

Soss

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We present Soss, a declarative probabilistic programming language embedded in the Julia language. Soss represents statistical models in terms of abstract syntax trees, and uses staged compilation for on-demand generation of “inference primitives” (random sampling, log-density, etc) without requiring casual users to worry about such details.

The approach taken by Soss makes it easy to extend to take advantage of other packages in the rapidly-growing Julia ecosystem. At the time of this writing, Soss users can choose from several inference back-ends and connect easily with larger systems SymPy and Gen.

1 INTRODUCTION

There are many approaches to building a Probabilistic Programming Language (“PPL”). Soss is distinct from most alternatives in a number of ways:

- Model specification in Soss is *declarative*; statements are represented not by the order entered by the user, but by the partial order given by their variable dependencies.
- Soss models are typically expressed in terms of *inference combinators* (like “For”) that combine distributions in some way to arrive at a new distribution.
- Soss models are *first-class*; models can be passed as arguments to other models, or used in place of distribution functions like `Normal` within a model.
- Soss models are *function-like*; abstractly, a model may be considered a function from its arguments to a joint distribution. In particular, specification of “observed variables” is separated from model definition.
- Values and distributions in Soss are represented internally as *abstract syntax trees*, keeping the internal representation close to that given by the user for maximum flexibility.
- Soss uses *staged compilation* for inference primitives like `rand` and `logpdf`, via novel *generalized generated functions* from `GeneralizedGenerated.jl`.
- Soss supports *model transformations*, functions that take a model and return another model.
- Soss supports *symbolic simplification*, making it easy to inspect or manipulate a symbolic representation of the log-density, or to use it to generate optimized code.
- Soss is *extensible*; users can define new inference primitives or model transformations externally, and use them as if they had been included in Soss.

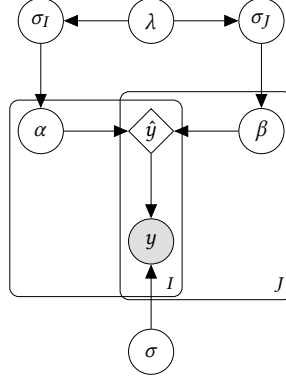
Finally, Soss uses Cockney rhyming slang to address the well-known challenge of naming things in computer science (“sauce pan” rhymes with “Stan”). Soss is open-source software.

2 MODELING IN SOSS

Figure 1 shows a plate diagram and Soss implementation of a simplified two-way ANOVA model. The Soss model `m` has arguments `I` and `J`. Each line of a model is either a *sample statement* (`v ~ rhs`) or an *assign statement* (`v = rhs`). In either case, `v` must be a valid Julia variable name, and `rhs` must be a valid Julia expression. For a sample statement, `rhs` should also be “distribution-like”, supporting `logpdf` and/or `rand` methods (depending on the inference algorithm to be used).

$i \in \{1, \dots, I\}$
 $j \in \{1, \dots, J\}$
 $\lambda \sim \text{Cauchy}_+(0, 1)$
 $\sigma_I \sim \text{Normal}_+(0, \lambda)$
 $\sigma_J \sim \text{Normal}_+(0, \lambda)$
 $\alpha_i \sim \text{Normal}(0, \sigma_I)$
 $\beta_j \sim \text{Normal}(0, \sigma_J)$
 $\hat{y}_{ij} = \alpha_i + \beta_j$
 $y_{ij} \sim \text{Normal}(\hat{y}_{ij}, \sigma)$

(a)



(b)

```

m = @model I,J begin
  λ ~ HalfCauchy()
  σI ~ HalfNormal(λ)
  σJ ~ HalfNormal(λ)

  α ~ Normal(0, σI) |> iid(I)
  β ~ Normal(0, σJ) |> iid(J)
  yhat = α .+ β'

  σ ~ HalfNormal()
  y ~ For(I,J) do i,j
    Normal(yhat[i,j], σ)
  end
end

```

(c)

Fig. 1. A generative model (a), plate diagram (b), and Soss model (c) for a two-way analysis of variance model. Note the specification that y will later be observed is not part of the model, but is instead given at inference time.

