Practice 3
Image Processing with Partial Differential Equations

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1 Introduction

In this Lab Assignment, the problem of Segmentation of grayscale images is tackled and implemented using a simple binary segmentation approach. This approach treats the segmentation task as the estimation of the contour of the object of interest from an image. The estimation task is first stated as a minimization problem of function. Particularly, this minimization problem was solved using the Chan-vese method, which uses the level set information for defining the problem, Chan et. which adds a convex approach, and a dual total variation formulation method was visited. The techniques will be explained in detail throughout the report.

2 Segmentation Technique Formulation

Figure 1 shows the main goal of this assignment. The image on the left side is the input image with 4 coins, and the image on the right side is the target image, the 4 coins segmented from the rest of the background. To achieve this result, The binary segmentation method estimates the binary mask ($\mu : x \in \omega \rightarrow 0, 1$) that separates the input image in two different areas. the foreground and the corresponding background part of the image.

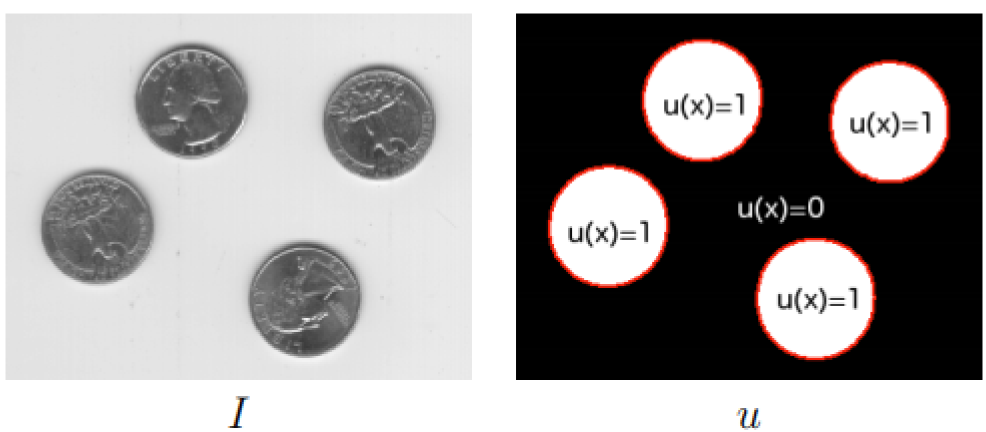

Figure 1: Left: Input Image, Right: Segmented Image

The Segmentation problem is Formulated As the minimization of:

$$(u*, c1*, c2*) = arg\, min \int |Du| + \lambda \int |I(x) - c1|^2 u(x)dx + \lambda \int |I(x) - c2|^2 (I - u(x))dx \tag{1}$$

The regularization term penalizes the perimeter of the segmented area. The parameter λ controls the influence of the regularization term. In order to minimize the data terms of (1.1), the pixels x which gray value I(x) is closer to c1 than to c2 will encourage to have u(x) = 1

3 Chan-Vese level set formulation

Chan and Vese use the concept of level set formulation to solve the binary segmentation problem. It uses a surface ($\phi : x \in \omega \rightarrow R$) as a representation of the level set. Assigning the image value to the foreground, contour, or image is done based on this value. When ($\phi = 0$) the level set shows the contour of the desired object in the image domain, when ($\phi < 0$) it is foreground image and ($\phi > 0$) represents the background image.

Different Initializations

The initial value of level set ϕ is an essential factor to consider when working with Chan-Vese level set techniques. Due to the fact that the method requires an initial mask of the contour to evolve and iteratively reach the goal of segmenting the object of interest. If the initialization value is not good or does not generalize the desired object, It can be trapped in the local minima and give an unwanted outcome. This effect of initialization can be clearly shown in the following figures.

