The IPhO Compendium

A Collection of Problems Presented
in The International Physics Olympiads

Contents

I IPhO (Warsaw, 1967)

Theoretical problem

Problem 1

A small ball with mass $M = 0.2\text{kg}$ rests on a vertical column with height $h = 5\text{m}$. A bullet with mass $m = 0.01\text{kg}$, moving with velocity $v_0 = 500\text{ms}^{-1}$, passes horizontally through the center of the ball. The ball reaches the ground at a distance $s = 20\text{m}$. Where does the bullet reach the ground? What part of the kinetic energy of the bullet was converted into heat when the bullet passed through the ball? Neglect resistance of the air, the size of the ball and the bullet. Assume that $g = 10\text{ms}^{-2}$.

Solution

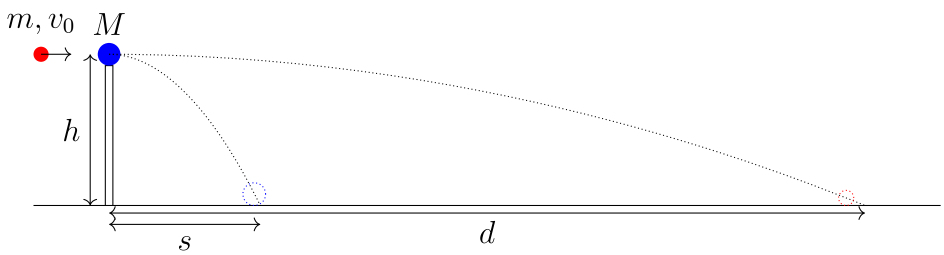


Figure 1: Sketch for Problem 1

We will use notation shown in Figure 1.

As no horizontal force acts on the system ball and bullet, the horizontal component of momentum of this system before collision and after collision must be the same,

$$mv_0 = mv' + MV$$

where v' and V are horizontal component of the velocity of the bullet and of the ball after collision, respectively.

So,

$$v = v_0 - \frac{M}{m}V$$

From conditions described in the text of the problem it follows that

$$v > V$$

After collision, both the ball and the bullet continue a free motion in the gravitational field with initial horizontal velocities v and V, respectively. Motion of the ball and motion of the bullet are continued for the same time,

$$t = \sqrt{\frac{2h}{g}}$$

It is the time of free fall from height h.

The distances passed by the ball and bullet during time t are

$$s = Vt \text{ and } d = vt$$

respectively. Thus,

$$V = s\sqrt{\frac{g}{2h}}$$

Therefore,

$$d = v_0\sqrt{\frac{2h}{g}} - \frac{M}{m}s \tag{1}$$

Numerically,

$$d = 100\text{m}$$

The total kinetic energy of the system was equal to the initial kinetic energy of the bullet,

$$E_0 = \frac{mv_0^2}{2}$$

Immediately after the collision, the total kinetic energy ot the system is equal to the sum of the kinetic energy of the bullet and the ball,

$$E_m = \frac{mv^2}{2} \text{ and } E_M = \frac{MV^2}{2}$$

Their difference, converted into heat, was

$$\Delta E = E_0 - (E_m + E_M)$$

It is the following part of the initial kinetic energy of the bullet,

$$p = \frac{\Delta E}{E_0} = 1 - \frac{E_m + E_M}{E_0}$$

By using expressions for energies and velocities (quoted earlier) we get

$$p = \frac{M}{m}\frac{s^2}{v_0^2}\frac{g}{2h}\left(2\frac{v_0}{s}\sqrt{\frac{2h}{g}} - \frac{M+m}{m}\right) \tag{2}$$

Numerically,

$$p = 92.8\%$$

Problem 2

Consider an infinite network consisting of resistors (resistance of each of them is r) as shown in Figure 2. Find the resultant resistance R_{AB} between points A and B.

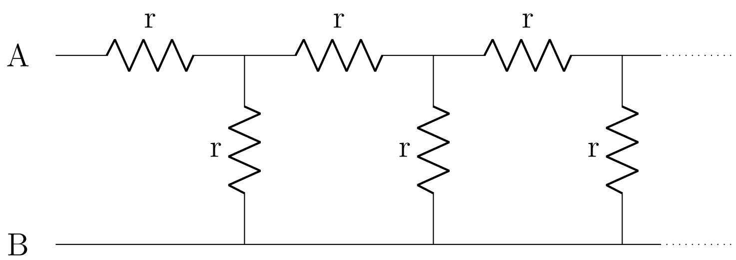


Figure 2: Sketch for Problem 2

Solution
It is easy to remark that after removing the left part of the work, shown in Figure 3 with the red square, then we receive a network that is identical with the initial network (it is result of the fact that the network is infinite).

Thus, we may use the equivalence shown graphically in Figure 4.

