volume} / \text{adv20}), 5)))$

Alpha#18: $(-1 * \text{rank}(((\text{stddev}(\text{abs}((\text{close} - \text{open})), 5) + (\text{close} - \text{open})) + \text{correlation}(\text{close}, \text{open}, 10))))$

Alpha#19: $((-1 * \text{sign}(((\text{close} - \text{delay}(\text{close}, 7)) + \text{delta}(\text{close}, 7)))) * (1 + \text{rank}((1 + \text{sum}(\text{returns}, 250)))))$

Alpha#20: $(((-1 * \text{rank}((\text{open} - \text{delay}(\text{high}, 1)))) * \text{rank}((\text{open} - \text{delay}(\text{close}, 1)))) * \text{rank}((\text{open} - \text{delay}(\text{low}, 1))))$

Alpha#21: (((sum(close, 8) / 8) + stddev(close, 8)) < (sum(close, 2) / 2)) ? (-1 * 1) : (((sum(close, 2) / 2) < ((sum(close, 8) / 8) - stddev(close, 8))) ? 1 : (((1 < (volume / adv20)) || ((volume / adv20) == 1)) ? 1 : (-1 * 1))))

Alpha#22: (-1 * (delta(correlation(high, volume, 5), 5) * rank(stddev(close, 20))))

Alpha#23: (((sum(high, 20) / 20) < high) ? (-1 * delta(high, 2)) : 0)

Alpha#24: (((delta((sum(close, 100) / 100), 100) / delay(close, 100)) < 0.05) || ((delta((sum(close, 100) / 100), 100) / delay(close, 100)) == 0.05)) ? (-1 * (close - ts_min(close, 100))) : (-1 * delta(close, 3)))

Alpha#25: rank(((((-1 * returns) * adv20) * vwap) * (high - close)))

Alpha#26: (-1 * ts_max(correlation(ts_rank(volume, 5), ts_rank(high, 5), 5), 3))

Alpha#27: ((0.5 < rank((sum(correlation(rank(volume), rank(vwap), 6), 2) / 2.0))) ? (-1 * 1) : 1)

Alpha#28: scale(((correlation(adv20, low, 5) + ((high + low) / 2)) - close))

Alpha#29: (min(product(rank(rank(scale(log(sum(ts_min(rank(rank((-1 * rank(delta((close - 1), 5))))), 2), 1))))), 1), 5) + ts_rank(delay((-1 * returns), 6), 5))

Alpha#30: (((1.0 - rank(((sign((close - delay(close, 1))) + sign((delay(close, 1) - delay(close, 2)))) + sign((delay(close, 2) - delay(close, 3)))))) * sum(volume, 5)) / sum(volume, 20))

Alpha#31: ((rank(rank(rank(decay_linear((-1 * rank(rank(delta(close, 10))))), 10))) + rank((-1 * delta(close, 3))) + sign(scale(correlation(adv20, low, 12))))

Alpha#32: (scale(((sum(close, 7) / 7) - close)) + (20 * scale(correlation(vwap, delay(close, 5), 230))))

Alpha#33: rank((-1 * ((1 - (open / close))^1)))

Alpha#34: rank(((1 - rank((stddev(returns, 2) / stddev(returns, 5)))) + (1 - rank(delta(close, 1)))))

Alpha#35: ((Ts_Rank(volume, 32) * (1 - Ts_Rank(((close + high) - low), 16))) * (1 - Ts_Rank(returns, 32)))

Alpha#36: (((((2.21 * rank(correlation((close - open), delay(volume, 1), 15))) + (0.7 * rank((open - close)))) + (0.73 * rank(Ts_Rank(delay((-1 * returns), 6), 5)))) + rank(abs(correlation(vwap, adv20, 6)))) + (0.6 * rank((((sum(close, 200) / 200) - open) * (close - open)))))

Alpha#37: (rank(correlation(delay((open - close), 1), close, 200)) + rank((open - close)))

Alpha#38: ((-1 * rank(Ts_Rank(close, 10))) * rank((close / open)))

Alpha#39: $((-1 * \text{rank}((\text{delta}(\text{close}, 7) * (1 - \text{rank}(\text{decay_linear}((\text{volume} / \text{adv20}), 9)))))) * (1 + \text{rank}(\text{sum}(\text{returns}, 250))))$

Alpha#40: $((-1 * \text{rank}(\text{stddev}(\text{high}, 10))) * \text{correlation}(\text{high}, \text{volume}, 10))$