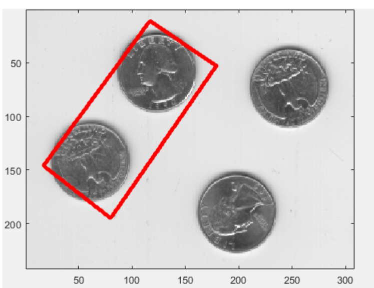

Figure 2: Initial mask

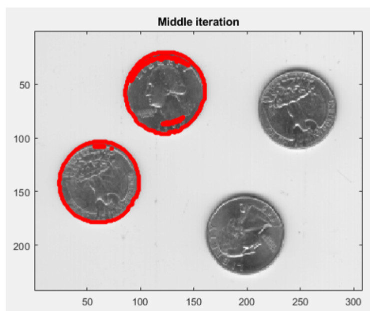

Figure 3: K=500 middle Iteration

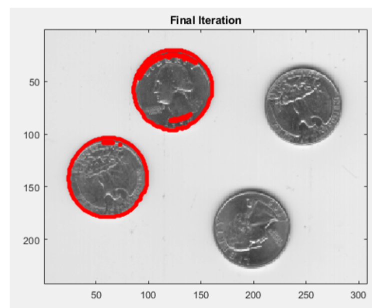

Figure 4: K=100 Final

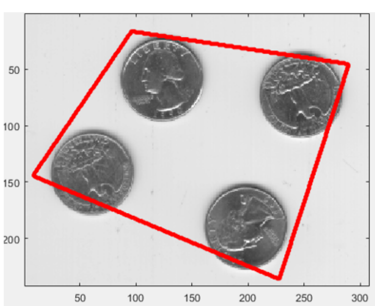

Figure 5: Initial mask

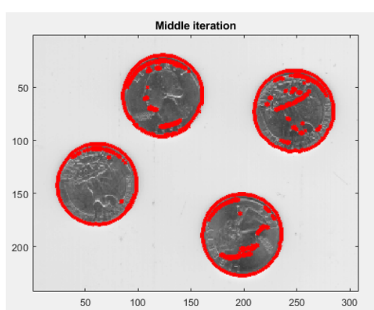

Figure 6: K=500 middle Iteration

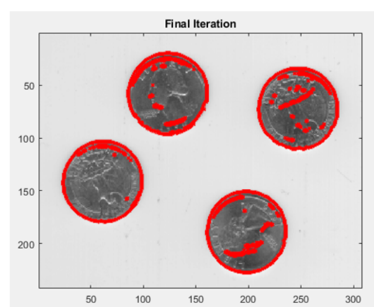

Figure 7: K=100 Final

Figure 8: Different Initialization of ϕ Illustration

Effect of the parameter η

η is the parameter of the Heaviside function $H(\phi(x))$. The effect of this parameter is experimented using different values. η affects the convergence speed of the task. When the value of η increases the segmentation converges slower. As can be observed from the following figures η value of 0.1 converges fast as compared to η value of 1.