Algebraically, this equivalence can be written as

$$R_{AB} = r + \frac{1}{\frac{1}{r} + \frac{1}{R_{AB}}}$$

Thus

$$R_{AB}^2 - rR_{AB} - r^2 = 0$$

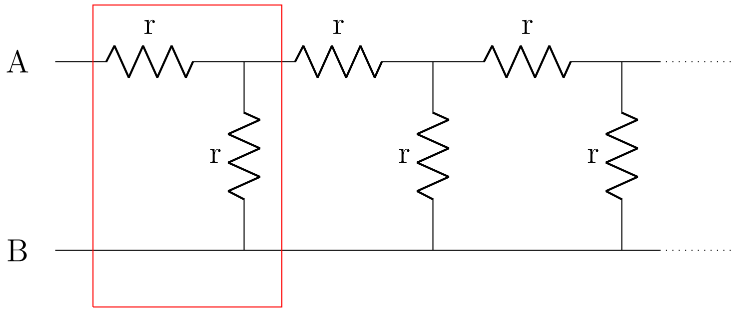


Figure 3: Auxiliary sketch for Problem 2

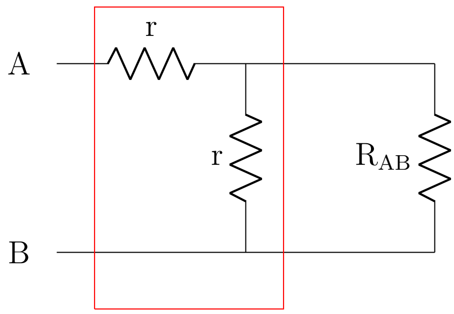


Figure 4: Auxiliary sketch for Problem 2

This equation has two solutions,

$$R_{AB} = \frac{1}{2}(1 \pm \sqrt{5})r$$

The solution corresponding to "-" in the above formula is negative, while resistance must be positive. So, we reject it. Finally, we receive

$$R_{AB} = \frac{1}{2}(1 + \sqrt{5})r \tag{3}$$

Problem 3

Consider two identical homogeneous balls, A and B, with the same initial temperatures. One of them is at rest on a horizontal plane, while the second one hangs on a thread (Figure 5). The same quantities of heat have been supplied to both balls. Are the final temperatures of the balls the same or not? Justify your answer. (All kinds of heat losses are negligible.)

Solution
As regards the text of the problem, the sentence *"The same quantities of heat*

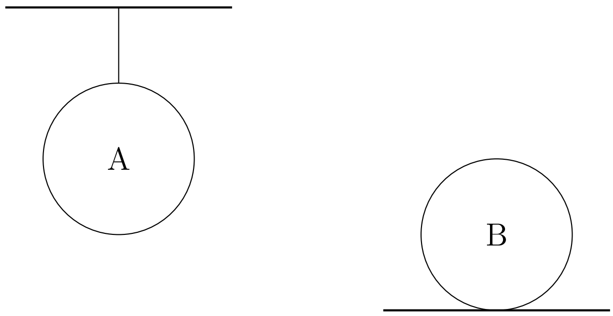


Figure 5: Sketch for Problem 3

have been supplied to both balls." is not too clear. We will follow intuitive understanding of this sentence, i.e. we will assume that both systems (A - the hanging ball and B - the ball resting on the plane) received the same portion of energy from outside. One should realize, however, that it is not the only possible interpretation.

When the balls are warmed up, their mass centers are moving as the radii of the balls are changing. The mass center of the ball A goes down, while the mass center of the ball B goes up. It is shown in Figure 6(scale is not conserved).

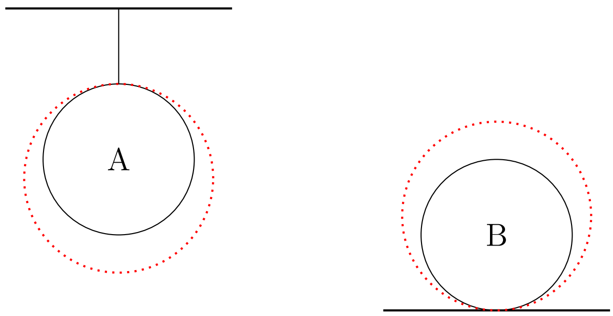


Figure 6: Auxiliary sketch for Problem 3

Displacement of the mass center corresponds to a change of the potential energy of the ball in the gravitational field.

In case of the ball A the potential energy decreases. From the 1st principle of thermodynamics, it corresponds to additional heating of the ball.

In case of the ball B the potential energy increases. From the 1st principle of thermodynamics it corresponds to some "losses of the heat provided" for performing a mechanical work necessary to rise the ball. The net result is that the final temperature of the ball B should be lower than the final temperature of the ball A.

The above effect is very small. For example, one my find (see later) that for balls made of lead, with radius 10cm, and portion of heat equal to 50kcal, the difference of the final temperature of the balls is of order 10^{-5}K. For spatial and time fluctuations, such small quantity practically cannot be measured.

