Data-Driven Optimal Control of Complex Networks

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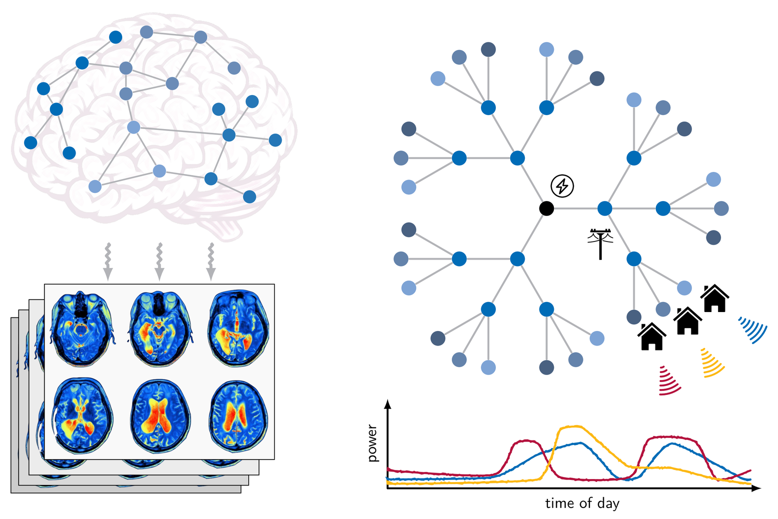
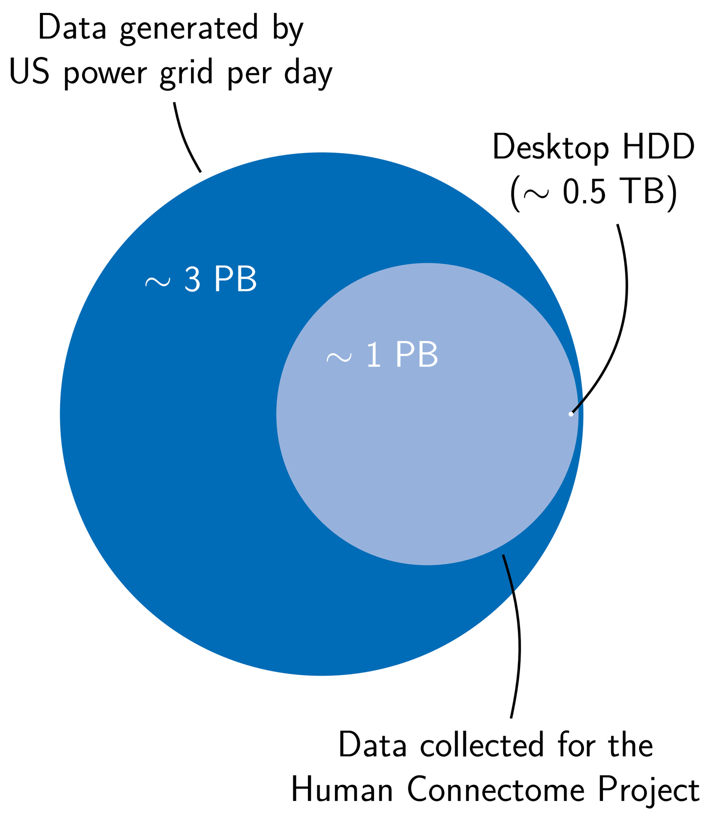
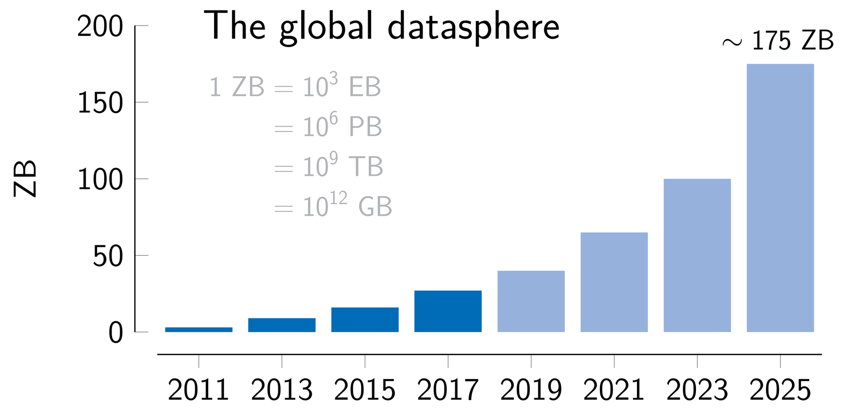
The big data revolution