Alpha#41: $((\text{high} * \text{low})^{0.5} - \text{vwap})$

Alpha#42: $(\text{rank}((\text{vwap} - \text{close})) / \text{rank}((\text{vwap} + \text{close})))$

Alpha#43: $(\text{ts_rank}((\text{volume} / \text{adv20}), 20) * \text{ts_rank}((-1 * \text{delta}(\text{close}, 7)), 8))$

Alpha#44: $(-1 * \text{correlation}(\text{high}, \text{rank}(\text{volume}), 5))$

Alpha#45: $(-1 * ((\text{rank}((\text{sum}(\text{delay}(\text{close}, 5), 20) / 20)) * \text{correlation}(\text{close}, \text{volume}, 2)) * \text{rank}(\text{correlation}(\text{sum}(\text{close}, 5), \text{sum}(\text{close}, 20), 2))))$

Alpha#46: $((0.25 < (((\text{delay}(\text{close}, 20) - \text{delay}(\text{close}, 10)) / 10) - ((\text{delay}(\text{close}, 10) - \text{close}) / 10))) ? (-1 * 1) : (((((\text{delay}(\text{close}, 20) - \text{delay}(\text{close}, 10)) / 10) - ((\text{delay}(\text{close}, 10) - \text{close}) / 10)) < 0) ? 1 : ((-1 * 1) * (\text{close} - \text{delay}(\text{close}, 1)))))$

Alpha#47: $(((((\text{rank}((1 / \text{close})) * \text{volume}) / \text{adv20}) * ((\text{high} * \text{rank}((\text{high} - \text{close}))) / (\text{sum}(\text{high}, 5) / 5))) - \text{rank}((\text{vwap} - \text{delay}(\text{vwap}, 5))))$

Alpha#48: $(\text{indneutralize}(((\text{correlation}(\text{delta}(\text{close}, 1), \text{delta}(\text{delay}(\text{close}, 1), 1), 250) * \text{delta}(\text{close}, 1)) / \text{close}), \text{Ind-Class.subindustry}) / \text{sum}(((\text{delta}(\text{close}, 1) / \text{delay}(\text{close}, 1))^2, 250))$

Alpha#49: $(((((\text{delay}(\text{close}, 20) - \text{delay}(\text{close}, 10)) / 10) - ((\text{delay}(\text{close}, 10) - \text{close}) / 10)) < (-1 * 0.1)) ? 1 : ((-1 * 1) * (\text{close} - \text{delay}(\text{close}, 1))))$

Alpha#50: $(-1 * \text{ts_max}(\text{rank}(\text{correlation}(\text{rank}(\text{volume}), \text{rank}(\text{vwap}), 5)), 5))$

Alpha#51: $(((((\text{delay}(\text{close}, 20) - \text{delay}(\text{close}, 10)) / 10) - ((\text{delay}(\text{close}, 10) - \text{close}) / 10)) < (-1 * 0.05)) ? 1 : ((-1 * 1) * (\text{close} - \text{delay}(\text{close}, 1))))$

Alpha#52: $((((-1 * \text{ts_min}(\text{low}, 5)) + \text{delay}(\text{ts_min}(\text{low}, 5), 5)) * \text{rank}(((\text{sum}(\text{returns}, 240) \text{ sum}(\text{returns}, 20)) / 220))) * \text{ts_rank}(\text{volume}, 5))$

Alpha#53: $(-1 * \text{delta}((((\text{close} - \text{low}) - (\text{high} - \text{close})) / (\text{close} - \text{low})), 9))$

Alpha#54: $((-1 * ((\text{low} - \text{close}) * (\text{open}^5))) / ((\text{low} - \text{high}) * (\text{close}^5)))$

Alpha#55: $(-1 * \text{correlation}(\text{rank}(((\text{close} - \text{ts_min}(\text{low}, 12)) / (\text{ts_max}(\text{high}, 12) - \text{ts_min}(\text{low}, 12))))), \text{rank}(\text{volume}), 6))$

Alpha#56: $(0 - (1 * (\text{rank}((\text{sum}(\text{returns}, 10) / \text{sum}(\text{sum}(\text{returns}, 2), 3))) * \text{rank}((\text{returns} * \text{cap}))))$

