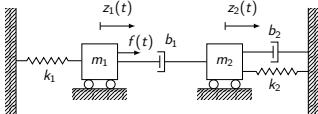


Esercizio 1



Rappresentazione interna o di stato con  $u(t) = f(t)$  e  $y(t) = [z_1 \ z_2]^\top$ ?

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$f(t) = \text{forza esterna} = \text{input}$

$\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \text{uscita}$

Rappresentazione di sistema:

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} \quad \begin{aligned} x_1(t) &= z_1(t) \\ x_2(t) &= z_2(t) \\ x_3(t) &= \dot{z}_1(t) \\ x_4(t) &= \dot{z}_2(t) \end{aligned}$$

Legge di Newton:

$$1) m_1 \ddot{z}_1 = f - k_1 z_1 - b_1 (\dot{z}_1 - \dot{z}_2)$$

$$2) m_2 \ddot{z}_2 = -k_2 z_2 - b_2 \dot{z}_2 + b_1 (\dot{z}_1 - \dot{z}_2)$$

$$\dot{x}_1 = \dot{z}_1 = x_3$$

$$\dot{x}_2 = \dot{z}_2 = x_4$$

$$\dot{x}_3 = \ddot{z}_1 = \frac{1}{m_1} (f - k_1 z_1 - b_1 \dot{z}_1 + b_1 \dot{z}_2) = \frac{1}{m_1} u - \frac{k_1}{m_1} x_1 - \frac{b_1}{m_1} x_3 + \frac{b_1}{m_1} x_4$$

$$\dot{x}_4 = \ddot{z}_2 = \frac{1}{m_2} (-k_2 z_2 - b_2 \dot{z}_2 + b_1 \dot{z}_1 - b_1 \dot{z}_2) = -\frac{k_2}{m_2} x_2 - \frac{b_1 + b_2}{m_2} x_4 + \frac{b_1}{m_2} x_3$$

$$\left\{ \begin{array}{l} \dot{x} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1}{m_1} & 0 & -\frac{b_1}{m_1} & \frac{b_1}{m_1} \\ 0 & -\frac{k_2}{m_2} & \frac{b_1}{m_2} & -\frac{b_1+b_2}{m_2} \end{bmatrix}}_F x + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1/m_1 \\ 0 \end{bmatrix}}_G u \end{array} \right.$$

$$\left. \begin{array}{l} y = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_H x + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_J u \end{array} \right.$$

Esercizio 2 [riadattato da Es. 1 tema d'esame 1 febbraio 2012]

$$\dot{x}(t) = Fx(t) + Gu(t) = \begin{bmatrix} 0 & 1 & 0 \\ 2\alpha & \alpha - 2 & 0 \\ 2 & 0 & 2 - \alpha^2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u(t), \quad \alpha \in \mathbb{R}$$

$$y(t) = Hx(t) = \begin{bmatrix} 1 & -2 & 0 \end{bmatrix} x(t)$$

1. Forma di Jordan  $F_J$  e i modi del sistema al variare di  $\alpha \in \mathbb{R}$ ?

2. Funzione di trasferimento  $W(s)$  al variare di  $\alpha \in \mathbb{R}$ ?

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$$F = \left[ \begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 2\alpha & \alpha-2 & 0 & 0 \\ 2 & 0 & 2-\alpha^2 & 0 \end{array} \right] \quad G = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad H = \begin{bmatrix} 1 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} F_{11} & 0 \\ F_{21} & F_{22} \end{bmatrix}$$

1) Forma di Jordan  $F_J$  e modi del sistema:

$$\lambda(F) = \lambda(F_{11}) \cup \lambda(F_{22})$$

(i) Calcolo autovalori  $F$ :

$$\Delta_{F_{11}}(\lambda) = \det(\lambda I - F_{11}) = \det \begin{bmatrix} \lambda & -1 \\ -2\alpha & \lambda - \alpha + 2 \end{bmatrix} = \lambda(\lambda - \alpha + 2) - 2\alpha$$

$$= \lambda^2 + (2 - \alpha)\lambda - 2\alpha$$

$$\lambda(F) = \{\alpha, -2, 2 - \alpha^2\}$$

1  $\alpha = -2$ :  $\lambda_1 = -2, v_1 = 3$

2  $\alpha = 2$ :  $\lambda_1 = -2, v_1 = 1 = g_1$

$\alpha = 2$ :  $\lambda_1 = -2, v_1 = 1 = g_1$

3  $\alpha = 2 - \alpha^2$ :  $\alpha^2 + \alpha - 2 = 0$   
 $(\alpha - 1)(\alpha + 2) = 0$