Calculation of the difference of the final temperatures was not required from the participants. Nevertheless, we present it here as an element of discussion.

We may assume that the work against the atmospheric pressure can be neglected. It is obvious that this work is small. Moreover, it is almost the same for both balls. So, it should not affect the difference of the temperatures substantially. We will assume that such quantities as specific heat of the lead and coefficient of thermal expansion of lead are constant (i.e. do not depend on temperature).

The heat used for changing the temperatures of balls may be written as

$$Q_i = mc\Delta t_i, \text{ where } i = A \text{ or } B$$

Here, m denotes the mass of ball, c the specific heat of lead, and Δt_i the change of the temperature of ball.

The changes of the potential energy of the balls are (neglecting signs)

$$\Delta E_i = mgr\alpha\Delta t_i, \text{ where } i = A \text{ or } B$$

Here, g denotes the gravitational acceleration, r initial radius of the ball, α coefficient of thermal expansion of lead. We assume here that the thread does not change its length.

Taking into account conditions described in the text of the problem and the interpretation mentioned at the beginning of the solution, we may write

$$Q = Q_A - \gamma\Delta E_A$$
$$Q = Q_B + \gamma\Delta E_B$$

γ denotes the thermal equivalent of work ($\approx 0.24\frac{\text{cal}}{\text{J}}$). In fact, γ is only a conversion ratio between calories and joules. If you use a system of units in which calories are not present, you may omit γ at all.

Thus,

$$Q = (mc - \gamma mgr\alpha)\Delta t_A, \text{ for the ball A,}$$
$$Q = (mc - \gamma mgr\alpha)\Delta t_B, \text{ for the ball B}$$

and

$$\Delta t_A = \frac{Q}{mc - \gamma mgr\alpha}, \ \Delta t_B = \frac{Q}{mc + \gamma mgr\alpha}$$

Finally, we get

$$\Delta t = \Delta t_A - \Delta t_B = \frac{2\gamma g r \alpha}{c^2 - (\gamma g r \alpha)^2 m} \frac{Q}{m} \approx \frac{2\gamma Q g r \alpha}{m c^2} \tag{4}$$

We neglected the term with α^2 as the coefficient α is very small.

Now we may put the numerical values, $Q = 50\text{kcal}$, $\gamma = 0.24\frac{\text{cal}}{\text{J}}$, $g \approx 9.8\text{ms}^{-2}$, $m \approx 47\text{kg}$ (mass of the lad ball with radius equal to 10cm), $r = 0.1\text{m}$, $c \approx 0.031\frac{\text{cal}}{\text{gK}}$, $\alpha \approx 29 \times 10^{-6}\text{K}^{-1}$. After calculations, we get $\Delta t \approx 1.5 \times 10^{-5}\text{K}$.

Problem 4[1]

A closed vessel with volume $V_0 = 10l$ contains dry air in the normal conditions ($t_0 = 0\,°\text{C}, p_0 = 1\text{atm}$). In some moment 3g of water were added to the vessel and the system was warmed up to $t = 100\,°\text{C}$. Find the pressure in the vessel. Discuss assumption you made to solve the problem.

Solution

The water added to the vessel evaporates. Assume that the whole portion of water evaporated. Then the density of water vapor in 100°C should be $0.300\frac{g}{l}$. It is less than the density of saturated vapor at 100°C, which equals to $0.597\frac{g}{l}$ (The students were allowed to use physical tables). So, at 100°C, the vessel contains air and unsaturated water vapor only (without any liquid phase).

Now we assume that both air and unsaturated water vapor behave as ideal gases. In view of Dalton law, the total pressure p in the vessel at 100°C is equal to the sum of partial pressures of the air p_a and unsaturated water vapor p_v,

$$p = p_a + p_v$$

As the volume of the vessel is constant, we may apply the Gay-Lussac law to the air. We obtain

$$p_a = p_0 \left(\frac{273 + t}{273} \right)$$

[1]The Organizing Committee prepared three theoretical problems. Unfortunately, at the time of the 1st Olympiad, the Romanian students from the last class had the entrance examinations at the universities. For that, Romania sent a team consisting of students from younger classes. They were not familiar with electricity. To give them a chance, the Organizers (under agreement of the International Board) added the fourth problem presented here. The student (not only from Romania) were allowed to choose three problems. The maximum possible scores for the problems were: 1st problem 10 points, 2nd problem 10 points, 3rd problem 10 points, and 4th problem 6 points. The fourth problem was solved by 8 students. Only four of them solved the problem for 6 points.