Alpha#57: $(0 - (1 * ((\text{close} - \text{vwap}) / \text{decay_linear}(\text{rank}(\text{ts_argmax}(\text{close}, 30)), 2))))$

Alpha#58: $(-1 * Ts_Rank(decay_linear(correlation(IndNeutralize(vwap, IndClass.sector), volume, 3.92795), 7.89291), 5.50322))$

Alpha#59: $(-1 * Ts_Rank(decay_linear(correlation(IndNeutralize(((vwap * 0.728317) + (vwap * (1 - 0.728317))), IndClass.industry), volume, 4.25197), 16.2289), 8.19648))$

Alpha#60: $(0 - (1 * ((2 * scale(rank((((close - low) - (high - close)) / (high - low)) * volume)))) scale(rank(ts_argmax(close, 10))))))$

Alpha#61: $(rank((vwap - ts_min(vwap, 16.1219))) < rank(correlation(vwap, adv180, 17.9282)))$

Alpha#62: $((rank(correlation(vwap, sum(adv20, 22.4101), 9.91009)) < rank(((rank(open) + rank(open)) < (rank(((high + low) / 2)) + rank(high)))) * -1)$

Alpha#63: $((rank(decay_linear(delta(IndNeutralize(close, IndClass.industry), 2.25164), 8.22237)) - rank(decay_linear(correlation(((vwap * 0.318108) + (open * (1 - 0.318108))), sum(adv180, 37.2467), 13.557), 12.2883))) * -1)$

Alpha#64: $((rank(correlation(sum(((open * 0.178404) + (low * (1 - 0.178404))), 12.7054), sum(adv120, 12.7054), 16.6208)) < rank(delta((((high + low) / 2) * 0.178404) + (vwap * (1 - 0.178404))), 3.69741))) * -1)$

Alpha#65: $((rank(correlation(((open * 0.00817205) + (vwap * (1 - 0.00817205))), sum(adv60, 8.6911), 6.40374)) < rank((open - ts_min(open, 13.635)))) * -1)$

Alpha#66: $((rank(decay_linear(delta(vwap, 3.51013), 7.23052)) + Ts_Rank(decay_linear((((low * 0.96633) + (low * (1 - 0.96633))) - vwap) / (open - ((high + low) / 2))), 11.4157), 6.72611)) * -1)$

Alpha#67: $((rank((high - ts_min(high, 2.14593)))^rank(correlation(IndNeutralize(vwap, IndClass.sector), IndNeutralize(adv20, IndClass.subindustry), 6.02936))) * -1)$

Alpha#68: $((Ts_Rank(correlation(rank(high), rank(adv15), 8.91644), 13.9333) < rank(delta(((close * 0.518371) + (low * (1 - 0.518371))), 1.06157))) * -1)$

Alpha#69: $((rank(ts_max(delta(IndNeutralize(vwap, IndClass.industry), 2.72412), 4.79344))^Ts_Rank(correlation(((close * 0.490655) + (vwap * (1 - 0.490655))), adv20, 4.92416), 9.0615)) * -1)$

Alpha#70: $((rank(delta(vwap, 1.29456))^Ts_Rank(correlation(IndNeutralize(close, IndClass.industry), adv50, 17.8256), 17.9171)) * -1)$

Alpha#71: $max(Ts_Rank(decay_linear(correlation(Ts_Rank(close, 3.43976), Ts_Rank(adv180, 12.0647), 18.0175), 4.20501), 15.6948), Ts_Rank(decay_linear((rank(((low + open) - (vwap + vwap)))^2), 16.4662), 4.4388))$

Alpha#72: $(rank(decay_linear(correlation(((high + low) / 2), adv40, 8.93345), 10.1519)) / rank(decay_linear(correlation(Ts_Rank(vwap, 3.72469), Ts_Rank(volume, 18.5188), 6.86671), 2.95011)))$

Alpha#73: (max(rank(decay_linear(delta(vwap, 4.72775), 2.91864)), Ts_Rank(decay_linear(((delta(((open * 0.147155) + (low * (1 - 0.147155))))), 2.03608) / ((open * 0.147155) + (low * (1 - 0.147155)))) * -1), 3.33829), 16.7411)) * -1)

Alpha#74: ((rank(correlation(close, sum(adv30, 37.4843), 15.1365)) < rank(correlation(rank(((high * 0.0261661) + (vwap * (1 - 0.0261661))))), rank(volume), 11.4791))) * -1)