$\alpha = 1$ :  $\lambda_1 = -2, v_1 = 1 = g_1$

$\alpha = 1$ :  $\lambda_2 = 1, v_1 = 2$

$$\lambda_{1,2} = \frac{-2 + \alpha \pm \sqrt{(2 - \alpha)^2 + 8\alpha}}{2}$$

$$= \frac{-2 + \alpha \pm \sqrt{(\alpha + 2)^2}}{2}$$

$$= \frac{-2 + \alpha \pm (\alpha + 2)}{2}$$

$$\frac{-1 + \alpha + \alpha + 2}{2} = \alpha$$

$$\frac{-2 + \alpha - \alpha - 2}{2} = -2$$

4  $\alpha \notin \{\pm 2, 1\}$ :  $\lambda_1 = \alpha, v_1 = 1 = g_1$   
 $\lambda_2 = -2, v_2 = 1 = g_2$   
 $\lambda_3 = 2 - \alpha^2, v_3 = 1 = g_3$

$F$  diagonalizzabile

(ii) Calcola delle molteplicità geometriche:

1  $\alpha = -2$ :  $\lambda_1 = -2$   $v_1 = \underline{3}$

$$g_1 = 3 - \text{rank}(\lambda_1 I - F) = 3 - \text{rank} \begin{bmatrix} -2 & -1 & 0 \\ -2\alpha & -2 & 0 \\ -2 & 0 & -4 + \alpha^2 \end{bmatrix}$$
$$= 3 - \text{rank} \begin{bmatrix} -2 & -1 & 0 \\ 4 & 2 & 0 \\ -2 & 0 & 0 \end{bmatrix} = 3 - 2 = 1$$

2  $\alpha = 2$ :  $\lambda_1 = -2$   $v_1 = g_1 = 1$   
 $\lambda_2 = 2$   $v_2 = \underline{2}$

$$g_2 = 3 - \text{rank}(\lambda_2 I - F) = 3 - \text{rank} \begin{bmatrix} 2 & -1 & 0 \\ -2\alpha & 4 - \alpha & 0 \\ -2 & 0 & \alpha^2 \end{bmatrix}$$
$$= 3 - \text{rank} \begin{bmatrix} 2 & -1 & 0 \\ 4 & 2 & 0 \\ -2 & 0 & 4 \end{bmatrix} = 3 - 2 = 1$$

$$v_1 = -2v_2 - \frac{1}{2}v_3$$

3  $\alpha = 1$ :  $\lambda_1 = -2$   $v_1 = g_1 = 1$   
 $\lambda_2 = 1$   $v_2 = \underline{2}$

$$g_2 = 3 - \text{rank}(\lambda_2 I - F) = 3 - \text{rank} \begin{bmatrix} 1 & -1 & 0 \\ -2\alpha & 3 - \alpha & 0 \\ -2 & 0 & -1 + \alpha^2 \end{bmatrix}$$
$$= 3 - \text{rank} \begin{bmatrix} 1 & -1 & 0 \\ -2 & 2 & 0 \\ -2 & 0 & 0 \end{bmatrix} = 3 - 2 = 1$$

(iii) Calcolo  $F_J$  e modi:

[1]  $\alpha = -2$ :  $\lambda_1 = -2$ ,  $v_1 = g_1 = 1$

$$F_J = \begin{bmatrix} -2 & 1 \\ -2 & 1 \\ -2 & \end{bmatrix} \quad \text{modi: } e^{-2t}, te^{-2t}, \frac{t^2}{2}e^{-2t}$$

[2]  $\alpha = 2$ :  $\lambda_1 = -2$ ,  $v_1 = g_1 = 1$   
 $\lambda_2 = 2$ ,  $v_2 = 2$ ,  $g_2 = 1$

$$F_J = \begin{bmatrix} -2 & & \\ & 2 & 1 \\ & & 2 \end{bmatrix} \quad \text{modi: } e^{-2t}, e^{2t}, te^{2t}$$

[3]  $\alpha = 1$ :  $\lambda_1 = -2$ ,  $v_1 = g_1 = 1$   
 $\lambda_2 = 1$ ,  $v_2 = 2$ ,  $g_2 = 1$

$$F_J = \begin{bmatrix} -2 & & \\ & 1 & 1 \\ & & 1 \end{bmatrix} \quad \text{modi: } e^{-2t}, e^t, te^t$$