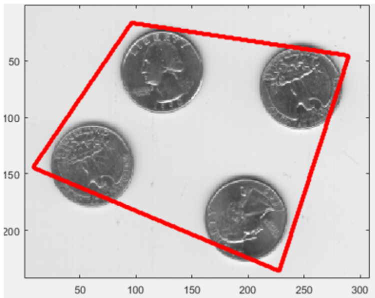

Figure 9: Initial mask

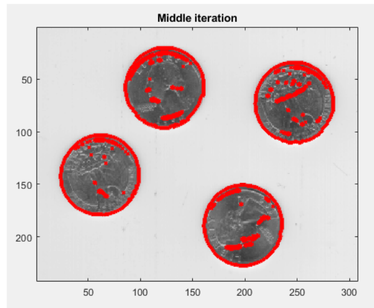

Figure 10: K=150, $\eta = 0.1$

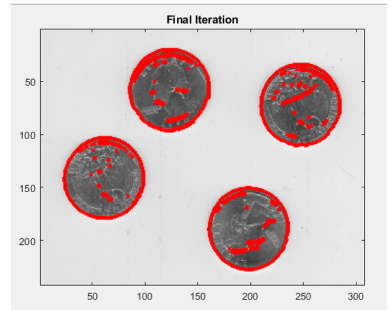

Figure 11: Final, $\eta = 0.1$

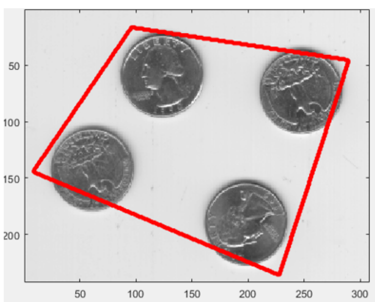

Figure 12: Initial mask

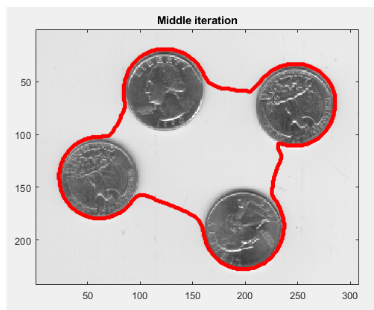

Figure 13: K=150, $\eta = 1$

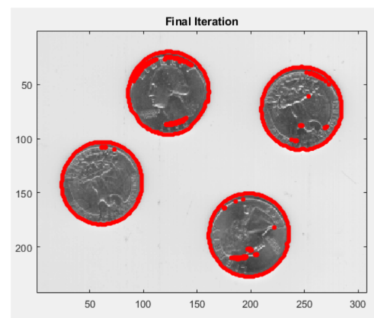

Figure 14: Final, $\eta = 1$

Figure 15: Effect of parameter η

Number of re initialization of ϕ n

Another important aspect to be considered when working with Chan is the update of the level set information to its 0 level set. This is done to make the iteration get back on track and for it not to get lost after some iterations. The parameter n here is used to determine after how many iterations to reset the level set. The effect of this was particularly observed in the case of the first type of iteration discussed in the previous section. It can be observed from figures 16-21, that when the value of n for reinitializing the level set to 0 was 10, the algorithm was stuck on the local minima and segment only the two coins. However, when the value of n was increased to 50, the algorithm was able to segment all the coins.

From this experiment, we can say that if the initial value of the mask is centered only to a certain object of interest, frequent updating of the ϕ will force it to only check the values of the pixels nearby and get stuck in local minima. While giving it a little bit freedom can help reach the global minima and segment the desired objects.