Alpha#75: (rank(correlation(vwap, volume, 4.24304)) < rank(correlation(rank(low), rank(adv50), 12.4413)))

Alpha#76: (max(rank(decay_linear(delta(vwap, 1.24383), 11.8259)), Ts_Rank(decay_linear(Ts_Rank(correlation(IndNeutralize(low, IndClass.sector), adv81, 8.14941), 19.569), 17.1543), 19.383)) * -1)

Alpha#77: min(rank(decay_linear((((high + low) / 2) + high) - (vwap + high)), 20.0451), rank(decay_linear(correlation(((high + low) / 2), adv40, 3.1614), 5.64125)))

Alpha#78: (rank(correlation(sum(((low * 0.352233) + (vwap * (1 - 0.352233))), 19.7428), sum(adv40, 19.7428), 6.83313))^rank(correlation(rank(vwap), rank(volume), 5.77492)))

Alpha#79: (rank(delta(IndNeutralize(((close * 0.60733) + (open * (1 - 0.60733))), IndClass.sector), 1.23438)) < rank(correlation(Ts_Rank(vwap, 3.60973), Ts_Rank(adv150, 9.18637), 14.6644)))

Alpha#80: ((rank(Sign(delta(IndNeutralize(((open * 0.868128) + (high * (1 - 0.868128))), IndClass.industry), 4.04545)))^Ts_Rank(correlation(high, adv10, 5.11456), 5.53756)) * -1)

Alpha#81: ((rank(Log(product(rank((rank(correlation(vwap, sum(adv10, 49.6054), 8.47743))^4), 14.9655))) < rank(correlation(rank(vwap), rank(volume), 5.07914))) * -1)

Alpha#82: (min(rank(decay_linear(delta(open, 1.46063), 14.8717)), Ts_Rank(decay_linear(correlation(IndNeutralize(volume, IndClass.sector), ((open * 0.634196) + (open * (1 - 0.634196))), 17.4842), 6.92131), 13.4283)) * -1)

Alpha#83: ((rank(delay(((high - low) / (sum(close, 5) / 5)), 2)) * rank(rank(volume))) / (((high low) / (sum(close, 5) / 5)) / (vwap - close)))

Alpha#84: SignedPower(Ts_Rank((vwap - ts_max(vwap, 15.3217)), 20.7127), delta(close, 4.96796))

Alpha#85: (rank(correlation(((high * 0.876703) + (close * (1 - 0.876703))), adv30, 9.61331))^rank(correlation(Ts_Rank(((high + low) / 2), 3.70596), Ts_Rank(volume, 10.1595), 7.11408)))

Alpha#86: ((Ts_Rank(correlation(close, sum(adv20, 14.7444), 6.00049), 20.4195) < rank(((open + close) - (vwap + open)))) * -1)

Alpha#87: (max(rank(decay_linear(delta(((close * 0.369701) + (vwap * (1 - 0.369701))), 1.91233), 2.65461)), Ts_Rank(decay_linear(abs(correlation(IndNeutralize(IndClass.industry), close, 13.4132)), 4.89768), 14.4535)) * -1)

Alpha#88: $\min(\text{rank}(\text{decay_linear}(((\text{rank}(\text{open}) + \text{rank}(\text{low})) - (\text{rank}(\text{high}) + \text{rank}(\text{close})))), 8.06882)), \text{Ts_Rank}(\text{decay_linear}(\text{correlation}(\text{Ts_Rank}(\text{close}, 8.44728), \text{Ts_Rank}(\text{adv60}, 20.6966), 8.01266), 6.65053), 2.61957))$

Alpha#89: $(\text{Ts_Rank}(\text{decay_linear}(\text{correlation}(((\text{low} * 0.967285) + (\text{low} * (1 - 0.967285)))), \text{adv10}, 6.94279), 5.51607), 3.79744) - \text{Ts_Rank}(\text{decay_linear}(\text{delta}(\text{IndNeutralize}(\text{vwap}, \text{IndClass.industry}), 3.48158), 10.1466), 15.3012))$

Alpha#90: $((\text{rank}((\text{close} - \text{ts_max}(\text{close}, 4.66719))))^{\text{Ts_Rank}(\text{correlation}(\text{IndNeutralize}(\text{adv40}, \text{IndClass.subindustry}), \text{low}, 5.38375), 3.21856)}) * -1)$