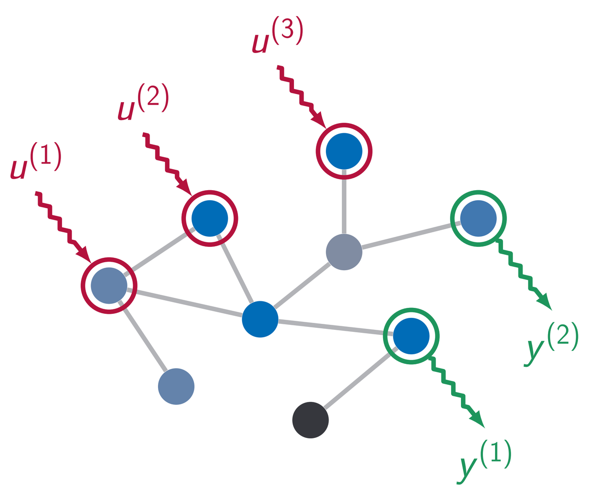






*Source: International Data Corporation | www.idc.com

Minimum-energy network control



network adjacency matrix input matrix

$$x(t+1) = A\,x(t) + B\,u(t)$$

$$y(t) = C\,x(t)$$

output matrix

Task

Steer the network from $y(0) = 0$ to $y(T) = y_f$ with minimum energy

if the network is output controllable

$$u^\star(t) = B^\top (A^\top)^{T-t-1} C^\top \mathcal{W}_T^{-1} y_f, \quad t = 0, 1, \dots, T-1$$

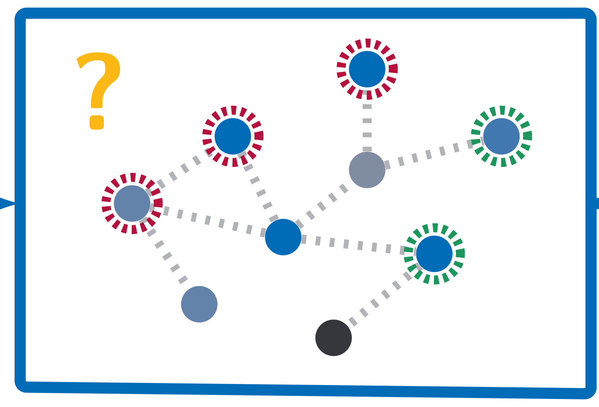
$$\mathcal{W}_T = \sum_{t=0}^{T-1} C A^t B B^\top (A^\top)^t C^\top = T\text{-steps output controllability Gramian}$$

Limitations

- $u^\star(t)$ requires ***exact knowledge*** of the network adjacency matrix
- $u^\star(t)$ ***numerically unreliable*** and ***expensive*** for large networks

Data-driven minimum-energy network control

$u_i(T-1)$
$\vdots$
$u_i(1)$
$u_i(0)$



$y_i(T)$

i-th experiment

Available data

$$U = \left[u_1\ u_2 \cdots u_i \cdots u_N\right] \qquad Y = \left[y_1\ y_2 \cdots y_i \cdots y_N\right]$$

Learning minimum-energy controls from data

$$u^\star = \left[u^\star(T-1)^\top \cdots u^\star(0)^\top\right]^\top = f(U, Y, y_f)\ ?$$

1. $\alpha^\star = \arg\min_\alpha \|U\alpha\|^2$ s. t. $Y\alpha = y_f$ → $u^\star = U\alpha^\star = (I - UK(UK)^\dagger)UY^\dagger y_f$
   if U full row rank — basis of $\ker(Y)$
2. $C^\star = \arg\min_C \|Y - CU\|_F^2$ → $u^\star = (C^\star)^\dagger y_f = (YU^\dagger)^\dagger y_f$
3. $M^\star = \arg\min_M \|MY - U\|_F^2$ → $u^\star \xrightarrow{N\to\infty} M^\star y_f = UY^\dagger y_f$
   if U_{ij} i.i.d. zero-mean r.v.'s