[4]  $\alpha \notin \{\pm 2, 1\}$      $\lambda_1 = -2$      $v_1 = g_1 = 1$   
                 $\lambda_2 = \alpha$      $v_2 = g_2 = 1$   
                 $\lambda_3 = 2 - \alpha^2$      $v_3 = g_3 = 1$

$$F_J = \begin{bmatrix} -2 & & \\ & \alpha & \\ & & 2 - \alpha^2 \end{bmatrix} \quad \text{modi: } e^{-2t}, e^{\alpha t}, e^{(2-\alpha^2)t}$$

2) Calcolo di  $W(s)$

$$F = \left[ \begin{array}{ccc|c} 0 & 1 & 0 & \\ 2\alpha & \alpha-2 & 0 & \\ \hline 2 & 0 & 2-\alpha^2 & \\ \hline F_{11} & 0 & \\ F_{21} & F_{22} \end{array} \right] \quad G = \left[ \begin{array}{c} 1 \\ 1 \\ 0 \end{array} \right] \xrightarrow{G_1} H = \left[ \begin{array}{c|c} \overbrace{1}^{H_1} & \\ \hline 1-2 & 0 \end{array} \right]$$

$$W(s) = H(sI - F)^{-1} G$$

$$\stackrel{|}{=} \left[ \begin{array}{cc} H_1 & 0 \end{array} \right] \left[ \begin{array}{cc} sI - F_{11} & 0 \\ -F_{21} & sI - F_{22} \end{array} \right]^{-1} \left[ \begin{array}{c} G_1 \\ 0 \end{array} \right]$$

$$\stackrel{|}{=} \left[ \begin{array}{cc} H_1 & 0 \end{array} \right] \left[ \begin{array}{cc} (sI - F_{11})^{-1} & 0 \\ * & (sI - F_{22})^{-1} \end{array} \right] \left[ \begin{array}{c} G_1 \\ 0 \end{array} \right]$$

$$\stackrel{|}{=} \left[ \begin{array}{cc} H_1 (sI - F_{11})^{-1} & 0 \end{array} \right] \left[ \begin{array}{c} G_1 \\ 0 \end{array} \right] = H_1 (sI - F_{11})^{-1} G_1$$

$$\stackrel{|}{=} \left[ \begin{array}{cc} 1 & -2 \end{array} \right] \left[ \begin{array}{cc} s & -1 \\ -2\alpha & s-\alpha+2 \end{array} \right]^{-1} \left[ \begin{array}{c} 1 \\ 1 \end{array} \right] = \left[ \begin{array}{cc} 1 & -2 \end{array} \right] \frac{1}{s(s-\alpha+2)-2\alpha} \left[ \begin{array}{cc} s-\alpha+2 & 1 \\ 2\alpha & s \end{array} \right] \left[ \begin{array}{c} 1 \\ 1 \end{array} \right]$$

$$\stackrel{|}{=} \frac{-s + 3 - 5\alpha}{s^2 + (2-\alpha)s - 2\alpha}$$

N.B.

$$\left[ \begin{array}{cc} A & 0 \\ B & C \end{array} \right]^{-1} = \left[ \begin{array}{cc} A^{-1} & 0 \\ * & C^{-1} \end{array} \right]$$

### Esercizio 3

$$\dot{x}(t) = Fx(t) = \begin{bmatrix} 0 & \alpha & 0 \\ \beta & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} x(t)$$

$$y(t) = Hx(t) = [1 \ 1 \ 1] x(t)$$

1. Per  $\alpha = \beta = 1$ ,  $x(0) \neq 0$  tali che  $y(t)$  non è divergente?

2. Per  $\alpha = 1$  e  $\beta = -1$ ,  $x(0) \neq 0$  tali che  $y(t)$  è limitata?

$$\begin{aligned} F &= \left[ \begin{array}{ccc|c} 0 & \alpha & 0 & 0 \\ \beta & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{array} \right] \quad H = [1 \ 1 \ 1] \\ &= \begin{bmatrix} F_{11} & 0 \\ 0 & F_{22} \end{bmatrix} \end{aligned}$$

