# MadDiff

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# Part I Introduction

## Introduction

Welcome to the documentation of MadDiff.jl

#### Note

This documentation page is under construction.

#### Note

This documentation is also available in PDF format.

## What is MadDiff?

MadDiff.jl is a simple algebraic modeling/differentiation package. MadDiff.jl constructs first and second derivative functions off-line (i.e., prior to calling the optimization solver) by applying operator overloading-based automatic differentiation on functions. The exact derivative functions can be obtained as results.

# **Bug reports and support**

Please report issues and feature requests via the Github issue tracker.

## Part II

**Quick Start** 

#### **Nonlinear Expressions**

MadDiff.jl provides a flexible user-interface for writing nonlinear expressions and evaluating the expressions and functions. For example,

```
using MadDiff

x = Variable()
p = Parameter()
expr = x[1]^2 + exp(x[2]^p[1])/2 + log(x[3]+p[2])
println(expr) # x[1]^2 + exp(x[2]^p[1])/2 + log(x[3] + p[2])

x0 = [0.,0.5,1.5]
p0 = [2,0.5]

f = function_evaluator(expr)
println("f(x0,p0) = $(f(x0,p0))") # f(x0,p0) = 1.3351598889038159

y0 = zeros(3)
g = gradient_evaluator(expr)
g(y0,x0,p0)
println("g(x0,p0) = $y0") # g(x0,p0) = [0.0, 0.6420127083438707, 0.5]
```

#### **Nonlinear Programming**

MadDiff.jl provides a simple user-interface for creating nonlinear prgogramming models and allows solving the created models using the solvers with NLPModels.jl interface (such as NLPModelsIpopt.jl and MadNLP.jl). The syntax is as follows:

#### Use with JuMP

MadDiff.jl can be used as an automatic differentiation backend. The syntax is as follows:

```
using MadDiff, JuMP, Ipopt

m = JuMP.Model(Ipopt.Optimizer)

@variable(m, x[i=1:1000], start=mod(i,2)==1 ? -1.2 : 1.)
@NLobjective(m, Min, sum(100(x[i-1]^2-x[i])^2+(x[i-1]-1)^2 for i=2:1000))
```

## Part III

How it works?