The pressure of the water vapor may be found from the equation of state of the ideal gas,

$$\frac{p_v V_0}{273 + t} = \frac{m}{\mu} R$$

where m denotes the mass of the vapor, μ the molecular mass of the water and R the universal gas constant. Thus,

$$p_v = \frac{m}{\mu} R \frac{273 + t}{V_0}$$

and finally

$$p = p_0 \frac{273 + t}{273} + \frac{m}{\mu} R \frac{273 + t}{V_0} \tag{5}$$

Numerically,

$$p = (1.366 + 0.516)\text{atm} \approx 1.88\text{atm}$$

Experimental problem

The following devices and materials are given: 1. Balance (without weights), 2. Calorimeter, 3. Thermometer, 4. Source of voltage, 5. Switches, 6. Wires, 7. Electric heater, 8. Stop-watch, 9. Beakers, 10. Water, 11. Petroleum, 12. Sand (for balancing).

Determine specific heat of petroleum. The specific heat of water is 1 cal/(g.°C). The specific heat of the calorimeter is 0.92 cal/(g.°C).

Discuss assumptions made in the solution.

Solution

The devices given to the students allowed using several methods. The students used the following three methods:

1. Comparison of velocity of warming up water and petroleum
2. Comparison of cooling down water and petroleum
3. Traditional heat balance

As no weights were given, the students had to use the sand to find portions of petrolem and water with masses equal to the mass of calorimeter.

First method: comparison of velocity of warming up

If the heater is inside water then both water and calorimeter are warming up. The heat taken by water and calorimeter is

$$Q_1 = m_w c_w \Delta t_1 + m_c c_c \Delta t_1$$

where m_w denotes mass of water, m_c mass of calorimeter, c_w specific heat of water, c_c specific heat of calorimeter, Δt_1 change of temperature of the system water + calorimeter.

On the other hand, the heat provided by the heater is equal

$$Q_2 = A\frac{U^2}{R}\tau_1$$

where A denotes the thermal equivalent of work, U voltage, R resistnce of the heater, τ_1 time of work of the heater in the water.

Of course,

$$Q_1 = Q_2$$

Thus

$$A\frac{U^2}{R}\tau_1 = m_w c_w \Delta t_1 + m_c c_c \Delta t_1$$

For petroleum in the calorimeter, we get a similar formula

$$A\frac{U^2}{R}\tau_2 = m_p c_p \Delta t_2 + m_c c_c \Delta t_2$$

where m_p denotes mass of petroleum, c_p specific heat of petroleum, Δt_2 change of temperature of the system water + petroleum, τ_2 time of work of the heater in the petroleum.

By dividing the last equations we get

$$\frac{\tau_1}{\tau_2} = \frac{m_w c_w \Delta t_1 + m_c c_c \Delta t_1}{m_p c_p \Delta t_2 + m_c c_c \Delta t_2}$$

It is convenient to perform the experiment by taking masses of water and petroleum equal to the mass of the calorimeter (for that we use the balance and the sand). For

$$m_w = m_p = m_c$$

the last formula can be written in a very simple form

$$\frac{\tau_1}{\tau_2} = \frac{c_w \Delta t_1 + c_c \Delta t_1}{c_p \Delta t_2 + c_c \Delta t_2}$$

Thus

$$c_c = \frac{\Delta t_1}{\tau_1}\frac{\tau_2}{\Delta t_2}c_w - \left(1 - \frac{\Delta t_1}{\tau_1}\frac{\tau_2}{\Delta t_2}\right)c_c$$

or

$$c_c = \frac{k_1}{k_2}c_w - \left(1 - \frac{k_1}{k_2}\right)c_c \tag{6}$$

where

$$k_1 = \frac{\Delta t_1}{\tau_1} \text{ and } k_2 = \frac{\Delta t_2}{\tau_2}$$

denote "velocities of heating" water and petroleum, respectively. These quantities can be determined experimentally by drawing graphs representing dependence Δt_1 and Δt_2 on time (τ). The experiment shows that these dependencies are linear. Thus, it is enough to take slopes of appropriate straight lines. The experimental setup given to the students allowed measurement of the specific heat of petroleum, equal to $0.53\frac{cal}{g^\circ\mathrm{C}}$, with accuracy about 1%.

Some students used certain mutations of this method by performing measurements at $\Delta t_1 = \Delta t_2$ or at $\tau_1 = \tau_2$. Then, of course, the error of the final result is greater (it is additionally affected by accuracy of establishing the conditions $\Delta t_1 = \Delta t_2$ or at $\tau_1 = \tau_2$).

Second method: comparison of velocity of cooling down

Some students initially heated the liquids in the calorimeter and later observed their cooling down. This method is based on the Newton's law of cooling. It says that the heat Q transferred during cooling in time τ is given by the formula

$$Q = h(t - \theta)s\tau$$

where t denotes the temperature of the body, θ the temperature of surrounding, s area of the body, and h certain coefficient characterizing properties of the surface. This formula is correct for small differences of temperatures $t - \theta$ only (small compared to t and θ in the absolute scale).

This method, like the previous one, can be applied in different versions. We will consider only one of them.