$$1) \underline{\alpha = \beta = 1}$$

$$y(t) = H e^{Ft} x(0)$$

$$\text{Calcolo di } e^{Ft}: \quad e^{Ft} = \begin{bmatrix} e^{F_{11}t} & 0 \\ 0 & e^{F_{22}t} \end{bmatrix} \rightarrow e^{F_{11}t}$$

$$\text{Calcolo } e^{F_{11}t} = T e^{F_0 t} T^{-1} \quad T^{-1} F T = F_D$$

$$(i) \text{ Autovettori di } F_{11}: \quad \Delta_{F_{11}}(\lambda) = \det(\lambda I - F_{11}) = \det \begin{bmatrix} \lambda & -1 \\ -1 & \lambda \end{bmatrix} = \lambda^2 - 1$$

$$\left. \begin{array}{ll} \lambda_1 = 1 & v_1 = 1 = g_1 \\ \lambda_2 = -1 & v_2 = 1 = g_2 \end{array} \right\} F_{11} \text{ diagonalizzabile}$$

(ii) Autovettori di  $F_{11}$ :

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \text{ autovettore di } \lambda_1 = 1: \quad (\lambda_1 I - F_{11}) v = 0 \quad \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left. \begin{array}{l} v_1 = v_2 \\ \parallel \end{array} \right\} v = \begin{bmatrix} \alpha \\ \alpha \end{bmatrix} \alpha \in \mathbb{R}$$

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \text{ autovettore di } \lambda_2 = -1: \quad (\lambda_2 I - F_{11}) v = 0 \quad \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\text{Matrice di cambio base: } T = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\left. \begin{array}{l} v_1 = -v_2 \\ \parallel \end{array} \right\} v = \begin{bmatrix} \alpha \\ -\alpha \end{bmatrix} \alpha \in \mathbb{R}$$

$$e^{Ft} = \begin{bmatrix} e^{F_{11}t} & 0 \\ 0 & e^{F_{22}t} \end{bmatrix} = \begin{bmatrix} T e^{F_{11,0}t} T^{-1} & 0 \\ 0 & e^{2t} \end{bmatrix} \quad \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} = \frac{1}{2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{aligned} T e^{F_{11,0}t} T^{-1} &= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} e^t & 0 \\ 0 & e^{-t} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} \\ &= \frac{1}{2} \begin{bmatrix} e^t & e^{-t} \\ e^t & -e^{-t} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} e^t + e^{-t} & e^t - e^{-t} \\ e^t - e^{-t} & e^t + e^{-t} \end{bmatrix} \end{aligned}$$

$$\begin{aligned} y(t) &= H e^{Ft} x(0) = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} e^t + e^{-t} & e^t - e^{-t} & 0 \\ e^t - e^{-t} & e^t + e^{-t} & 0 \\ 0 & 0 & 2e^{2t} \end{bmatrix} \begin{bmatrix} x_{0,1} \\ x_{0,2} \\ x_{0,3} \end{bmatrix} \\ &= \begin{bmatrix} e^t & e^t & e^{2t} \end{bmatrix} \begin{bmatrix} x_{0,1} \\ x_{0,2} \\ x_{0,3} \end{bmatrix} \\ &= e^t x_{0,1} + e^t x_{0,2} + e^{2t} x_{0,3} \quad x(0) = \begin{bmatrix} \gamma \\ -\gamma \\ 0 \end{bmatrix} \quad \gamma \in \mathbb{R} \\ &= 0 \quad \leftarrow \end{aligned}$$

$$2) \alpha = 1, \beta = -1$$

$$F = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad e^{Ft} = \begin{bmatrix} e^{\alpha_1 t} & 0 \\ 0 & e^{\alpha_2 t} \end{bmatrix}$$

$$= \begin{bmatrix} \cos t & \sin t & 0 \\ -\sin t & \cos t & 0 \\ 0 & 0 & e^{2t} \end{bmatrix}$$

$$y(t) = H e^{Ft} x(0) = [1 \ 1 \ 1] \begin{bmatrix} \cos t & \sin t & 0 \\ -\sin t & \cos t & 0 \\ 0 & 0 & e^{2t} \end{bmatrix} \begin{bmatrix} x_{0,1} \\ x_{0,2} \\ x_{0,3} \end{bmatrix}$$

$$= [\cos t - \sin t \quad \cos t + \sin t \quad e^{2t}] \begin{bmatrix} x_{0,1} \\ x_{0,2} \\ x_{0,3} \end{bmatrix}$$

$$x(0) = \begin{bmatrix} y_1 \\ y_2 \\ 0 \end{bmatrix} \quad y_1, y_2 \in \mathbb{R}$$