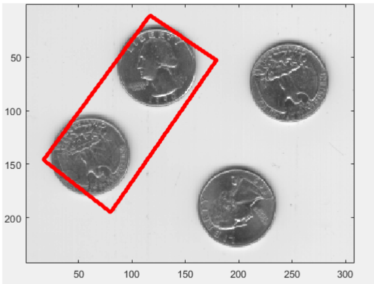

Figure 16: Initial mask

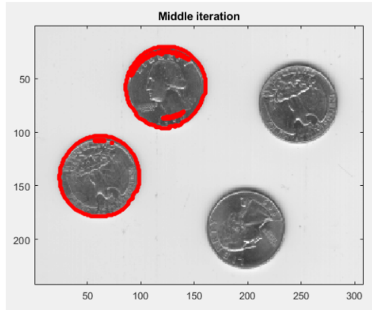

Figure 17: K=500, n=10

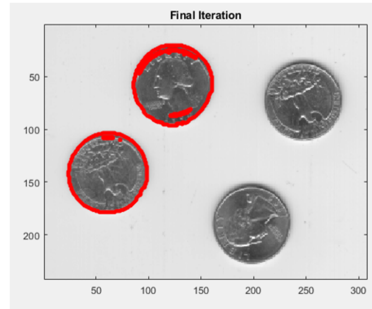

Figure 18: K=1000 Final

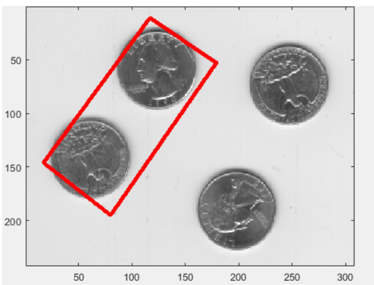

Figure 19: Initial MAsk

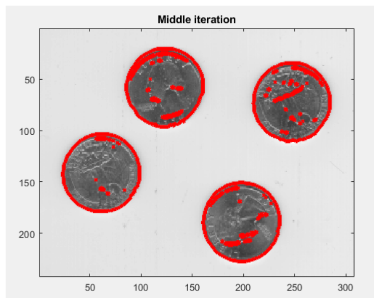

Figure 20: K=500, n=50

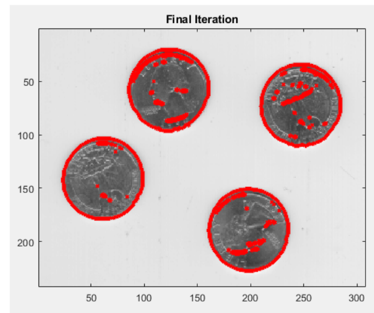

Figure 21: K=1000 Final

Figure 22: Effect of n: first figs: n=10, Second figs: n=50

Noisy Image

The method was tested using a noisy image with a standard deviation of 30. As it can be detected from the following figure, The estimation of a contour approach was able to detect the object of interest with some artifacts and errors.

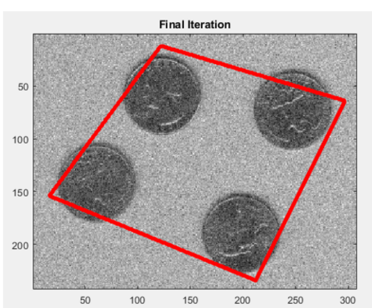

Figure 23: Initial mask

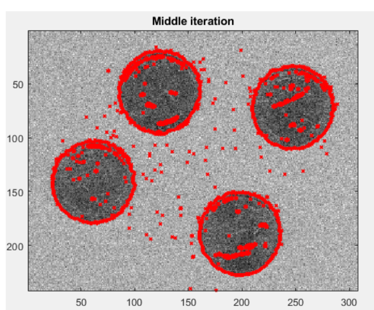

Figure 24: K=500 middle Iteration

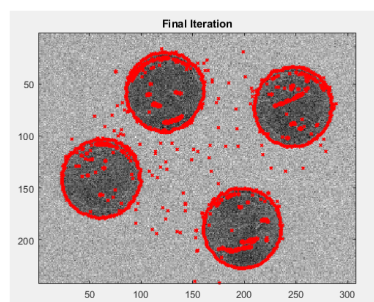

Figure 25: K=100 Final

Figure 26: Segmentation in noisy image

4 Chan, Esedoglu and Nikolova convex formulation

Another insight into solving the minimization problem was introduced by Chan, Esedoglu, and Nikolova. They added relaxation energy to the minimization problem, instead of totally relying on the level set information. It lets u(x) to take its values from continuous interval[0;1] and is convex function.

The result of the Chan, Esedoglu, and Nikolova convex formulation using η =1 , K=1000, is demonstrated in the following figures.