2.1 Joint Distributions

A Soss model is *function-like*, taking values for its arguments and returning a *joint distribution*. For example, given m from Figure 1, we can build the joint distribution $d=m(I=2, J=3)$. This supports the usual `rand` and `logpdf` functions, as well as some others provided by Soss described in the following sections. For example,¹

```

julia> d = m(I=2, J=3); rand(d) # or just rand(m(I=2, J=3))
(I = 2, J = 3, yhat = [-0.52 -1.5 0.0037; -0.81 -1.85 -0.28], σ = 2.5, λ = 1.3, σJ = 0.72, σI = 0.41,
β = [-0.54, -1.5, -0.023], α = [0.026, -0.26], y = [-0.45 -2.0 2.2; -3.4 -3.9 2.3])

```

2.2 Inference Primitives

A given inference algorithm can be expressed in terms of a collection of functions on a model or joint distribution. For example, we often need to evaluate the log-density or its gradient or draw random samples. In Soss, these *inference primitives* generate model-specific code at execution time.

At this writing, the following inference primitive are available:

rand draws a random point or sample from a given distribution.

particles draws a *systematic sample*, as described in [Douc et al. 2005].

logpdf takes a distribution and a point, and returns the log-density at that point.

xform takes a distribution and some known data, and returns a bijection from \mathbb{R}^n to the space of unknowns. This is useful for algorithms like Hamiltonian Monte Carlo [Neal 2011].

symlogpdf takes a model, and returns a SymPy [Meurer et al. 2017] representation of the log-density, with some additional Julia-based simplifications.

codegen calls `symlogpdf`, and uses the result to generate a more efficient version of `logpdf`.

In the above, a *distribution* refers to a joint distribution, while *point* refers to a named tuple in the space space of a distribution.

cite MonteCarloMeasurements.jl and TransformVariables.jl

¹This and future outputs are lightly edited for space, for example rounding floating point values.

The set of inference primitives is not fixed, but can be extended by users. New primitives can be added in outside packages, and does not require making changes to the Soss codebase.

2.3 Inference

Together with automatic differentiation, the above inference primitives enable a wide variety of inference algorithms. Rather than building such algorithms specific to Soss, we defer this to external packages. So far, our focus has been Hamiltonian Monte Carlo [Neal 2011], as implemented in `DynamicHMC.jl` [Papp and Piibeleht 2019] and `AdvancedHMC.jl` [Ge et al. 2018]; Soss has interfaces to these in the `dynamicHMC` and `advancedHMC` functions, respectively.

2.4 Functions on Models

Soss includes several functions for querying sets of variables associated with a model. As described above, models are function-like, with the inputs given by **arguments**. In the body of a model every statement declares that some variable is either **sampled** or **assigned**. The sampled and assigned variables together comprise the **parameters**, and all of the above are **variables**. Each of these is a Soss function that takes a model and returns a vector of distinct Julia symbols.

Soss ignores the ordering of statements given by the user, instead working in terms of the DAG ordering. You can access this graph with `digraph`, or use `poset` to get the partial order. Finally, `toposort` returns a topologically-ordered vector of the parameters. Each of these three functions takes a model as input.

Soss represents and manipulates variable dependencies using `SimpleGraphs`, `SimplePosets`, and `SimplePartitions` from [Scheinerman 2019].

2.5 Model Transformations

`Do` implements Pearl’s `do` operator for causal intervention [Pearl 1995].²

`predictive` is like `Do`, but only returns the strongly-connected components containing the specified variables. This is useful for sampling from the posterior predictive distribution.

`markovBlanket` takes a model and a variable, and returns a model representing the Markov blanket at the given variable.

3 IMPLEMENTATION

```
struct Model{A,B,M}
  args  :: Vector{Symbol}
  vals  :: NamedTuple
  dists :: NamedTuple
  retn  :: Union{Nothing, Symbol, Expr}
end
```

The ability to generate arbitrary Julia code makes it difficult to measure performance in Soss. When a performance bottleneck is found, developers or users can ask Soss to generate something different. This design places no a priori constraints on performance.

Restructure my writing?

Besides the purely static code generation via regular macros happening in parsing time, Soss heavily uses a mechanism of "zero-cost" runtime code generation originated by Julia’s generated functions [Bezanson et al. 2012], a.k.a staged functions in the original paper and earlier versions of Julia Programming Language.

²Our `Do` operator is capitalized because `do` is a keyword in Julia.

The generated functions provide the capability of performing code generation during type inference, generating programs computed by the body of function(a.k.a, generator), once and only once for each combination of argument types.