## Part IV

## **API Manual**

#### **MadDiffCore**

#### 4.1 MadDiffCore

```
MadDiffCore.MadDiffCore - Module.
   MadDiffCore
   Core algorithm for MadDiff.
MadDiffCore.AbstractExpression - Type.
   | AbstractExpression{T <: AbstractFloat}
   Abstract type for expression, gradient, hessian, entry, and field evaluators.
    source
MadDiffCore.Constant - Type.
   Constant{T <: AbstractFloat} <: Expression{T}</pre>
   Expression for constants.
   Constant(x::T) where T <: AbstractFloat</pre>
   Returns a Constant with value x.
   Example
    julia> e = Constant(1.)
    julia> non_caching_eval(e, [1.,2.,3.])
    1.0
    source
MadDiffCore.Constant - Method.
   Constant{T}(x::R) where {T <: AbstractFloat, R <: Real}
   Returns a Constant\{T,R\} whose value is x.
    source
```

```
MadDiffCore.Entry - Type.
   Entry{T <: AbstractFloat}</pre>
   Abstract type for entry evaluators.
    source
MadDiffCore.Expression - Type.
   Expression{T <: AbstractFloat}</pre>
   Abstract type for expression evaluators.
    source
MadDiffCore.Expression1 - Type.
   | Expression1{T <: AbstractFloat, F <: Function ,E <: Expression{T}} <: Expression{T}
   Expression for univariate function
    source
MadDiffCore.Expression2 - Type.
   | Expression2{T <: AbstractFloat, F <: Function, E1, E2} <: Expression{T}
   Expression for bivariate function
    source
MadDiffCore.ExpressionIfElse - Type.
   ExpressionIfElse{T,E0 <: Expression{T}, E1, E2} <: Expression{T}</pre>
   Expression for ifelse
    source
MadDiffCore.ExpressionSum - Type.
   | ExpressionSum{T <: AbstractFloat, E <: Expression{T}, I} <: Expression{T}
   Expression for a summation of Expressions
   source
MadDiffCore.Field - Type.
   | Field{T <: AbstractFloat}
   Abstract type for field evaluators.
   source
MadDiffCore.Gradient - Type.
   Gradient{T <: AbstractFloat}</pre>
```

```
Abstract type for gradient evaluators.
   source
MadDiffCore.Gradient - Method.
   Gradient(e :: Expression{T}) where T
   Returns the Gradient of an absraction e.
   source
MadDiffCore.Gradient0 - Type.
   | GradientO{T <: AbstractFloat} <: Gradient{T}
   Gradient of Variable.
   source
MadDiffCore.Gradient1 - Type.
   | Gradient1{T <: AbstractFloat, F, D1 <: Gradient} <: Gradient{T}
   Gradient of Expression1.
   source
MadDiffCore.Gradient2 - Type.
   | Gradient2{T <: AbstractFloat, F,D1 <: Gradient,D2 <: Gradient} <: Gradient{T}
   Gradient of Expression2.
   source
MadDiffCore.Gradient2F1 - Type.
   Gradient2F1{T <: AbstractFloat, F, D1 <: Gradient, R<: Real} <: Gradient{T}
   Gradient of Expression2 whose first argument is <: Real.
   source
MadDiffCore.Gradient2F2 - Type.
   | Gradient2F2{T <: AbstractFloat, F,D1 <: Gradient, R<: Real} <: Gradient{T}
   Gradient of Expression2 whose second argument is <: Real.
    source
MadDiffCore.GradientIfElse - Type.
   GradientIfElse{T, G1, G2} <: Gradient{T}</pre>
   Gradient of ExpressionIfElse
    source
MadDiffCore.GradientNull - Type.
```

```
GradientNull{T <: AbstractFloat} <: Gradient{T}</pre>
   Gradient of Parameter or Constant.
    source
MadDiffCore.GradientSum - Type.
   GradientSum{T <: AbstractFloat,D <: Gradient{T},I} <: Gradient{T}
   Gradient of ExpressionSum.
   source
MadDiffCore.Hessian - Type.
   Hessian{T <: AbstractFloat}</pre>
   Abstract type for hessian evaluators.
    source
MadDiffCore.Hessian02 - Type.
   | Hessian02{T,H11,H12,H21,H22} <: Hessian{T}
   Hessian of '
    source
MadDiffCore.Hessian11 - Type.
   | Hessian11{T,F,H1,H11} <: Hessian{T}
   Hessian of Expression1
   source
MadDiffCore.Hessian11F1 - Type.
   Hessian11F1{T,F,H1,H11,R} <: Hessian{T}</pre>
   Hessian of Expression2 whose first argument is <: Real.
   source
MadDiffCore.Hessian11F2 - Type.
   Hessian11F2{T,F,H1,H11,R} <: Hessian{T}</pre>
   Hessian of Expression2 whose second argument is <: Real.
    source
MadDiffCore.HessianNull - Type.
   | HessianNull{T} <: Hessian{T} end
   Hessian of linear expressions (e.g., Variable, Expression2{T, typeof(*), Int64, Variable{T}} where T)
    source
```

```
MadDiffCore.Parameter - Type.
   Parameter{T <: AbstractFloat} <: Expression{T}</pre>
   Expression for parameters.
   source
MadDiffCore.Parameter - Method.
   Parameter(n::Int)
   Returns a Parameter{Float64} whose index is n
   Example
    julia> e = Parameter(3)
    julia> non_caching_eval(e, [1.,2.,3.], [4.,5.,6.])
    6.0
   source
MadDiffCore.Parameter - Method.
   Parameter{T}(n::Int) where T <: AbstractFloat
   Returns a Parameter\{T\} whose index is n.
    source
MadDiffCore.Variable - Type.
   Variable{T <: AbstractFloat} <: Expression{T}</pre>
   Expression for variables.
   source
MadDiffCore.Variable - Method.
   Variable(n::Int)
   Returns a Variable{Float64} whose index is n
   Example
    julia> e = Variable(2)
    julia> non_caching_eval(e, [1.,2.,3.])
    2.0
    source
MadDiffCore.Variable - Method.
   Variable{T}(n::Int) where T <: AbstractFloat
   Returns a Variable\{T\} whose index is n.
    source
```