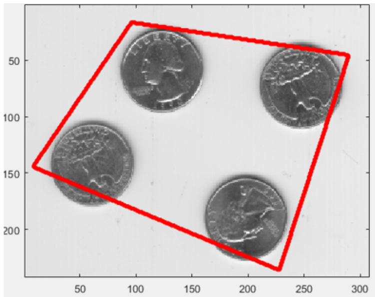

Figure 27: Initial mask

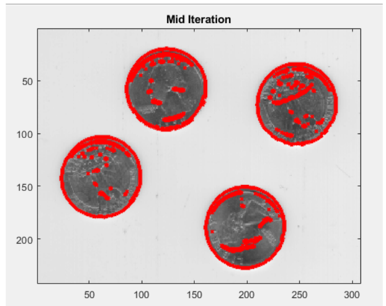

Figure 28: K=500 middle Iteration

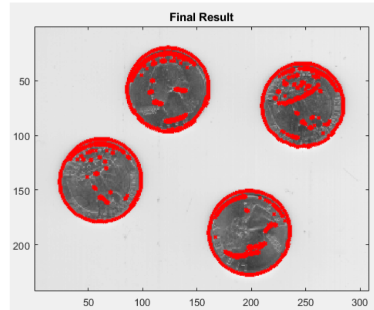

Figure 29: K=100 Final

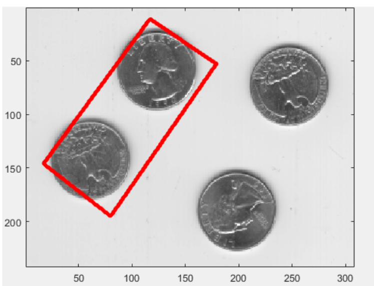

Figure 30: Initial mask

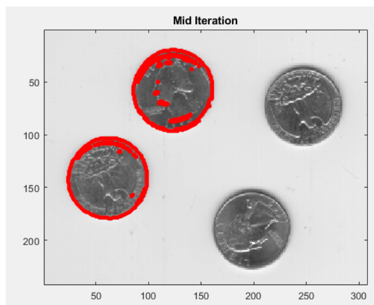

Figure 31: K=500 middle Iteration

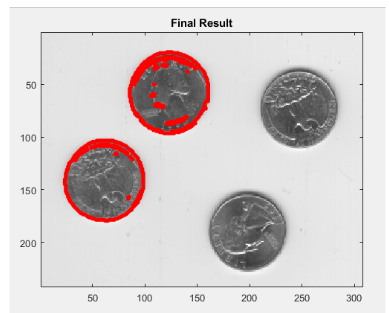

Figure 32: K=100 Final

Figure 33: Different Initialization of ϕ

Initializations Effect

This method yields similar results to the first one. The sensibility to the initialization can be seen from the above figure. It can be seen that the second initialization was stuck in the local minima. However, as explained before the convergence can be led to the global minima by reinitializing the value of ϕ at larger iteration intervals.

Comparision of the two methods

The comparison between using the level set information and convexity can be explained using the energy function which is plotted in figure 36. Chan, Esedoglu and Nikolova convex method converge fast after around 75 iterations, while the model with level set information takes more time up to 200 iterations to reach convergence.

5 Dual Formulation of the Total Variation

In this, the dual formulation of Total variation is considered which a key point for dealing with non-differentiability when $|\nabla u|$ is0.

The results obtained using this approach are similar to the previous one. The key difference that can be dictated is the effect of the initialization of ϕ. In Chan, Vese method the initialization had a big