Data-driven network control with noisy data

$$\tilde{U} = U + W \qquad \tilde{Y} = Y + V$$

W_{ij} i.i.d. $\mathbb{E}[W_{ij}] = 0$, $\mathbb{E}[W_{ij}^2] = \sigma_W^2$

V_{ij} i.i.d. $\mathbb{E}[V_{ij}] = 0$, $\mathbb{E}[V_{ij}^2] = \sigma_V^2$

1 2 3 → Typically biased and not asymptotically correct

Correction step

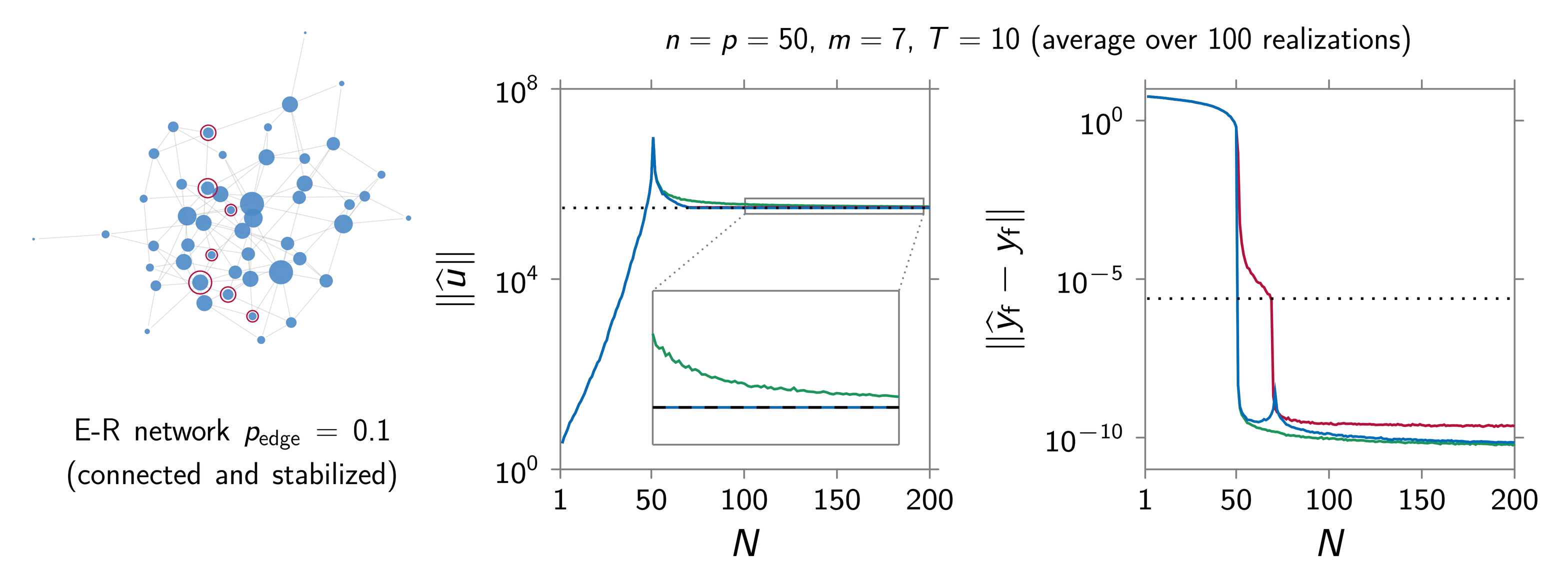
$$\tilde{Y}^\dagger \to \tilde{Y}^\top(\tilde{Y}\tilde{Y}^\top - N\sigma_V^2 I)^\dagger$$
$$\tilde{U}^\dagger \to \tilde{U}^\top(\tilde{U}\tilde{U}^\top - N\sigma_W^2 I)^\dagger$$