#### **MadDiffModels**

#### 5.1 MadDiffModels

MadDiffModels.MadDiffModels - Module.

```
MadDiffModels
```

MadDiffModels is a submodule of MadDiff. MadDiffModels allows modeling nonlinear optimization problem of the following form:

```
minimize: f(x)

subject to: xl \le x \le xu

gl \le g(x) \le gu,
```

#### where:

- $x \in R^n$  is the decision variable.
- f : R^n -> R is the objective function
- g : R^n -> R^m is the constraint mapping.

The model is constructed as an NLPModel (see https://github.com/JuliaSmoothOptimizers/NLPModels.jl), and can be solved with various NLP solvers such as:

- MadNLP (https://github.com/MadNLP/MadNLP.jl)
- Ipopt (https://github.com/JuliaSmoothOptimizers/NLPModelsIpopt.jl)
- Knitro (https://github.com/JuliaSmoothOptimizers/NLPModelsKnitro.jl)

source

MadDiffModels.Constraint - Type.

```
Constraint
```

A constraint index of MadDiffModel.

source

```
MadDiffModels.MadDiffModel - Type.
```

```
MadDiffModel{T <: Real}
```

**Example** 

```
A mathematical model of a nonlinaer program.
    source
MadDiffModels.MadDiffModel - Method.
    | MadDiffModel()
    Creates an empty MadDiffModel{Float64}.
    Example m = MadDiffModel(linear_solver = "ma27")
    source
MadDiffModels.MadDiffModel - Method.
    |MadDiffModel{T}()
    Creates an empty MadDiffModel{T}.
    Example m = MadDiffModel{Float32}()
    source
MadDiffModels.ModelComponent - Type.
   ModelComponent
    A model component (eitehr a variable or a parameter) of MadDiffModel.
    source
MadDiffModels.constraint - Method.
   constraint(m::MadDiffModel, e::MadDiffCore.Expression; lb=0., ub=0.)
    Adds a constraint to MadDiffModel. Example "' m = MadDiffModel()
    x = [variable(m) \text{ for } i=1:3] \text{ constraint}(m, x[1]^2 + 2*sin(x[2]) - exp(x[3]) >= 0) \text{ constraint}(m, x[1]^4 + 2*sin(x[2]) - exp(x[3]) >= 0)
    x[2]^4 x[3]^4; lb = 0.1, ub = 1.)
    source
MadDiffModels.dual - Method.
   dual(c::Constraint)
    Retrun the dual of constraint c.
    source
MadDiffModels.instantiate! - Method.
   instantiate!(m::MadDiffModel; sparse = true)
    Instantiates the model m. The model must be instantiated before solving.
```

```
using MadDiff, NLPModelsIpopt
    m = MadDiffModel()
    x = [variable(m) for i=1:3]
    objective(m, x[1]^2 + x[2]^2 + \sin(x[3]))
    constraint(m, 3x[2]^2 \ll 1.)
    instantiate!(m)
    ipopt(m)
   source
MadDiffModels.lower_bound - Method.
   |lower_bound(c::Constraint)
   Retrun the lower bound of constraint c.
    source
MadDiffModels.lower_bound - Method.
   lower_bound(x::ModelComponent{V}) where V <: MadDiffCore.Variable</pre>
   Retrun the lower bound of variable x.
    source
MadDiffModels.objective - Method.
   objective(m::MadDiffModel, e::MadDiffCore.Expression
   Sets the objective function for MadDiffModel. Only minimization is supported. Example
    m = MadDiffModel()
    x = [variable(m) for i=1:3]
    objective(m, x[1]^2 + x[2] + sin(x[3]))
   source
MadDiffModels.parameter - Method.
   parameter(m::MadDiffModel{T}, val)
   Creates a parameter for MadDiffModel with value val. Example
    m = MadDiffModel()
    p = parameter(m, 0.5)
   source
MadDiffModels.set_lower_bound - Method.
   set_lower_bound(c::Constraint,val)
```

```
Set the lower bound of constraint c to val.
    source
MadDiffModels.set_lower_bound - Method.
    | set_lower_bound(x::ModelComponent{V},val) where V <: MadDiffCore.Variable
    Set the lower bound of variable 'x' to val.
    source
MadDiffModels.set_upper_bound - Method.
   set_upper_bound(c::Constraint,val)
