

# Starting Off

We can use an ANOVA test to see if a continuous variable is independent of a categorical variable.

What statistical measure would we use to measure if there is a relationship between two continuous variables?

X Chi-square test

# Goal -> implement chi-squared

- > Use cases
- > Review key concepts
- > When to use the chi-square test
- > What is chi-square
- > Calculate!

# Let's review

1. The fundamentals of the sampling distributions for the sample mean and the sample proportion.
2. We illustrated how these sampling distributions form the basis for estimation (confidence intervals) and testing for one mean or one proportion.
3. Then we extended the discussion to analyzing situations for two variables; one a response and the other an explanatory. When both variables were categorical we compared two proportions; when the explanatory was categorical, and the response was quantitative, we compared two means.
4. The explanatory variable is categorical with more than two levels, and the response is quantitative (Analysis of Variance or ANOVA)
5. Next, we will take a look at other methods and discuss how they apply to situations where:
  - both variables are categorical with at least one variable with more than two levels (Chi-square Test of Independence)
  - both variables are quantitative (Linear Regression)

# Let's review

## Parametric tests:

- Require assumptions about population characteristics: normality of the underlying distribution, homogeneity of variance, known mean / variance.
- Examples: F, z, t tests, Chi-square

## Nonparametric tests:

- Do not require assumptions about population characteristics.
- Can be used with very skewed distributions or when the population variance is not homogeneous.
- Can be used with ordinal or nominal data.
- Examples: Chi-square, Wilcoxon, and Kruskal-Wallis tests

Nonparametric tests are less powerful than parametric tests, so we don't use them when parametric tests are appropriate. But if the assumptions of parametric tests are violated, we use nonparametric tests.

# What is chi-squared used for?

Statistical evidence of **association or relationship** between **categorical variables**

**Goodness of fit** - does observed frequency distribution differ from a theoretical distribution.

**Homogeneity** - is two populations distributions for a categorical variable are the same.

**Independence** - determine if two variables are independent of each other.

<https://courses.lumenlearning.com/wmopen-concepts-statistics/chapter/chi-square-tests-review/>

<https://www.quora.com/Whats-the-difference-between-a-chi-squared-test-of-homogeneity-and-a-chi-squared-test-of-independence-When-is-it-appropriate-to-use-each>

# What is Chi-Squared Test of Independence

Pearson's chi-squared test is used to determine whether there is a statistically significant difference between the expected frequencies and the observed frequencies in one or more categories of a so-called contingency table.

$H_0$ : Variable A and Variable B are independent.

$H_a$ : Variable A and Variable B are not independent.

Chi-square test is like the z-test for two independent proportions, but when you have more than two groups that you are comparing against.

Is the rate of survival different for different types of cancer (colon, breast, throat, etc.)?

# Check for understanding

## **Should we use the chi-square tests in the following situations?**

1. Do first generation college students study the same majors (Business, Art History, Computer Science, etc.) as legacy students?
2. Do children who receive vaccinations have a higher incidence of autism than children who don't?
3. Is the average highway gas mileage the same for 3 different types of cars?
4. 6 months after graduation, do students from different data science programs (Flatiron, General Assembly, Metis) have different job placement rates?