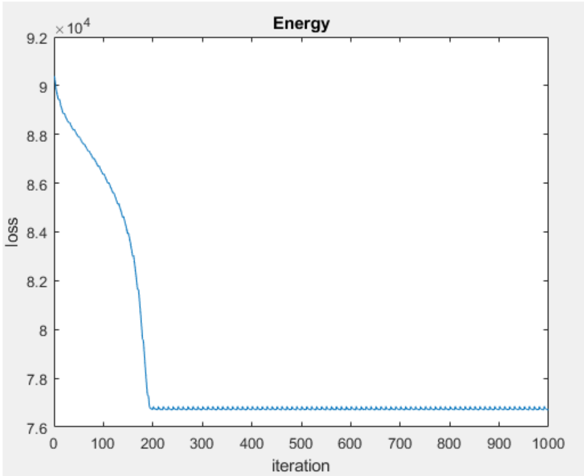

Figure 34: Chan Vese

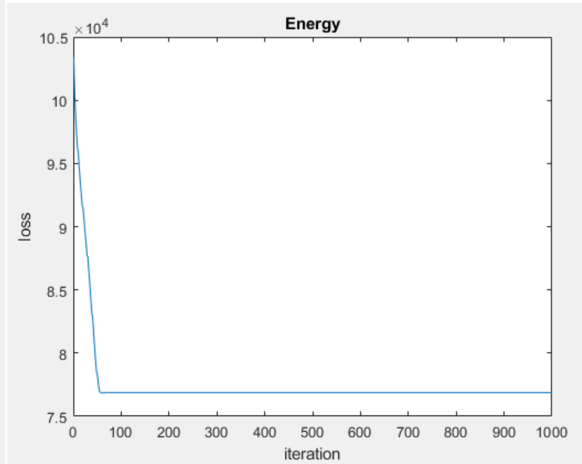

Figure 35: Nikolova Convex

Figure 36: Energy propagation for Chan Vese and Nikolova Convex

effect which leads the output to get stuck in the local minimum when there is frequent resetting of ϕ. While Here in the dual Formulation we can see in the following figure, regardless of the initialization the result converges to the desired outcome.

The second comparison is the convergence speed. This dual formulation converges faster than the previous approach. This is displayed in the Figure 44.

