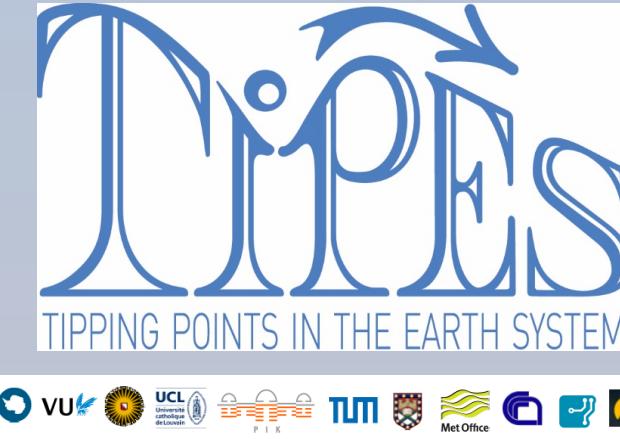


Tipping points and abrupt climate change: A comparison of advanced analysis methods for paleoclimate records



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1. MOTIVATION

Introduction

- Present-day global warming could lead to crossing of tipping points (TPs), bringing dramatic changes to the climate system.
- Records of past climate exhibit abrupt transitions, which may represent TPs in the Earth system's past.
- Identifying and comparing TPs in paleoclimate records is complicated as the records vary in their resolution, time spans, and periodicities.
- As a result, TPs in paleoclimate records are often not properly identified.
- An objective, automated methodology for detecting abrupt transitions is crucial for identifying and comparing TPs.

