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Mathematics Grade 6 Geometry
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P2. Median In A Right Angle Triangle

Problem. Consider the right angle triangle $\triangle ABC$, $m(\angle A) = 90°$ and M the middle of BC as shown in Figure 1. Prove that $AM = \frac{BC}{2}$.

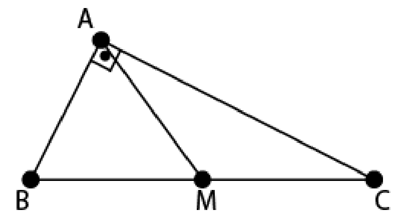

FIGURE 1. Right angle triangle and median.

Proof:

The proof will be completed by showing that $\triangle BAM$ is isosceles that is $AM \cong BM$. Since M is the middle point of BC we will conclude $AM = BM = \frac{BC}{2}$.

To prove that $AM \cong BM$ it is best to use an auxiliary construction. To relate AM and BM it is best to use the perpendicular from M to AB. E is a point on AB such that $ME \perp AB$.

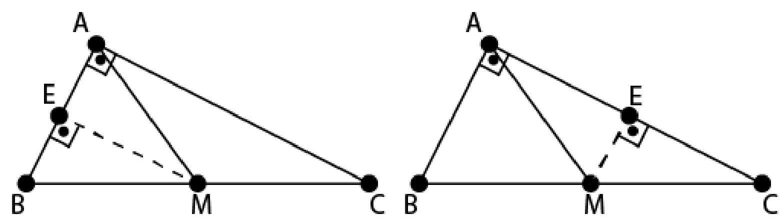

FIGURE 2. Auxiliary conditions choices.

To prove that $AM \cong BM$ the most helpful pair of similar triangles is: $\triangle ABC$ and $\triangle EBM$;

Consider the similarity of these triangles and $BM = \frac{BC}{2}$. This implies E is the middle point of AB.

Because $AE \cong BE$ and they have a right angle it can be stated that triangle $\triangle AEM$ is congruent with $\triangle BEM$.

The case of congruence observed is side-angle-side.

One of the consequences of the triangles congruence is that $AM \cong BM$.

We conclude: $AM = BM = \frac{BC}{2}$.

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