    Set the upper bound of constraint c to val.
    source
MadDiffModels.set_upper_bound - Method.
   \begin{tabular}{ll} | set\_upper\_bound(x::ModelComponent\{V\},val) & where V <: MadDiffCore.Variable \\ \end{tabular}
    Set the upper bound of variable 'x' to val.
    source
MadDiffModels.setvalue - Method.
   | setvalue(p::ModelComponent{P},val) where P <: MadDiffCore.Parameter
    Set the value of parameter 'p' to val.
    source
MadDiffModels.setvalue - Method.
   | setvalue(x::ModelComponent{V},val) where V <: MadDiffCore.Variable
    Set the value of variable 'x' to val.
    source
MadDiffModels.upper_bound - Method.
   upper_bound(c::Constraint)
    Retrun the upper bound of constraint c.
    source
MadDiffModels.upper_bound - Method.
   upper_bound(x::ModelComponent{V}) where V <: MadDiffCore.Variable</pre>
    Retrun the upper bound of variable x.
    source
MadDiffModels.value - Method.
```

```
value(p::ModelComponent{P}) where P <: MadDiffCore.Parameter</pre>
    Return the value of parameter p.
    source
MadDiffModels.value - Method.
    \label{eq:value} \begin{tabular}{ll} | value(x::ModelComponent\{V\}) & where V <: MadDiffCore.Variable \\ \end{tabular}
    Return the value of variable x.
    source
MadDiffModels.variable - Method.
    variable(m::MadDiffModel{T}; lb=-Inf, ub=Inf, start=0.)
    Creates a variable for MadDiffModel.
    Example
    m = MadDiffModel()
    x = variable(m; lb = -1, ub = 1, start = 0.5)
    source
NLPModels.cons! - Method.
    | NLPModels.cons!(m::MadDiffModel,x::AbstractVector,y::AbstractVector)
    Evaluate the constraints of m at x and store the result in the vector y.
    source
NLPModels.get_ifix - Function.
    get_ifix(m::MadDiffModel)
    Return the value ifix from MadDiffModel.
    source
NLPModels.get_ifree - Function.
    get_ifree(m::MadDiffModel)
    Return the value ifree from MadDiffModel.
    source
NLPModels.get_iinf - Function.
    get_iinf(m::MadDiffModel)
    Return the value iinf from MadDiffModel.
    source
NLPModels.get_ilow - Function.
```

```
get_ilow(m::MadDiffModel)
   Return the value ilow from MadDiffModel.
    source
NLPModels.get_irng - Function.
   get_irng(m::MadDiffModel)
   Return the value irng from MadDiffModel.
   source
NLPModels.get_islp - Function.
   get_islp(m::MadDiffModel)
   Return the value islp from MadDiffModel.
    source
NLPModels.get_iupp - Function.
   get_iupp(m::MadDiffModel)
   Return the value iupp from MadDiffModel.
    source
NLPModels.get_jfix - Function.
   get_jfix(m::MadDiffModel)
   Return the value jfix from MadDiffModel.
   source
NLPModels.get jfree - Function.
   get_jfree(m::MadDiffModel)
   Return the value jfree from MadDiffModel.
   source
NLPModels.get_jinf - Function.
   get_jinf(m::MadDiffModel)
   Return the value jinf from MadDiffModel.
    source
NLPModels.get_jlow - Function.
   get_jlow(m::MadDiffModel)
   Return the value jlow from MadDiffModel.
    source
```

```
NLPModels.get_jrng - Function.
   get_jrng(m::MadDiffModel)
   Return the value jrng from MadDiffModel.
    source
NLPModels.get_jupp - Function.
   get_jupp(m::MadDiffModel)
   Return the value jupp from MadDiffModel.
    source
NLPModels.get_lcon - Function.
   get_lcon(m::MadDiffModel)
   Return the value Icon from MadDiffModel.
    source
NLPModels.get_lin - Function.
   get_lin(m::MadDiffModel)
   Return the value lin from MadDiffModel.
    source
NLPModels.get_lin_nnzj - Function.
   get_lin_nnzj(m::MadDiffModel)
   Return the value lin_nnzj from MadDiffModel.
    source
NLPModels.get_lvar - Function.