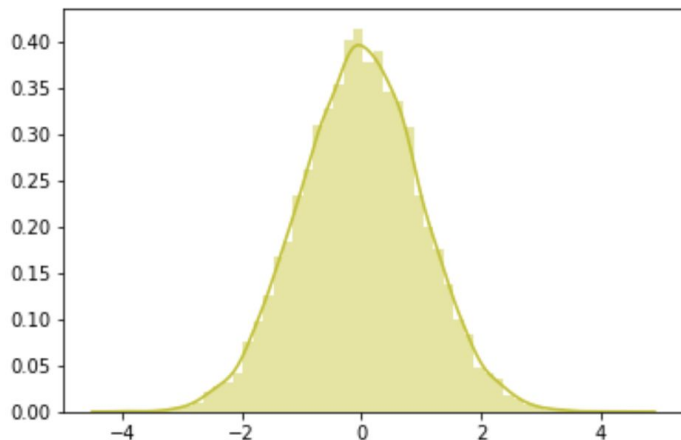


# What is chi-squared part 2

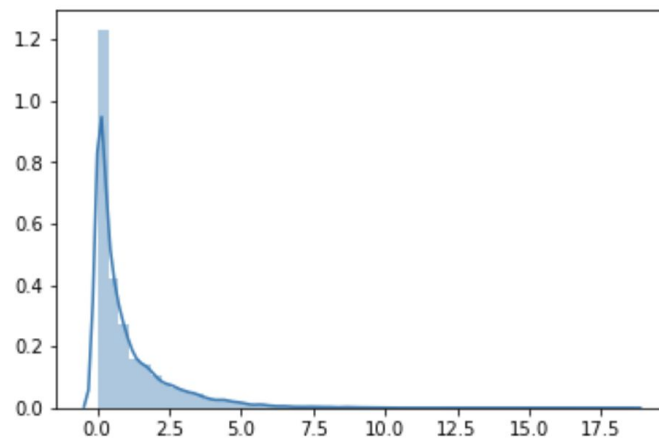
Cool graphs

**Normal distribution**  $\rightarrow$  square the random variable  $\rightarrow$  **chi-square**

pdf



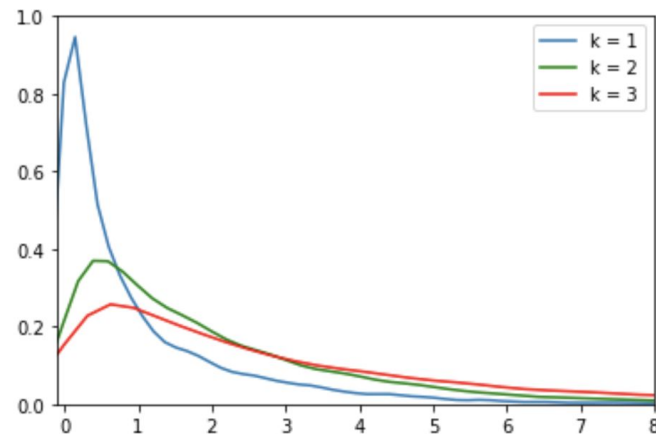
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# To use chi-squared

Every statistical test has assumptions, for chi-squared

- Data should be frequencies, not percentages
- Categories are mutually exclusive
- The observations are assumed be independent of each other - you randomly sampled your observations.
- More than 80% of cells must have a  $>5$  count



# Calculating

First, are we comparing categorical variables?

1. Null and alternative hypotheses
2. Chi-squared test statistic
3. Degrees of freedom
4. Confidence level
5. Compare all three ^
6. Reject or fail to reject the null

The primary method for displaying the summarization of categorical variables is called a contingency table. When we have two measurements on our subjects that are both the categorical, the contingency table is sometimes referred to as a two-way table.

# What is chi-squared?

Start with a random variable  $Y$ , make a normal distribution  $Z = \sum \frac{(Y - \mu)}{\sigma}$

Square the random variable  $Z^2 = \sum \left( \frac{Y - \mu}{\sigma} \right)^2$

Summing the random variables gives the distribution  $Q = \sum_{i=1}^k Z_i^2$

with one parameter -> **degrees of freedom**  $Q \sim \chi^2(k)$

# 1. define the question

A sample of students have the option of ice cream or cake after school. Is there a relationship between grade level and snack? Test the hypothesis with a significance of 5%

	Ice cream	Cake	<b>total</b>
3rd grade	18	11	<b>29</b>
4th grade	15	16	<b>31</b>
5th grade	9	15	<b>24</b>
<b>total</b>	<b>42</b>	<b>42</b>	<b>84</b>

$\alpha =$

This is a 3x2

Contingency Table

## 2a. find chi-squared

The chi-squared test statistic is

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

$\alpha$  is 0.05

	Ice cream	Cake	total
3rd grade	O = 18 E =	O = 11 E =	<b>29</b>
4th grade	O = 15 E =	O = 16 E =	<b>31</b>
5th grade	O = 9 E =	O = 15 E =	<b>24</b>
<b>total</b>	<b>42</b>	<b>42</b>	<b>84</b>

$\alpha = 0.05$

## 2b. find expected values

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

**O** is observed values (that are given)

**E** is expected values (row total x column total)/sample size

	Ice cream	Cake	<b>total</b>
3rd grade	O = 18 E =	O = 11 E =	<b>29</b>
4th grade	O = 15 E =	O = 16 E =	<b>31</b>
5th grade	O = 9 E =	O = 15 E =	<b>24</b>
<b>total</b>	<b>42</b>	<b>42</b>	<b>84</b>

$$\alpha = 0.05$$

## 2c. find chi-squared test statistic

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

Now calculate  $\chi^2$

	Ice cream	Cake	total
3rd grade	O = 18 E = 14.5	O = 11 E = 14.5	<b>29</b>
4th grade	O = 15 E = 15.5	O = 16 E = 15.5	<b>31</b>
5th grade	O = 9 E = 12	O = 15 E = 12	<b>24</b>
<b>total</b>	<b>42</b>	<b>42</b>	<b>84</b>

$$\alpha = 0.05$$

$$\chi^2 = 3.222$$



### 3. degrees of freedom

degrees of freedom = (number of rows - 1)(number of columns - 1)