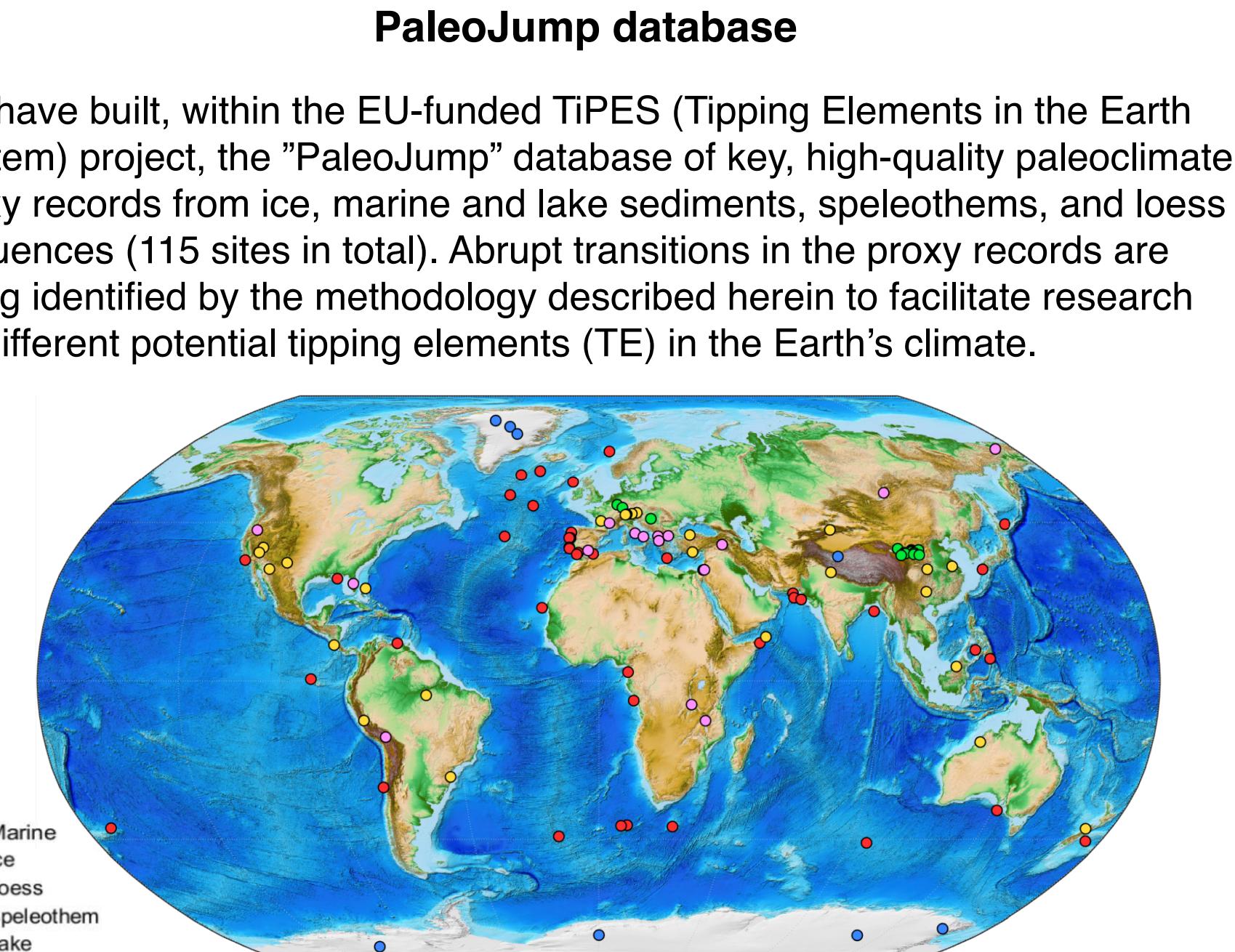


Figure 1: Map showing the 115 sites in the PaleoJump database.

2. METHODOLOGY

Kolmogorov-Smirnov test

Our methodology is based on the nonparametric Kolmogorov-Smirnov (KS) test (e.g. Massey et al., 1951). This test is applied to compare two samples of a paleoclimate record.

The KS statistic (D_{KS}) quantifies the difference between the empirical distribution functions of the two samples. An abrupt transition is identified if D_{KS} is greater than a critical value:

$$D_{KS} > c(\alpha) \sqrt{\frac{n_1 + n_2}{n_1 n_2}}$$

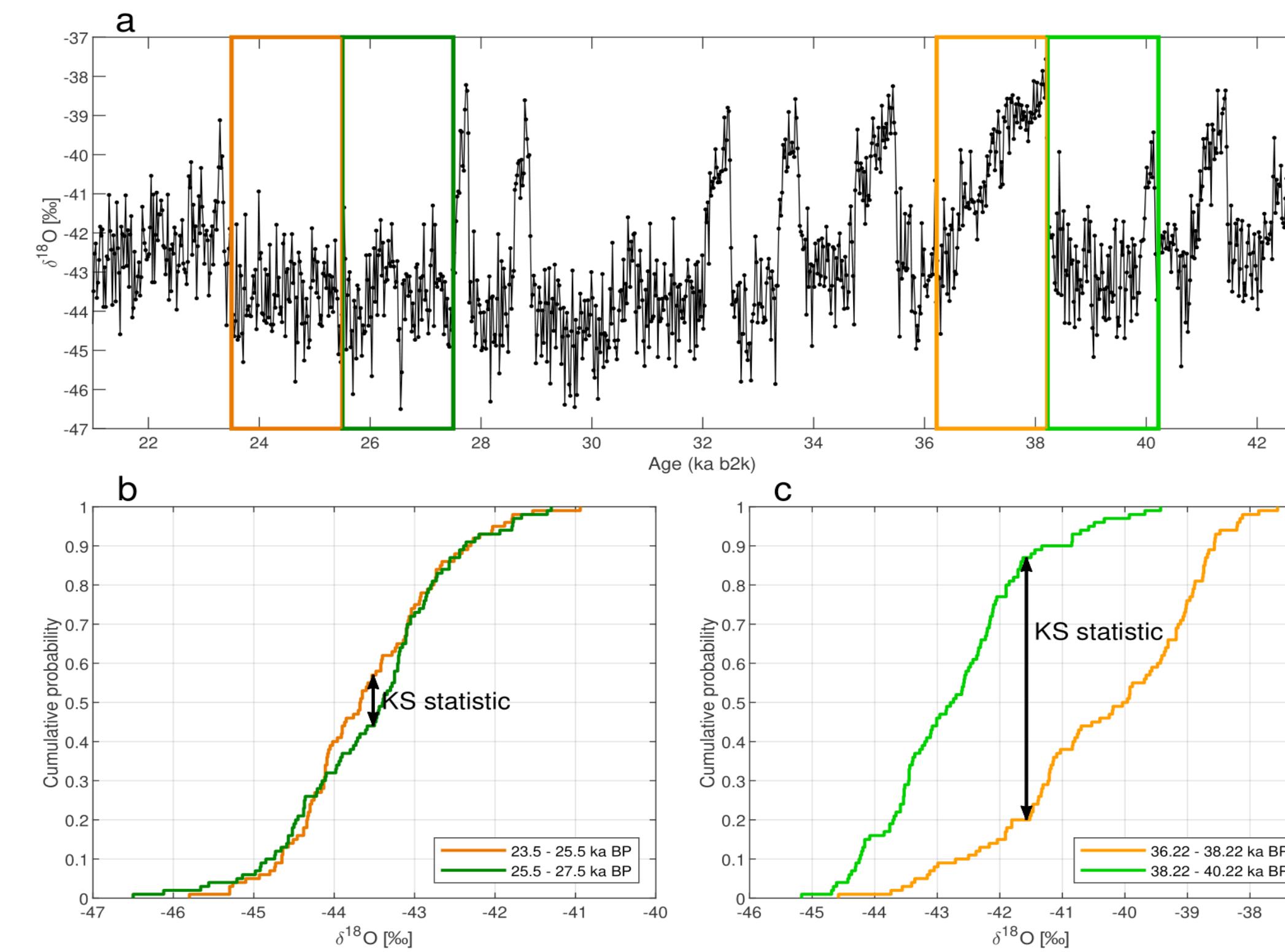


Figure 2: (a) Snapshot of the NGRIP $\delta^{18}\text{O}$ record (21–43 ka b2k). The green and orange rectangles correspond to the sample windows used for evaluating the KS statistic. (b, c) Empirical distribution functions of the two pairs of samples. The length of the black arrow is equal to the KS statistic D_{KS} .

Other key components of the methodology

- varying window size:
 $w_i = w_{\min} \left(\frac{w_{\max}}{w_{\min}} \right)^{(i-1)/(N_w-1)}$
- minimum rate-of-change threshold:
 $\frac{|\bar{x}_1 - \bar{x}_2|}{\sigma_1} > \sigma_c \quad \text{and} \quad \frac{|\bar{x}_1 - \bar{x}_2|}{\sigma_2} > \sigma_c$
- long term trends in maxima and minima