   get_lvar(m::MadDiffModel)
   Return the value Ivar from MadDiffModel.
   source
NLPModels.get minimize - Function.
   | get_minimize(m::MadDiffModel)
   Return the value minimize from MadDiffModel.
   source
NLPModels.get_name - Function.
   get_name(m::MadDiffModel)
```

```
Return the value name from MadDiffModel.
   source
NLPModels.get_ncon - Function.
   get_ncon(m::MadDiffModel)
   Return the value ncon from MadDiffModel.
   source
NLPModels.get_nlin - Function.
   get_nlin(m::MadDiffModel)
   Return the value nlin from MadDiffModel.
   source
NLPModels.get_nln - Function.
   get_nln(m::MadDiffModel)
   Return the value nln from MadDiffModel.
   source
NLPModels.get_nln_nnzj - Function.
   get_nln_nnzj(m::MadDiffModel)
   Return the value nln_nnzj from MadDiffModel.
   source
NLPModels.get nlvb - Function.
   get_nlvb(m::MadDiffModel)
   Return the value nlvb from MadDiffModel.
   source
NLPModels.get_nlvc - Function.
   get_nlvc(m::MadDiffModel)
   Return the value nlvc from MadDiffModel.
    source
NLPModels.get_nlvo - Function.
   get_nlvo(m::MadDiffModel)
   Return the value nlvo from MadDiffModel.
    source
NLPModels.get_nnln - Function.
```

```
get_nnln(m::MadDiffModel)
   Return the value nnln from MadDiffModel.
    source
NLPModels.get_nnzh - Function.
   get_nnzh(m::MadDiffModel)
   Return the value nnzh from MadDiffModel.
   source
NLPModels.get_nnzj - Function.
   get_nnzj(m::MadDiffModel)
   Return the value nnzj from MadDiffModel.
    source
NLPModels.get_nnzo - Function.
   get_nnzo(m::MadDiffModel)
   Return the value nnzo from MadDiffModel.
    source
NLPModels.get_nvar - Function.
   get_nvar(m::MadDiffModel)
   Return the value nvar from MadDiffModel.
   source
NLPModels.get ucon - Function.
   get_ucon(m::MadDiffModel)
   Return the value ucon from MadDiffModel.
   source
NLPModels.get_uvar - Function.
   get_uvar(m::MadDiffModel)
   Return the value uvar from MadDiffModel.
    source
NLPModels.get_x0 - Function.
   get_x0(m::MadDiffModel)
   Return the value x0 from MadDiffModel.
    source
```

```
NLPModels.get_y0 - Function.
   get_y0(m::MadDiffModel)
   Return the value y0 from MadDiffModel.
    source
NLPModels.grad! - Method.
    NLPModels.grad!(m::MadDiffModel,x::AbstractVector,y::AbstractVector)
   Evaluate the gradient of m at x and store the result in the vector y.
    source
NLPModels.hess_coord! - Method.
    NLPModels.hess coord!(m::MadDiffModel,x::AbstractVector,lag::AbstractVector,z::AbstractVector;
    \hookrightarrow obj_weight = 1.0)
   Evaluate the Lagrangian Hessian of m at primal x, dual lag, and objective weight obj_weight and store
   the result in the vector zin sparse coordinate format.
    source
NLPModels.hess_structure! - Method.
   | NLPModels.hess_structure!(m::MadDiffModel,I::AbstractVector{T},J::AbstractVector{T})
   Evaluate the structure of the Lagrangian Hessian and store the result in I and J in sparse coordinate format.
    source
NLPModels.jac coord! - Method.
   | NLPModels.jac_coord!(m::MadDiffModel,x::AbstractVector,J::AbstractVector)
   Evaluate the constraints Jacobian of m at x and store the result in the vector J in sparse coordinate format.
    source
NLPModels.jac structure! - Method.
   NLPModels.jac_structure!(m::MadDiffModel,I::AbstractVector{T},J::AbstractVector{T})
   Evaluate the structure of the constraints Jacobian and store the result in I and J in sparse coordinate
   format.
    source
NLPModels.obj - Method.
   NLPModels.obj(m::MadDiffModel,x::AbstractVector)
   Return the objective value of m at x.
    source
```