Further, Julia enables type inference and compiler optimizations equivalent to non-runtime ones in runtime for the callsites of generated functions, hence we gain runtime code generation without losing performance.

To take advantage of this, Soss designs a system to encode sufficient information for generating actual codes, into the types, or objects that can be dispatched like types, which allows the use of generated functions here.

Unfortunately, in current stage, there're some implementation restrictions to Julia's generated functions, and generated functions cannot generate arbitrary Julia codes, where some advanced programming constructions are missing, such as generators and function-related stuffs like closures(including functions with no free variables), multiple dispatch, etc.

To address this, we introduced the works of easing the restrictions of Julia generated functions, cite GG here?

which allow us to generate closures from generated functions, and make the programs sufficiently powerful.

Programs that can be generated by Soss are turing complete, as all constructs of Lambda Calculus can be generated.

4 EXTENDING SOSS

5 RELATED WORK

The idea of code generation and symbolic simplification in an embedded PPL goes back to *Passage* [Scherrer et al. 2012].

Hakaru [Narayanan et al. 2016] takes a more ambitious symbolic approach in a stand-alone PPL, allowing a wider variety of program transformations.

Soss began with a goal of representing models with continuous, fixed-dimensionality parameter spaces, inspired by *Stan* [Carpenter et al. 2017].

Gen [Cusumano-Towner et al. 2019] was developed independently from Soss, but takes a similar approach (and distinct from most PPLs) in its representation of a model as a function from its arguments to a "trace" (to use Gen's terminology). In both Soss and Gen's static DSL, this trace is a mapping from variable names to values. The similarity of these systems makes interoperability relatively straightforward, as demonstrated in *SossGen* (<https://github.com/cscherrer/SossGen.jl>).

Turing [Ge et al. 2018]

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```

197   ⟨model⟩  ⊨  @model ⟨args⟩ begin ⟨statements⟩ end
198   ⟨args⟩   ⊨  λ | ⟨Symbol⟩ | ⟨Symbol⟩, ⟨args⟩
199
200   ⟨statements⟩ ⊨ ⟨statement⟩ | ⟨statement⟩ Julia Line Sep ⟨statements⟩ | ⟨statements⟩ ⟨retn⟩
201   ⟨statement⟩ ⊨ ⟨assign⟩ | ⟨sample⟩
202   ⟨assign⟩   ⊨  ⟨Symbol⟩ = ⟨Expr⟩
203   ⟨sample⟩   ⊨  ⟨Symbol⟩ ~ ⟨Measure⟩
204
205   ⟨retn⟩     ⊨  return ⟨Expr⟩
206   ⟨Symbol⟩   ⊨  Julia Symbol
207   ⟨Expr⟩     ⊨  Julia Expr
208   ⟨Measure⟩  ⊨  Probability measure (see text)
209
210

```

Fig. 2. Backus-Naur Form representation for a user-specified model.

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A MODEL SYNTAX

B SYMBOLIC SIMPLIFICATION

C EXTENDED EXAMPLE

Break this up! This is just a staging area

```

240 julia> using Revise, Soss, Random
241
242 julia> Random.seed!(1);
243
244 julia> m = @model I, J begin
245     λ ~ HalfCauchy()

```

$$\ell = -3 \log(2) - 5 \log(\pi) - \sigma^2$$

$$-4 \log(\lambda) - 2 \log(\lambda^2 + 1) - \frac{\sigma_I^2}{\lambda^2} - \frac{\sigma_J^2}{\lambda^2}$$

$$+ \sum_{i=1}^I \left(-\log(\sigma_I) - \frac{\log \pi}{2} - \frac{\log 2}{2} - \frac{\alpha_i^2}{2\sigma_I^2} \right)$$

$$+ \sum_{j=1}^J \left(-\log(\sigma_J) - \frac{\log \pi}{2} - \frac{\log 2}{2} - \frac{\beta_j^2}{2\sigma_J^2} \right)$$

$$+ \sum_{\substack{1 \leq i \leq I \\ 1 \leq j \leq J}} \left(-\log(\sigma) - \frac{\log \pi}{2} - \frac{\log 2}{2} - \frac{(y_{ij} - \hat{y}_{ij})^2}{2\sigma^2} \right)$$

(a) Direct implementation

$$\ell = -7.8 - 0.92I - 0.92J - 0.92IJ - \sigma^2$$

$$-4 \log \lambda - 2 \log(\lambda^2 + 1) - IJ \log \sigma$$

$$-I \log \sigma_I - J \log \sigma_J - \frac{\sigma_I^2}{\lambda^2} - \frac{\sigma_J^2}{\lambda^2}$$

$$- \frac{1}{2\sigma_I^2} \sum_{i=1}^I \alpha_i^2 - \frac{1}{2\sigma_J^2} \sum_{j=1}^J \beta_j^2$$

$$- \frac{1}{2\sigma^2} \sum_{\substack{1 \leq i \leq I \\ 1 \leq j \leq J}} (y_{ij} - \hat{y}_{ij})^2$$

(b) After symbolic simplification

Fig. 3. Symbolic log-density of m before (a) and after (b) simplification steps.

julia> Soss.sourceRand()(m)

quote