Consider the situation when cooling of water and petroleum is observed in the same calorimeter (containing initially water and later petroleum). The head lost by the system water + calorimeter is

$$\Delta Q_1 = (m_w c_w + m_c c_c)\Delta t$$

where Δt denotes a change of the temperature of the system during certain period τ_1. For the system petroleum + calorimeter, under assumption that the change in the temperature Δt is the same, we have

$$\Delta Q_2 = (m_p c_p + m_c c_c)\Delta t$$

Of course, the time corresponding to Δt in the second case will be different. Let it be τ_2.

From the Newton's law we get

$$\frac{\Delta Q_1}{\Delta Q_2} = \frac{\tau_1}{\tau_2}$$

Thus

$$\frac{\tau_1}{\tau_2} = \frac{(m_w c_w + m_c c_c)}{m_p c_p + m_c c_c}$$

If we conduct the experiment at

$$m_w = m_p = m_c$$

then we get

$$c_p = \frac{T_2}{T_1} c_w - \left(1 - \frac{T_2}{T_1}\right) \tag{7}$$

As cooling is rather a very slow process, this method gives the result with definitely greater error.

Third method: heat balance

This method is rather typical. The students heated the water in the calorimeter to certain temperature t_1 and added the petroleum with the temperature t_2. After reaching the thermal equilibrium, the final temperature was t. From the thermal balance (neglecting the heat losses) we have

$$(m_w c_w + m_c c_c)(t_1 - t) = m_p c_p (t - t_2)$$

If, like previously, the experiment is conducted at

$$m_w = m_p = m_c$$

then

$$c_p = (c_w + c_c) \frac{t_1 - t}{t - t_2} \tag{8}$$

In this methods, the heat losses (when adding the petroleum to the water) always played a substantial role.

The accuracy of the result equal or better than 5% can be reached by using any of the methods described above. However, one should remark that in the first method it was easiest. The most common mistake was neglecting the heat capacity of the calorimeter. This mistake increased the error additionally by about 8%.

II IPhO (Budapest, 1968)

Theoretical problem

Problem 1

On an inclined plane of 30°, a block, mass $m_2 = 4$kg, is joined by a light cord to a solid cylinder, mass $m_1 = 8$kg, radius $r = 5$cm (Figure 7). Find the acceleration if the bodies are released. The coefficient of friction between the block and the inclined plane $\mu = 0.2$. Friction at the bearing and rolling friction are negligible.

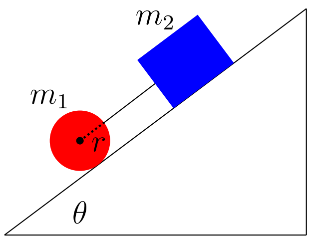


Figure 7: Sketch for Problem 1

Solution

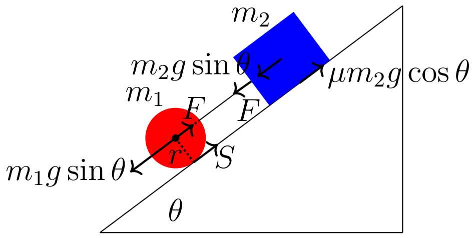


Figure 8: Auxiliary sketch for Problem 1

If the cord is stressed, the cylinder and the block are moving with the same acceleration a. Let F be the tension in the cord, S the frictional force between the cylinder and the inclined plane (Figure 8). The angular acceleration of the cylinder is $\alpha = \frac{a}{r}$. The net force causing the acceleration of the block

$$m_2 a = m_2 g \sin\theta - \mu m_2 g \cos\theta + F$$

and the net force causing the acceleration of the cylinder

$$m_1 a = m_1 g \sin\theta - S - F$$

The equation of motion for the rotation of the cylinder

$$Sr = \alpha I = \frac{a}{r} I$$

where I is the moment of inertia of the cylinder, Sr is the torque of the frictional force.

Solving the system of equations we get

$$a = g\frac{(m_1 + m_2)\sin\theta - \mu m_2 \cos\theta}{m_1 + m_2 + \frac{I}{r^2}} \tag{9}$$

$$S = \frac{I}{r^2} g \frac{(m_1 + m_2)\sin\theta - \mu m_2 \cos\theta}{m_1 + m_2 + \frac{I}{r^2}} \tag{10}$$

$$F = m_2 g \frac{\mu\left(m_1 + \frac{I}{r^2}\right)\cos\theta - \frac{I\sin\theta}{r^2}}{m_1 + m_2 + \frac{I}{r^2}} \tag{11}$$

The moment of inertia of a solid cylinder is $I = \frac{m_1 r^2}{2}$. Using the given numerical values,

$$a = 3.25\text{ms}^{-2}$$
$$S = 13.01\text{N}$$
$$F = 0.192\text{N}$$

Discussion

The condition for the system to start moving is $a > 0$. Inserting $a = 0$ into Equation (9), we obtain the limit for angle θ_1

$$\tan\theta_1 = \mu\frac{m_2}{m_1 + m_2} = \frac{\mu}{3} = 0.0667$$
$$\theta_1 \approx 3.81^\circ$$