#### **MadDiffMOI**

#### 6.1 MadDiffMOI

```
MadDiffM0I.MadDiffM0I - Module.
   |MadDiffM0I
   MadDiffMOI is a submodule of MadDiff. MadDifMOI allows solving nonlinear optimization problems speci-
   fied by MathOptInterface (https://github.com/jump-dev/JuMP.jl/tree/od/moi-nonlinear).
   source
MadDiffCore.Expression - Method.
   Expression(ex::MOI.Nonlinear.Expression; subex = nothing)
   Create a MadDiff.Expression from MOI.Expression.
    source
MadDiffCore.SparseNLPCore - Method.
   | MadDiffCore.SparseNLPCore(nlp_data::MOI.Nonlinear.Model)
   Create MadDiffCore.SparseNLPCore from MOI.Nonlinear.Model.
    source
MadDiffMOI.MadDiffAD - Type.
   | MadDiffAD() <: MOI.Nonlinear.AbstractAutomaticDifferentiation
   A differentiation backend for MathOptInterface based on MadDiff
    source
MadDiffMOI.MadDiffEvaluator - Type.
   | MadDiffEvaluator <: MOI.AbstractNLPEvaluator
   A type for callbacks for MathOptInterface's nonlinear model.
MathOptInterface.NLPBlockData - Method.
```

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```
MOI.NLPBlockData(evaluator::MadDiffEvaluator)
   Create MOI.NLPBlockData from MadDiffEvaluator
    source
MathOptInterface.Nonlinear.Evaluator - Method.
   MOI.Nonlinear.Evaluator(model::MOI.Nonlinear.Model, ::MadDiffAD, ::Vector(MOI.VariableIndex))
   Create a MOI.Nonlinear.Evaluator from MOI.Nonlinear.Model using MadDiff's AD capability.
    source
MathOptInterface.eval_constraint - Method.
   | MOI.eval_constraint(evaluator::MadDiffEvaluator, g, x)
    Evaluate the gradient of evaluator at x and store the result in the vector g.
    source
MathOptInterface.eval_constraint_jacobian - Method.
   MOI.eval_constraint_jacobian(evaluator::MadDiffEvaluator, J, x)
   Evaluate the constraints Jacobian of evaluator at x and store the result in the vector J in sparse coordinate
   format.
    source
MathOptInterface.eval_hessian_lagrangian - Method.
   |MOI.eval_hessian_lagrangian(evaluator::MadDiffEvaluator, Η, x, σ, μ)
   Evaluate the Lagrangian Hessian of evaluator at primal x, dual \mu, and objective weight \sigma and store the
   result in the vector H in sparse coordinate format.
    source
MathOptInterface.eval objective - Method.
   MOI.eval_objective(evaluator::MadDiffEvaluator, x)
    Return the objective value of evaluator at x.
    source
MathOptInterface.eval_objective_gradient - Method.
   | MOI.eval_objective_gradient(evaluator::MadDiffEvaluator, g, x)
   Evaluate the constraints of evaluator at x and store the result in the vector g.
    source
MathOptInterface.hessian_lagrangian_structure - Method.
   MOI.hessian_lagrangian_structure(evaluator::MadDiffEvaluator)
```

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Return the structure of the Lagrangian Hessian in  $Vector{Tuple{Int,Int}}$  format.

source

 ${\tt MathOptInterface.jacobian\_structure-Method}.$ 

| MOI.jacobian\_structure(evaluator::MadDiffEvaluator)

Return the structure of the constraints Jacobian in  $Vector{Tuple{Int,Int}}$  format.

source