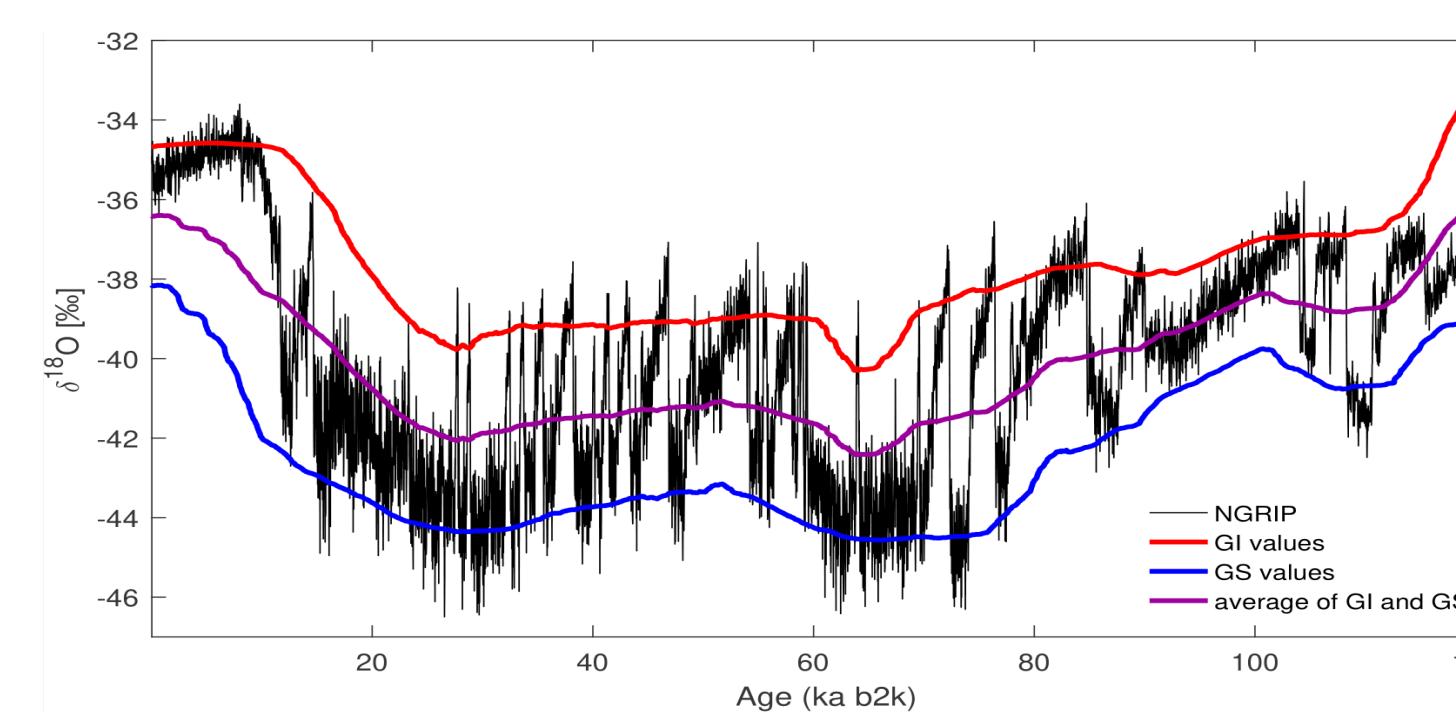


Figure 3: NGRIP $\delta^{18}\text{O}$ record (black line). Red line: calculated interstadial values; blue line: calculated stadial values; purple line: average between red and blue lines.

Receiver operating characteristic

We use the receiver operating characteristic (ROC) analysis to compare the diagnostic ability of different classifiers used in our methodology and to optimize the KS method's parameters.

3. TRANSITION DETECTION

Kolmogorov-Smirnov test

The KS method properly identifies the abrupt transitions of Greenland's NGRIP ice core record described in Rasmussen et al. (2014). Precise dates of transitions are found for different time scales, including glacial-interglacial transitions, Dansgaard-Oeschger events, and shorter sub-events. Please see details on the KS and RQA method, as well as on their comparison, in Bagniewski, W., M. Ghil, and D.-D. Rousseau, 2021, *Chaos*, **31**(11), doi: <https://doi.org/10.1063/5.0062543>.