Alpha#91: $((\text{Ts_Rank}(\text{decay_linear}(\text{decay_linear}(\text{correlation}(\text{IndNeutralize}(\text{close}, \text{IndClass.industry}), \text{volume}, 9.74928), 16.398), 3.83219), 4.8667) \text{rank}(\text{decay_linear}(\text{correlation}(\text{vwap}, \text{adv30}, 4.01303), 2.6809))) * -1)$

Alpha#92: $\min(\text{Ts_Rank}(\text{decay_linear}((((\text{high} + \text{low}) / 2) + \text{close}) < (\text{low} + \text{open})), 14.7221), 18.8683), \text{Ts_Rank}(\text{decay_linear}(\text{correlation}(\text{rank}(\text{low}), \text{rank}(\text{adv30}), 7.58555), 6.94024), 6.80584))$

Alpha#93: $(\text{Ts_Rank}(\text{decay_linear}(\text{correlation}(\text{IndNeutralize}(\text{vwap}, \text{IndClass.industry}), \text{adv81}, 17.4193), 19.848), 7.54455) / \text{rank}(\text{decay_linear}(\text{delta}(((\text{close} * 0.524434) + (\text{vwap} * (1 - 0.524434)))), 2.77377), 16.2664)))$

Alpha#94: $((\text{rank}((\text{vwap} - \text{ts_min}(\text{vwap}, 11.5783))))^{\text{Ts_Rank}(\text{correlation}(\text{Ts_Rank}(\text{vwap}, 19.6462), \text{Ts_Rank}(\text{adv60}, 4.02992), 18.0926), 2.70756)}) * -1)$

Alpha#95: $(\text{rank}((\text{open} - \text{ts_min}(\text{open}, 12.4105))) < \text{Ts_Rank}((\text{rank}(\text{correlation}(\text{sum}(((\text{high} + \text{low}) / 2), 19.1351), \text{sum}(\text{adv40}, 19.1351), 12.8742))^5), 11.7584))$

Alpha#96: $(\max(\text{Ts_Rank}(\text{decay_linear}(\text{correlation}(\text{rank}(\text{vwap}), \text{rank}(\text{volume}), 3.83878), 4.16783), 8.38151), \text{Ts_Rank}(\text{decay_linear}(\text{Ts_ArgMax}(\text{correlation}(\text{rank}(\text{open}), \text{rank}(\text{adv60}), 4.13242), 3.65459), 12.6556), 14.0365), 13.4143)) * -1)$

Alpha#97: $((\text{rank}(\text{decay_linear}(\text{delta}(\text{IndNeutralize}(((\text{low} * 0.721001) + (\text{vwap} * (1 - 0.721001)))), \text{IndClass.industry}), 3.3705), 20.4523)) - \text{Ts_Rank}(\text{decay_linear}(\text{Ts_Rank}(\text{correlation}(\text{Ts_Rank}(\text{low}, 7.87871), \text{Ts_Rank}(\text{adv60}, 17.255), 4.97547), 18.5925), 15.7152), 6.71659)) * -1)$

Alpha#98: $(\text{rank}(\text{decay_linear}(\text{correlation}(\text{vwap}, \text{sum}(\text{adv5}, 26.4719), 4.58418), 7.18088)) \text{rank}(\text{decay_linear}(\text{Ts_Rank}(\text{Ts_ArgMin}(\text{correlation}(\text{rank}(\text{open}), \text{rank}(\text{adv15}), 20.8187), 8.62571), 6.95668), 8.07206)))$

Alpha#99: $((\text{rank}(\text{correlation}(\text{sum}(((\text{high} + \text{low}) / 2), 19.8975), \text{sum}(\text{adv60}, 19.8975), 8.8136)) < \text{rank}(\text{correlation}(\text{low}, \text{volume}, 6.28259)))) * -1)$

Alpha#100: $(0 - (1 * (((1.5 * \text{scale}(\text{indneutralize}(\text{indneutralize}(\text{rank}((((\text{close} - \text{low}) - (\text{high} - \text{close})) / (\text{high} - \text{low})) * \text{volume})), \text{IndClass.subindustry}), \text{IndClass.subindustry}))) \text{scale}(\text{indneutralize}((\text{correlation}(\text{close}, \text{rank}(\text{adv20}), 5) - \text{rank}(\text{ts_argmin}(\text{close}, 30))), \text{IndClass.subindustry}))) * (\text{volume} / \text{adv20}))))$

Alpha#101: $((\text{close} - \text{open}) / ((\text{high} - \text{low}) + .001))$

Appendix B: GARCH-M

We also tried to fit a combination of factor model and GARCH-M model. However, the fitted results were not satisfactory and we could not calculate t statistics for the coefficients in some conditions. Thus we were unable to use it for prediction. We still include the model here.