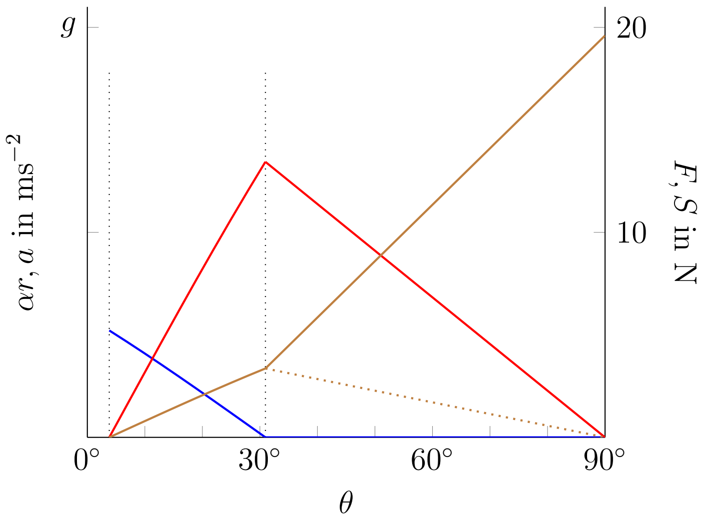


Figure 9: Auxiliary sketch for the discussion section of Problem 1

For the cylinder separately $\theta_1 = 0$ and for the block separately $\theta_1 = \tan^{-1}\mu = 11.31°$. See Figure 9! If the cord is not stretched the bodies move separately. We obtain the limit by inserting $F = 0$ into Equation (11),

$$\tan\theta_2 = \mu\left(1 + \frac{m_1 r^2}{I}\right) = 3\mu = 0.6$$
$$\theta_2 \approx 30.96°$$

The condition for the cylinder to slip is that the value of S (calculated from Equation (11) taking the same coefficient of friction) exceeds the value of $\mu m_1 g \cos\theta$. This gives the same value for θ_3 as we had for θ_2. The acceleration of the centers of the cylinder and the block is the same $g(\sin\theta - \mu\cos\theta)$, the frictional force at the bottom of the cylinder is $\mu m_1 g \cos\theta$, the peripheral acceleration of the cylinder is $\mu \frac{m_1 r^2}{I} g \cos\theta$.

Problem 2

There are 300cm^3 toluene of 0°C temperature in a glass and 110cm^3 toluene of 100°C temperature in another glass. Find the final volume after the two liquids are mixed. The coefficient of volume expansion of toluene $\beta = 0.001(°\mathrm{C})^{-1}$. Neglect the loss of heat.

Solution

If the volume at temperature t_1 is V_1, then the volume at temperature 0°C is

$$V_{10} = \frac{V_1}{1+\beta t_1}$$

In the same way, if the volume at temperature t_2 is V_2, we have

$$V_{20} = \frac{V_2}{1+\beta t_2}$$

Furthermore, if the density of the liquid at 0°C is ρ, then the masses are

$$m_1 = V_{10}\rho \text{ and } m_2 = V_{20}\rho$$

respectively.

After mixing the liquids, the temperature is

$$t = \frac{m_1 t_1 + m_2 t_2}{m_1 + m_2}$$

The volumes at this temperature are

$$V_{10}(1+\beta t) \text{ and } V_{20}(1+\beta t)$$

The sum of the volumes after mixing

$$\begin{aligned} V_{10}(1+\beta t) + V_{20}(1+\beta t) &= V_{10} + V_{20} + \beta(V_{10}+V_{20})t \\ &= V_{10} + V_{20} + \beta\Big(\frac{m_1+m_2}{d}\Big)\Big(\frac{m_1 t_1 + m_2 t_2}{m_1+m_2}\Big) \\ &= V_{10} + V_{20} + \beta\Big(\frac{m_1 t_1 + m_2 t_2}{d}\Big) \\ &= V_{10} + \beta V_{10} t_1 + V_{20} + \beta V_{20} t_2 \\ &= V_{10}(1+\beta t_1) + V_{20}(1+\beta t_2) \\ &= V_1 + V_2 \end{aligned}$$

The sum of the volumes is constant. In our case, it is 410cm^3. The result is valid for any number of quantities of toluene, as the mixing can be done successively adding always one more glass of liquid to the mixture.

Problem 3

Parallel light rays are falling on the plane surface of a semi-cylinder made of glass, at an angle of 45°, in such a plane which is perpendicular to the axis of the semi-cylinder (Figure 10). Index of refraction is $\sqrt{2}$. Where are the rays emerging out of the cylindrical surface?