	Ice cream	Cake	<b>total</b>
3rd grade	O = 18 E = 14.5	O = 11 E = 14.5	<b>29</b>
4th grade	O = 15 E = 15.5	O = 16 E = 15.5	<b>31</b>
5th grade	O = 9 E = 12	O = 15 E = 12	<b>24</b>
<b>total</b>	<b>42</b>	<b>42</b>	<b>84</b>

$$\alpha = 0.05$$

$$\chi^2 = 3.222$$

$$\text{df} =$$

## 4. + 5. compare confidence interval

Use a [table](#) of **degrees of freedom** and **chi-squared statistic** to compare

	Ice cream	Cake	total
3rd grade	O = 18 E = 14.5	O = 11 E = 14.5	<b>29</b>
4th grade	O = 15 E = 15.5	O = 16 E = 15.5	<b>31</b>
5th grade	O = 9 E = 12	O = 15 E = 12	<b>24</b>
<b>total</b>	<b>42</b>	<b>42</b>	<b>84</b>

$$\alpha = 0.05$$

$$\chi^2 = 3.222$$

$$df = 2$$

$$CV = 5.9915$$

## 6. Reject the null?

If the chi-squared statistic > critical value, reject the null hypothesis  $Q_k \geq \chi^2_{k,\alpha}$

Else, fail to reject the null hypothesis

	Ice cream	Cake	total
3rd grade	O = 18 E = 14.5	O = 11 E = 14.5	<b>29</b>
4th grade	O = 15 E = 15.5	O = 16 E = 15.5	<b>31</b>
5th grade	O = 9 E = 12	O = 15 E = 12	<b>24</b>
<b>total</b>	<b>42</b>	<b>42</b>	<b>84</b>

$$\alpha = 0.05$$

$$\chi^2 = 3.222$$

$$\text{df} = 2$$

$$\text{CV} = 5.9915$$

# chi-squared in action

Are floaties and paddleboards distributed equally among new york beaches?

	Brighton	Coney island	Rockaway	Fort Tilden	<b>total</b>
floaties		3	31	13	
paddle boards	15	28		5	<b>71</b>
<b>total</b>	<b>17</b>		<b>54</b>	<b>18</b>	<b>120</b>

$\alpha =$

$\chi^2 =$

df =

critical value =

# chi-squared in code

## $\chi^2$ Test with scipy

```
In [8]: 1 # chi-squared test with similar proportions
        2 from scipy.stats import chi2_contingency
        3 from scipy.stats import chi2
```

```
In [9]: 1 # contingency table
        2 table = [10, 20, 30],
        3           [6, 9, 17]]
        4 print(table)
```

```
[[10, 20, 30], [6, 9, 17]]
```

```
In [10]: 1 stat, p, dof, expected = chi2_contingency(table)
        2 print('dof=%d' % dof)
        3 print(expected)
```

```
dof=2
```

```
[[10.43478261 18.91304348 30.65217391]
 [ 5.56521739 10.08695652 16.34782609]]
```

# chi-squared in code

```
In [11]: 1 # interpret test-statistic
          2 prob = 0.95
          3 critical = chi2.ppf(prob, dof)
```

```
In [12]: 1 print('probability=%.3f, critical=%.3f, stat=%.3f' % (prob, critical, stat))
          2 if abs(stat) >= critical:
          3     → print('Dependent (reject H0)')
          4 else:
          5     → print('Independent (fail to reject H0)')
```

probability=0.950, critical=5.991, stat=0.272  
Independent (fail to reject H0)

```
In [13]: 1 # interpret p-value
          2 alpha = 1.0 - prob
          3 print('significance=%.3f, p=%.3f' % (alpha, p))
          4 if p <= alpha:
          5     → print('Dependent (reject H0)')
          6 else:
          7     → print('Independent (fail to reject H0)')
```

significance=0.050, p=0.873  
Independent (fail to reject H0)

# Closing Comments

CRISP-DM breaks the process of **data mining** into six major phases:

- Business Understanding
- Data Understanding
- Data Preparation
- Modeling
- Evaluation
- Deployment

The sequence of the phases is not strict and moving back and forth between different phases as it is always required. The arrows in the process diagram indicate the most important and frequent dependencies between phases. The outer circle in the diagram symbolizes the cyclic nature of data mining itself. A data mining process continues after a solution has been deployed. The lessons learned during the process can trigger new, often more focused business questions, and subsequent data mining processes will benefit from the experiences of previous ones.

Where do you think statistical tests should be utilized within this process?