Model Introduction The GARCH-M (Generalized Autoregressive Conditional Heteroskedasticity-in-Mean) model extends the traditional GARCH framework by incorporating conditional volatility directly into the mean equation. This is particularly relevant in financial applications, where asset returns may depend on their own volatility due to risk-return tradeoffs. In addition to the GARCH structure, our model includes observable factors that may influence asset returns.

Mean Equation The GARCH-M model assumes the following form for the conditional mean of returns:

$$R_t = \alpha + \beta^\top F_t + \lambda \cdot \sqrt{h_t} + \varepsilon_t$$

- R_t : Asset return at time t
- α : Intercept term
- β : Vector of factor loadings
- F_t : Vector of observable factors at time t
- λ : Coefficient for the conditional standard deviation (volatility-in-mean effect)
- h_t : Conditional variance
- ε_t : Zero-mean error term

Variance Equation The conditional variance h_t follows a GARCH(p, q) process:

$$h_t = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j h_{t-j}$$

- ω : Constant term
- α_i : ARCH parameters (response to past squared residuals)
- β_j : GARCH parameters (response to past variances)

Estimation Method The parameters of the GARCH-M model are estimated by maximizing the log-likelihood function under the assumption of normally distributed errors:

$$\mathcal{L}(\theta) = -\frac{1}{2} \sum_t \left[\log(h_t) + \frac{\varepsilon_t^2}{h_t} \right]$$

The parameter vector θ includes:

$$\theta = (\alpha, \beta, \lambda, \omega, \alpha_1, \dots, \alpha_p, \beta_1, \dots, \beta_q)$$

Numerical optimization (e.g., SLSQP) is used to estimate θ , with constraints imposed to ensure positivity and stationarity of the variance process:

- $\omega > 0, \alpha_i \geq 0, \beta_j \geq 0$
- $\sum \alpha_i + \sum \beta_j < 1$ (for covariance stationarity)

Significance Testing After estimation, the Hessian matrix of the log-likelihood is approximated numerically. Standard errors are computed from the inverse of the Hessian:

$$\text{Var}(\hat{\theta}) \approx H^{-1} \Rightarrow \text{SE}(\hat{\theta}_i) = \sqrt{(H^{-1})_{ii}}$$

Then, the t-statistics and p-values are calculated as:

$$t_i = \frac{\hat{\theta}_i}{\text{SE}(\hat{\theta}_i)}, \quad p_i = 2(1 - \Phi(|t_i|))$$

However, we find that the calculated t-statistics are not valid as the standard errors are not robust. As we are not able to improve the standard errors, we cannot use them to conduct robust hypothesis tests. This is a limitation of our current approach.

Implementation To implement the GARCH-M model, we should determine the orders of the GARCH process (p, q). We simply apply GARCH model on the return rates to determine the orders as the GARCH-M model is a special case of the GARCH model which may have similar autocorrelation structures. We use the AIC and BIC criteria to select the optimal orders. The selected orders are then used in the GARCH-M model. We also filter the factors to select a subset of factors to include in the model. As we mentioned in factor modeling, we have a large number of factors which may lead to overfitting and make the calculation costly. To avoid this, we conduct a preliminary analysis to select a subset of factors. Then we fit the GARCH-M model using the selected factors and orders. The model is estimated using maximum likelihood estimation (MLE) with numerical

optimization methods. For each stock, we use a period of 500 days to fit the model and then predict the return rate of next day. And we apply the model to all stocks using rolling window approach. The model is re-estimated every day.

Appendix C: Responsibility

- Yanzhong Huang:
 - Data collection and preprocessing
 - Factor modeling and selection
 - ARIMA and GARCH model
 - Report writing
- Qinqin Huang:
 - GARCH-M model
 - Time Series Analysis
 - Report writing
- Yongyi Tang:
 - Time Series Analysis
 - Trading Strategy and Backtesting
 - Report writing
- Haoyue Tang:
 - Data collection and preprocessing
 - ARIMA and GARCH model
 - Report writing

References

- Zura Kakushadze, 2015. Formulaic Alphas