```

σ = rand(HalfNormal())
λ = rand(HalfCauchy())
σJ = rand(HalfNormal(λ))
σI = rand(HalfNormal(λ))
β = rand(iid(J, Normal(0, σJ)))
α = rand(iid(I, Normal(0, σI)))
yhat = α .+ β'
y = rand(For(I, J) do i, j
    Normal(yhat[i, j], σ)
end)
(I=I, J=J, yhat=yhat, λ=λ, σI=σI,
σJ=σJ, α=α, β=β, σ=σ, y=y)
```

end

(a) rand

julia> Soss.sourceLogpdf()(m)

quote

```

_ℓ = 0.0
_ℓ += logpdf(HalfNormal(), σ)
_ℓ += logpdf(HalfCauchy(), λ)
_ℓ += logpdf(HalfNormal(λ), σJ)
_ℓ += logpdf(HalfNormal(λ), σI)
_ℓ += logpdf(Normal(0, σJ) |> iid(J), β)
_ℓ += logpdf(Normal(0, σI) |> iid(I), α)
yhat = α .+ β'
_ℓ += logpdf(For(I, J) do i, j
    Normal(yhat[i, j], σ)
end, y)
return _ℓ
```

end

(b) logpdf

Fig. 4. Generated code when calling rand(m) and logpdf(m, x).

```

σI ~ HalfNormal(λ)
σJ ~ HalfNormal(λ)
α ~ Normal(0, σI) |> iid(I)
β ~ Normal(0, σJ) |> iid(J)
yhat = α .+ β'
σ ~ HalfNormal()
y ~ For(I, J) do i, j
    Normal(yhat[i, j], σ)
end
end;
```

julia> truth = rand(m(I=2, J=3)); pairs(truth);

julia> post = dynamicHMC(m(I=2, J=3), (y=truth.y,));

```

295 julia> postpred = predict(m(I=2,J=3), post) |> particles;
296
297 julia> postpred.yhat
298 2×3 Array{Particles{Float64,1000},2}:
299  0.241 ± 0.26  0.527 ± 0.3  -0.556 ± 0.38
300  0.22 ± 0.26  0.506 ± 0.32  -0.577 ± 0.37
301
302 julia> postpred.y
303 2×3 Array{Particles{Float64,1000},2}:
304  0.224 ± 0.58  0.529 ± 0.58  -0.58 ± 0.64
305  0.223 ± 0.56  0.483 ± 0.58  -0.58 ± 0.61
306
307 julia> m.args
308 2-element Array{Symbol,1}:
309  :I
310  :J
311
312 julia> pairs(m.vals)
313 pairs(::NamedTuple) with 1 entry:
314   :yhat => :( $\alpha$  .+  $\beta$ ')
315
316 julia> pairs(m.dists)
317 pairs(::NamedTuple) with 7 entries:
318   : $\lambda$  => :(HalfCauchy())
319   : $\sigma I$  => :(HalfNormal( $\lambda$ ))
320   : $\sigma J$  => :(HalfNormal( $\lambda$ ))
321   : $\alpha$  => :(Normal(0,  $\sigma I$ ) |> iid(I))
322   : $\beta$  => :(Normal(0,  $\sigma J$ ) |> iid(J))
323   : $\sigma$  => :(HalfNormal())
324   :y => :(For(I, J) do i, j...
325
326
327
328
329
330
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334
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338
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342
343

```