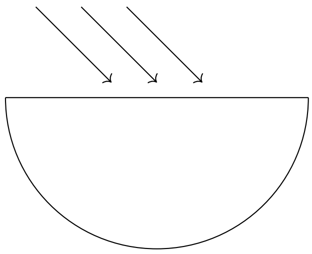


Figure 10: Sketch for Problem 3

Solution
Let us use angle θ to describe the position of the rays in the glass (Figure 11). According to the law of refraction, $\frac{\sin 45^\circ}{\sin \beta} = \sqrt{2}$, $\sin \beta = 0.5$, $\beta = 30^\circ$. The refracted angle is 30° for all of the incoming rays. We have to investigate what happens if θ changes from 0° to 180°.

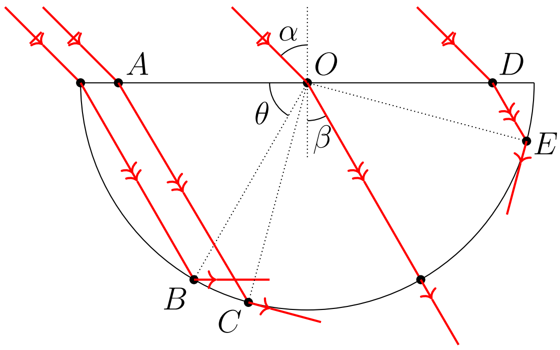


Figure 11: Auxiliary Sketch for Problem 3

It is easy to see that θ ($\angle AOB$) can not be less than 60°. The critical angle is given by $\sin \beta_{critical} = \frac{1}{n} = \frac{\sqrt{2}}{2}$, hence $\beta_{critical} = 45^\circ$. In the case of total internal reflection, $\angle ACO = 45^\circ$, hence $\theta = 180^\circ - 60^\circ - 40^\circ = 75^\circ$. If θ is more than 75°, the rays can emerge the cylinder. Increasing the angle we reach the critical angle again if $\angle OED = 45^\circ$. Thus, the rays are leaving the glass cylinder if $75^\circ < \theta < 165^\circ$. CE, arc of the emerging rays, subtends a central angle of 90°.

Experimental problem

Three closed boxes (black boxes) with two plug sockets on each are present for investigation. The participants have to find out, without opening the boxes, what kind of elements are in them and measure their characteristic properties. AC and DC meters (their internal resistance and accuracy are given) and AC 50Hz and DC sources are put at the participants' disposal.

Solution

No voltage is observed at any of the plug sockets therefore none of the boxes contains a source.

Measuring the resistances using first AC then DC, one of the boxes gives the same result. Conclusion: the box contains a simple resistor. Its resistance is determined by measurement.

One of the boxes has a very great resistance for DC but conducts AC well. It contains a capacitor, the value can be computed as $C = \frac{1}{\omega X_C}$.

The third box conducts both AC and DC, its resistance for AC is greater. It contains a resistor and an inductor connected in series. The values of the resistance and the inductance can be computed from the measurements.

III IPhO (Brno, 1969)

Theoretical problem

Problem 1

Figure 12 shows a mechanical system consisting of three carts A, B and C of masses $m_1 = 0.3$kg, $m_2 = 0.2$kg, $m_3 = 1.5$kg, respectively. Carts B and A are connected by a light taut inelastic string which passes over a light smooth pulley attaches to the cart C as shown. For this problem, all resistive and frictional forces may be ignored as may the moments of inertia of the pulley and of the wheels of all three carts. Take the acceleration due to gravity g to be 9.81ms^{-2}.

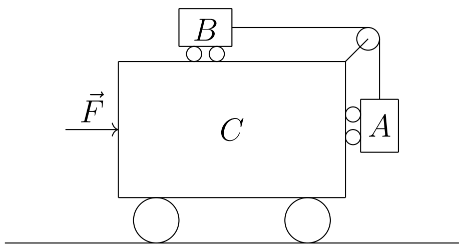


Figure 12: Sketch for Problem 1

A horizontal force $\vec{F}$ is now applied to cart C as shown. The size of $\vec{F}$ is such that carts A and B remain at rest relative to cart C. Find the tension in the string connecting carts A and B. Determine the magnitude of $\vec{F}$.

Later, cart C is held stationary, while carts A and B are released from rest. Determine the accelerations of carts A and B. Calculate also the tension in the string.

Solution

Case 1. The force $\vec{F}$ has so big magnitude that the carts A and B remain at the rest with respect to the cart C, i.e. they are moving with the same acceleration as the cart C is. Let $\vec{G_1}$, $\vec{T_1}$ and $\vec{T_2}$ denote forces acting on particular carts as shown in the Figure 13 and let us write the equations of motion for the carts A and B and also for whole mechanical system. Note that certain internal forces (viz. normal reactions) are not shown.