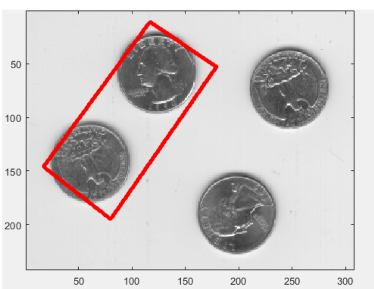

Figure 37: Initial mask

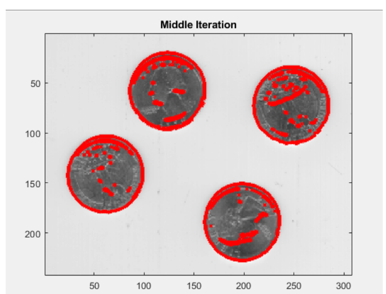

Figure 38: K=500 middle Iteration

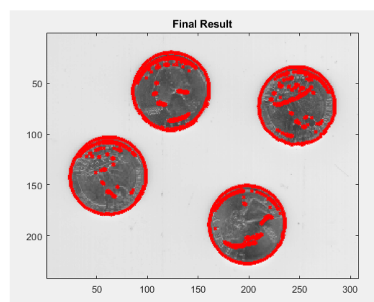

Figure 39: K=100 Final

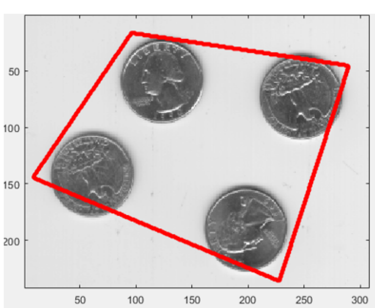

Figure 40: Initial mask

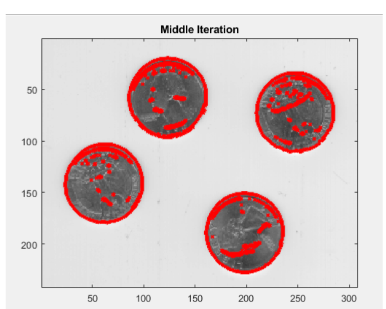

Figure 41: K=500 middle Iteration

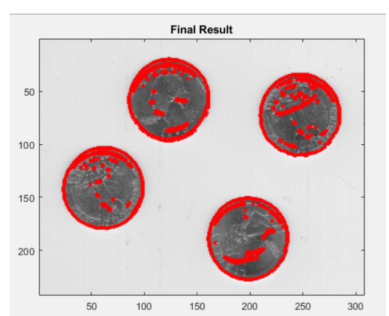

Figure 42: K=100 Final

Figure 43: Different Initialization of ϕ

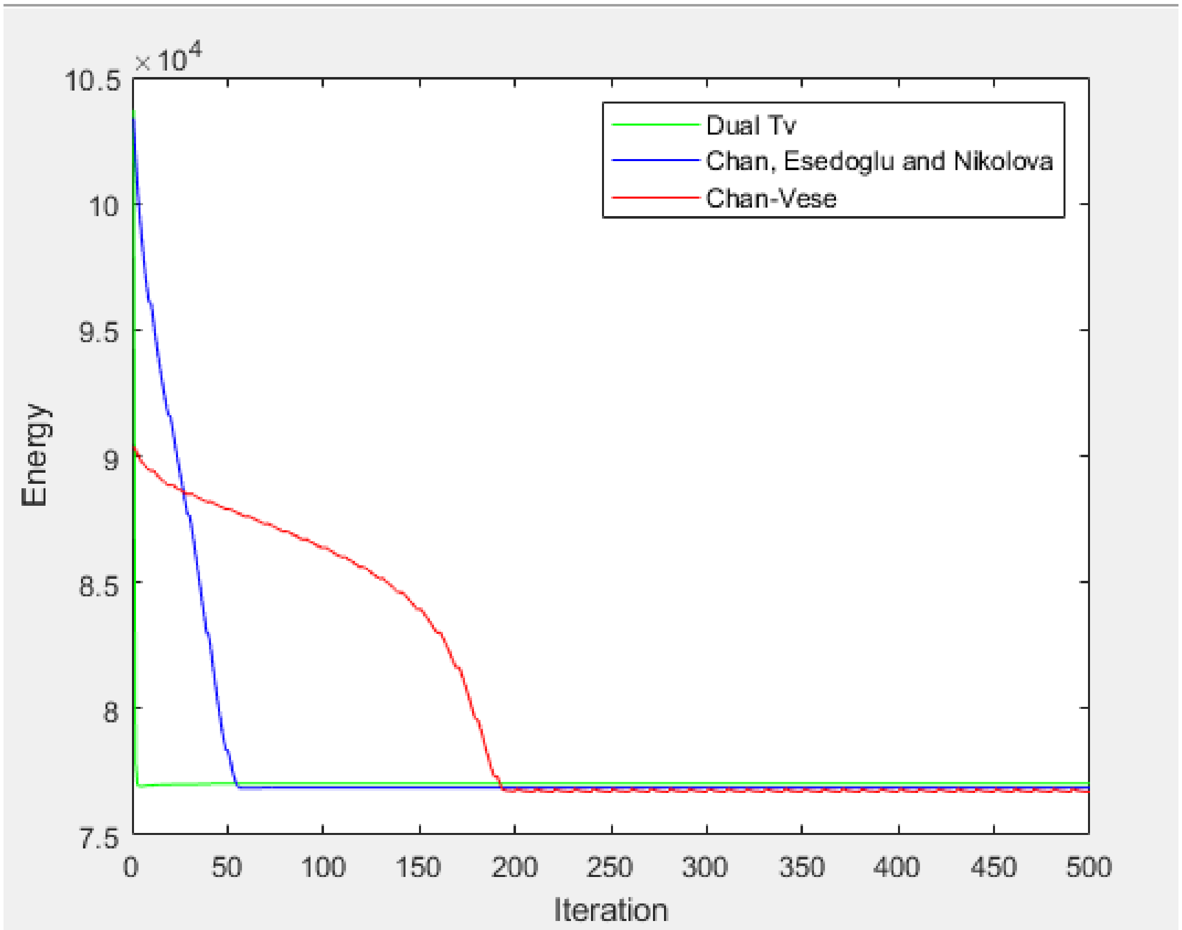

Figure 44: Energy Function for the three methods

6 Conclusion

To conclude the Matlab work, the binary segmentation problem is addressed as an estimation of the contour of an object of interest from an image and was modeled as a minimization problem. Different algorithms like Chan-Ves, Chan, Esedoglu, and Nikolova, and Dual Formulation Techniques were used. They attempt to solve the minimization problem using different assumptions. The first uses the level set information to model it and is dependant on the initialization. The second algorithm formulates the problem as a convex formulation which is limited by the non-differentiability problem. The third algorithm addresses this non-differntiability problem based on the dual total variation. Comparison among the visited methods was assessed using the energy function. The Dual total variation gives fast convergence speed in relative to the others. It at the same time was robust for handling different initialization effects.