(almeno uno fra  $y_1$  e  $y_2 \neq 0$ )

$$x(t+1) = Fx(t) + Gu(t) = \begin{bmatrix} -3 & 1 \\ 0 & 2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = x(t)$$

1. Modi del sistema e loro carattere?

2. Matrice di trasferimento  $W(z)$ ?3. Evoluzione del sistema per  $x(0) = \begin{bmatrix} \frac{1}{10} \\ 0 \end{bmatrix}$  e  $u(t) = \delta(t)$ ,  $t \geq 0$ ?

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$$F = \left[ \begin{array}{cc|c} -3 & 1 & \\ \hline 0 & 2 & \end{array} \right] \quad G = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

1) Modi e carattere.

$$\text{Autovalori di } F: \lambda_1 = -3 \quad v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = g_1 \\ \lambda_2 = 2 \quad v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = g_2$$

Forma di Jordan:  $F_J = \begin{bmatrix} -3 & \\ & 2 \end{bmatrix}$  modi:  $(-3)^t, 2^t$  divergenti

$$2) W(z) = H(zI - F)^{-1} G$$

$$= \begin{bmatrix} z+3 & -1 \\ 0 & z-2 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{(z+3)(z-2)} \begin{bmatrix} z-2 & 1 \\ 0 & z+3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{(z+3)(z-2)} \begin{bmatrix} 1 \\ z+3 \end{bmatrix} = \begin{bmatrix} \frac{1}{(z+3)(z-2)} \\ \frac{1}{z-2} \end{bmatrix}$$

$$3) x(0) = \begin{bmatrix} 1/10 \\ 0 \end{bmatrix} \quad u(t) = \delta(t) = \begin{cases} 1 & t=0 \\ 0 & t \neq 0 \end{cases}$$

$$X(z) = \underbrace{z(zI - F)^{-1} x(0)}_{X_L(z)} + \underbrace{W(z) U(z)}_{X_F(z)}$$

$$\frac{1}{10} (-3)^t$$

$\uparrow$   
 $z^{-1}$

$$X_L(z) = \frac{z}{(z+3)(z-2)} \begin{bmatrix} z-2 & 1 \\ 0 & z+3 \end{bmatrix} \begin{bmatrix} 1/10 \\ 0 \end{bmatrix} = \frac{z}{(z+3)(z-2)} \begin{bmatrix} \frac{1}{10}(z-2) \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{10} \frac{z}{z+3} \\ 0 \end{bmatrix}$$

$$X_f(z) = W(z) \overset{''}{U}(z) = \begin{bmatrix} 1 \\ \frac{1}{(z+3)(z-2)} \\ \frac{1}{z-2} \end{bmatrix} = \begin{bmatrix} X_{f,1}(z) \\ X_{f,2}(z) \end{bmatrix}$$

Calcolo di  $x_{f,1}(t)$ :

$$X_{f,1}'(z) = \frac{X_{f,1}(z)}{z} = \frac{1}{z(z+3)(z-2)} = \frac{A_1}{z} + \frac{A_2}{z+3} + \frac{A_3}{z-2}$$

$$A_1 = -\frac{1}{6}, \quad A_2 = \frac{1}{15}, \quad A_3 = \frac{1}{10}$$

$$X_{f,1}(z) = z X_{f,1}'(z) = A_1 + \frac{A_2 z}{z+3} + \frac{A_3 z}{z-2}$$

$\mathcal{Z}^{-1} \downarrow \quad \mathcal{Z}^{-1} \downarrow \quad \mathcal{Z}^{-1} \downarrow$

$$A_1 \delta(t) \quad A_2 (-3)^t \quad A_3 2^t$$

Calcolo di  $x_{f,2}(t)$ :

$$X_{f,2}'(z) = \frac{X_{f,2}(z)}{z} = \frac{1}{z(z-2)} = \frac{A_1}{z} + \frac{A_2}{z-2} \quad A_1 = -\frac{1}{2}, \quad A_2 = \frac{1}{2}$$

$$X_{f,2} = z X_{f,2}'(z) = A_1 + \frac{A_2 z}{z-2}$$

$\mathcal{Z}^{-1} \downarrow \quad \mathcal{Z}^{-1} \downarrow$

$$A_1 \delta(t) \quad A_2 2^t$$