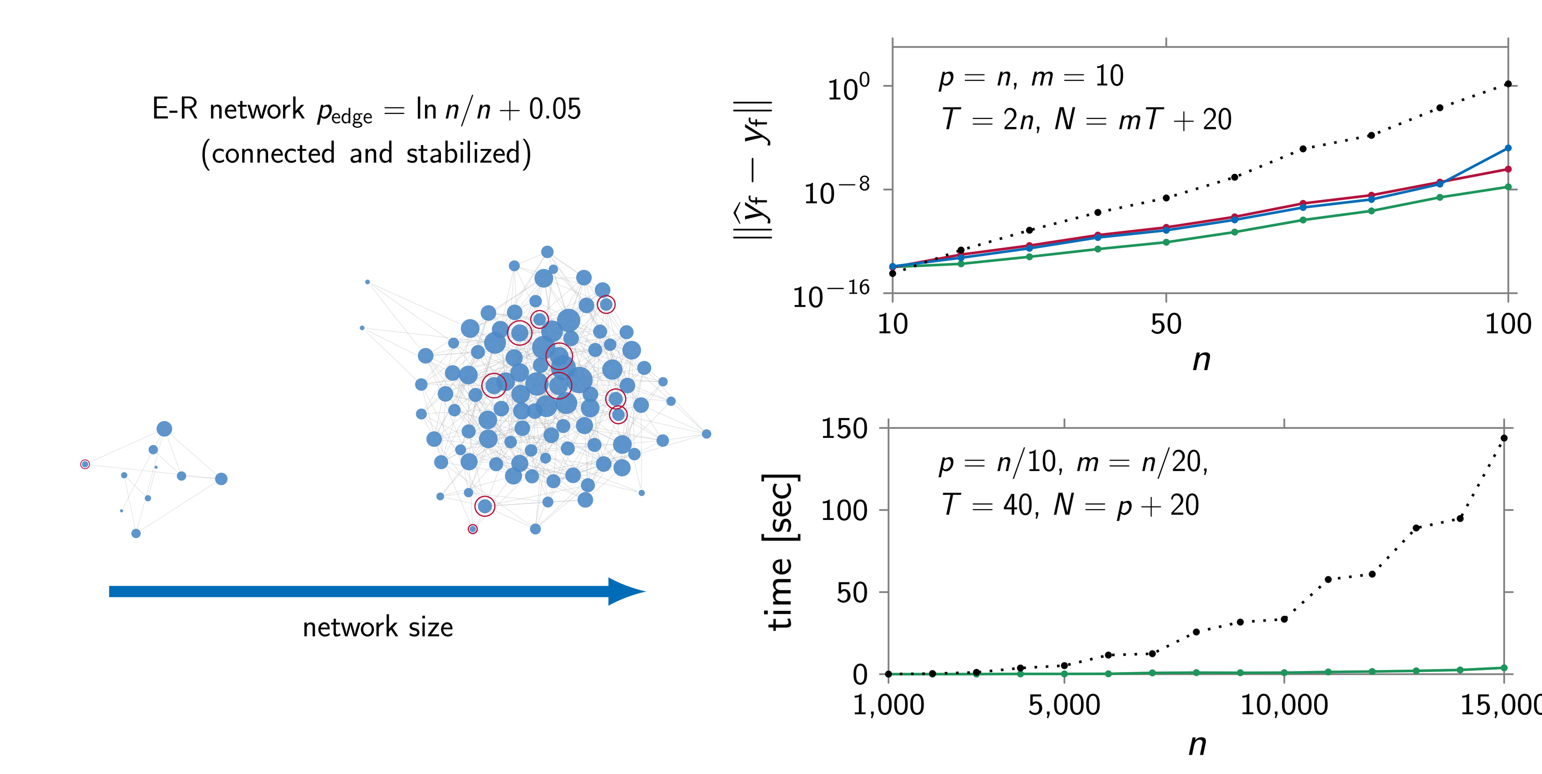
→ Asymptotically correct

Numerical performance

⋯ Model-based — 1 — 2 — 3

n = network size, m = # control nodes, p = # output nodes





*Software: Matlab 2018b | Hardware: 2.6 GHz Intel Core i5, 8GB RAM

Take-home message

Minimum-energy network controls can be computed directly from ***non-optimal*** and ***noisy data*** via closed-form expressions that are ***numerically more reliable*** and ***cheaper*** than model-based ones

Future work

- Nonasymptotic error bounds?
- Robustness to attacks?
- Network classification from data-driven control metrics?

G. Baggio, V. Katewa, F. Pasqualetti, "Data-Driven Minimum-Energy Controls for Linear Systems", *IEEE Control Systems Letters*, 3(3), pp. 589–594, 2019.