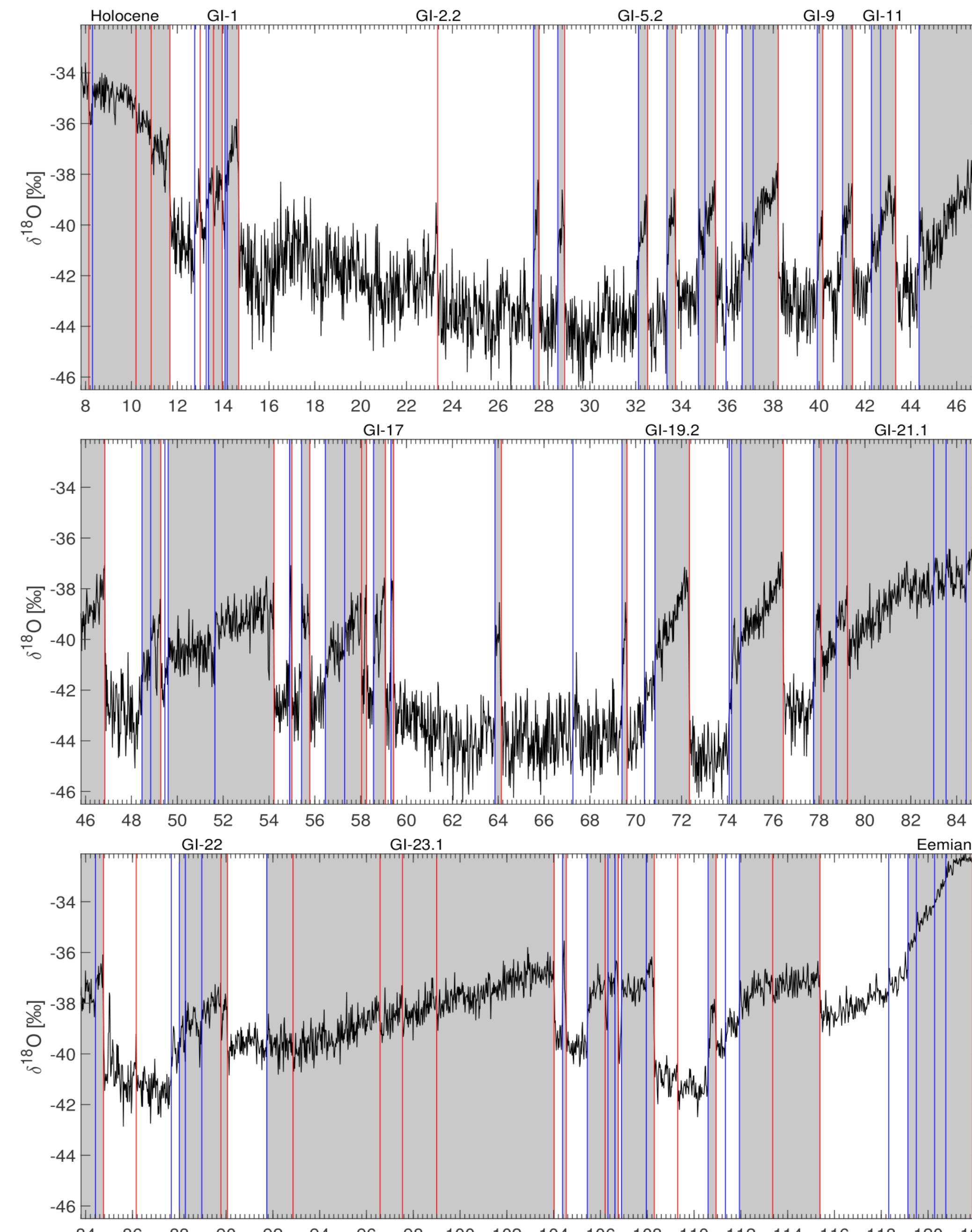


Figure 4: NGRIP $\delta^{18}\text{O}$ ice core record. Vertical bars represent abrupt transitions detected with the KS method: warming (red) and cooling (blue). Grey shaded areas represent interstadial (warm) events.

4. SENSITIVITY OF TRANSITION DETECTION TO WINDOW SIZE

The augmented KS test uses a variable window length to find transitions at different time scales. Here we show a general comparison of the two methods, including the effect of the window length on their performance. The KS test shows more consistency between the window sizes than the recurrence analysis. It is therefore the more robust method for establishing precise dates of abrupt transitions.

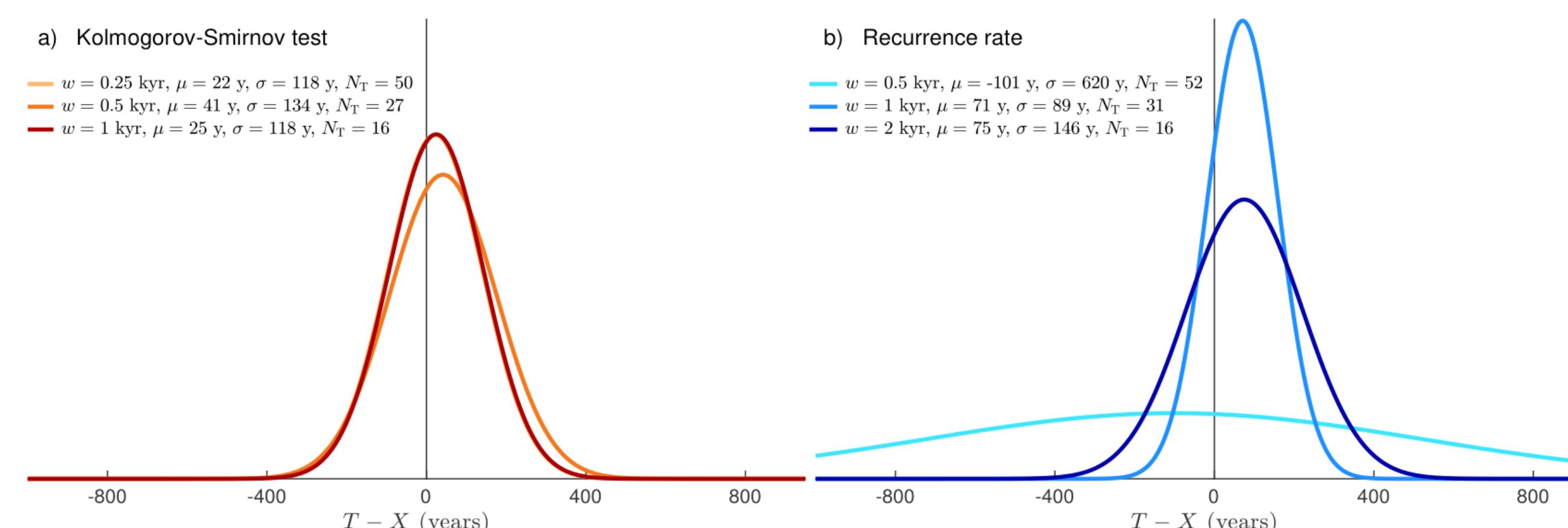


Figure 6: Normal distributions of $T - X$ for different window lengths w , where T = detected transitions, X = transitions defined by Rasmussen et al. (2014)

Recurrence analysis

We compare transitions detected using the KS method with the results of recurrence quantification analysis (RQA, Marwan et al., 2007). By evaluating the recurrence rate (RR) in a sliding window, we identify changes in the time series. While this method identifies the major transitions found with the KS test, properly recognizing the transition points becomes challenging at time scales shorter than the window length. Still, recurrence analysis can effectively identify regime shifts in paleoclimate records. Different types of shifts will be identified depending on the time scale that is investigated. Please visit the iPoster version of this presentation for a video showing a compilation of recurrence plots at different time scales.

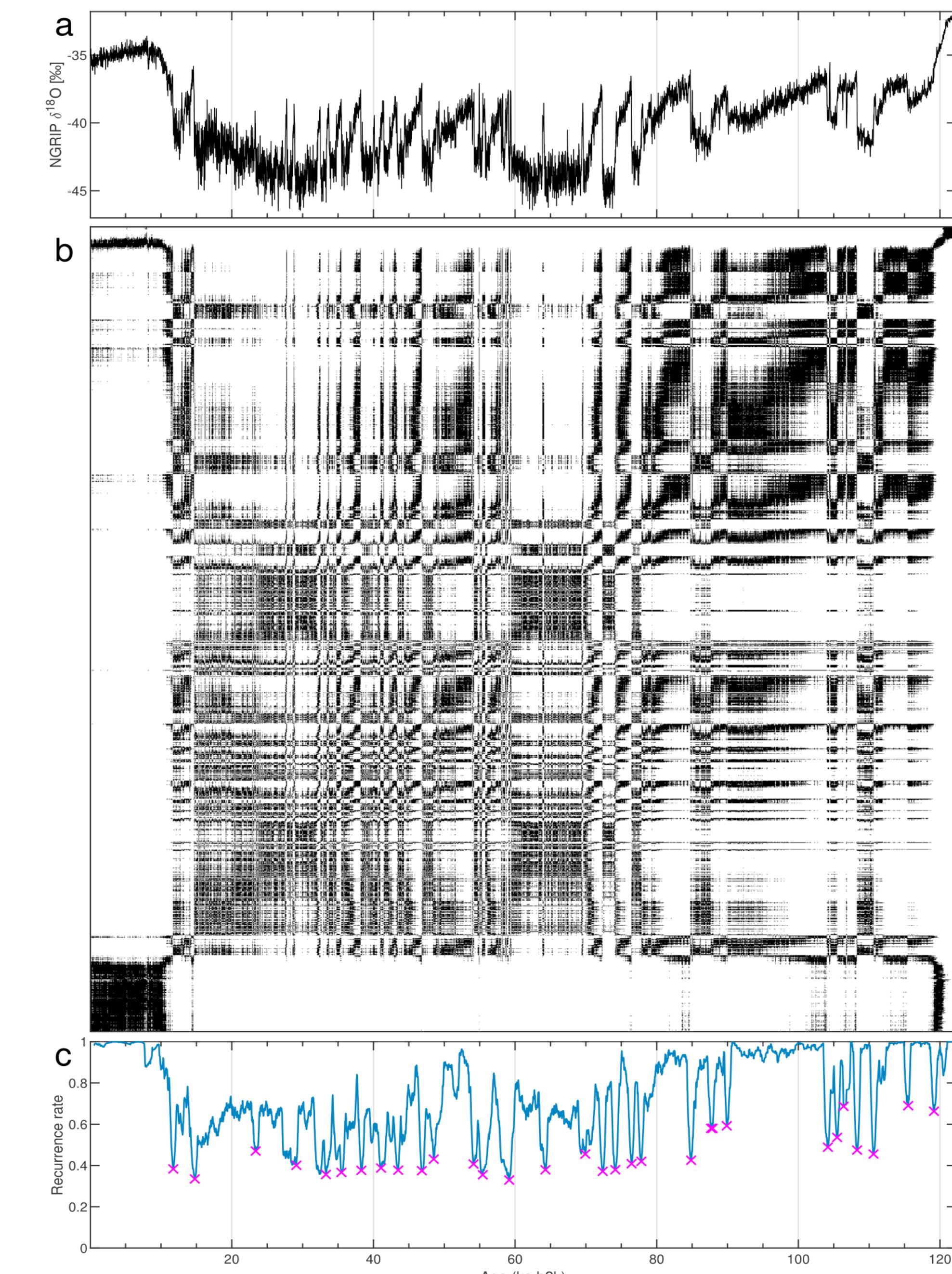


Figure 5: (a) NGRIP $\delta^{18}\text{O}$ time series; (b) recurrence plot; (c) recurrence rate. Magenta crosses represent local RR minima and they correspond to abrupt transitions.

5. SUMMARY

- An objective, automated transition detection methodology is crucial for identifying TPs and facilitates the comparison of paleoclimate records.
- The augmented KS test is very effective in detecting abrupt transitions in paleoproxy records and allows identifying precise dates of individual transitions.
- Recurrence analysis is useful in identifying regime changes and helps establish particularly important transitions. - The two methodologies complement each other.

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