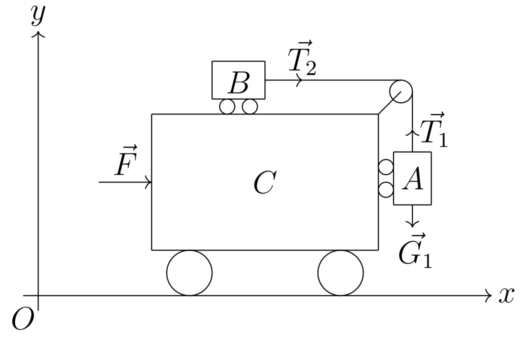


Figure 13: Auxiliary Sketch for Problem 1

The cart B is moving in the coordinate system Oxy with an acceleration a_x. The only force acting on the cart B is the force $\vec{T}_2$, thus

$$T_2 = m_2 a_x$$

Since $\vec{T}_1$ and $\vec{T}_2$ denote tensions in the same cord, their magnitudes satisfy

$$T_1 = T_2$$

The forces $\vec{T}_1$ and $\vec{G}_1$ act on the cart A in the direction of the y-axis. Since, according to condition 1, the carts A and B are at rest with respect to the cart C, the acceleration in the direction of the y-axis equals to zero, $a_y = 0$, which yields

$$T_1 - m_1 g = 0$$

Consequently,

$$T_1 = T_2 = m_1 g \tag{12}$$

So, the motion of the whole mechanical system is described by the equation

$$F = (m_1 + m_2 + m_3) a_x$$

because forces between the carts A and C and also between the carts B and C are internal forces with respect to the system of all three bodies. Let us remark here that also the tension $\vec{T}_2$ is the internal force with respect to the system of all bodies, as can be easily seen from the analysis of forces acting on the pulley. From equations regarding tensions in the cord, we obtain

$$a_x = \frac{m_1}{m_2} g$$

Substituting the last result, we arrive at

$$F = (m_1 + m_2 + m_3)\frac{m_1}{m_2}g \tag{13}$$

Numerically,$T_2 = T_1 = 0.3 \times 9.81\text{N} = 2.94\text{N}$, $F = 2 \times \frac{3}{2} \times 9.81\text{N} = 29.4\text{N}$.

Case 2. If the cart C is immovable then the cart A moves with an acceleration a_y and the cart B with an acceleration a_x. Since the cord is inextensible (i.e. it cannot lengthen), the equality

$$a_x = -a_y = a$$

holds true. Then the equation of motion for the carts A, respectively B, can be written in the following form

$$\begin{aligned} T_1 &= G_1 - m_1 a \\ T_2 &= m_2 a \end{aligned}$$

The magnitudes of the tensions in the cord again satisfy

$$T_1 = T_2$$

These yield

$$(m_1 + m_2)a = m_1 g$$

Using the last result, we can calculate

$$a = a_x = -a_y = \frac{m_1}{m_1 + m_2}g \tag{14}$$

$$T_1 = T_2 = \frac{m_2 m_1}{m_2 + m_1}g \tag{15}$$

Numerically, $a = a_x = \frac{3}{5} \times 9.81\text{ms}^{-2} = 5.89\text{ms}^{-2}$, $T_1 = T_2 = 1.18\text{N}$.

Problem 2

Water of mass m_2 is contained in a copper calorimeter of mass m_1. Their common temperature is t_2. A piece of ice of mass m_3 and temperature $t_3 < 0°\text{C}$ is dropped into the calorimeter. Determine the temperature and masses of water and ice in the equilibrium state for general values of m_1, m_2, m_3, t_2 and t_3. Write equillibrium equations for all possible processes which have to be considered. Find the final temperature and final masses of water and ice for $m_1 = 1.00\text{kg}$, $m_2 = 1.00\text{kg}$, $m_3 = 2.00\text{kg}$, $t_2 = 10°\text{C}$, $t_3 = -20°\text{C}$.

Neglect the energy losses, assume the normal barometric pressure. Specific heat of copper is $c_1 = 0.1\frac{\text{kcal}}{\text{kg}°\text{C}}$, specific heat of water $c_2 = 1\frac{\text{kcal}}{\text{kg}°\text{C}}$, specific heat of ice $c_3 = 0.492\frac{\text{kcal}}{\text{kg}^\circ\text{C}}$, latent heat of fusion of ice $l = 78.7\frac{\text{kcal}}{\text{kg}}$. Take 1cal = 4.2J.

Solution
We use the following notation: t temperature of the final equillibrium state, $t_0 = 0^\circ$C the melting point of ice under normal pressure conditions, M_2 final mass of water, M_3 final mass of ice, $m_2^{'} \le m_2$ mass of water which freezes to ice, $m_3^{'} \le m_3$ mass of ice which melts to water.

Generally, four possible processes and corresponding equilibrium states can occur:

1. $t_0 < t < t_2$, $m_2^{'} = 0$, $m_3^{'} = m_3$, $M_2 = m_2 + m_3$, $M_3 = 0$.
   Unknown final temperature t can be determined from the equation

$$(m_1c_1 + m_2c_2)(t_2 - t) = m_3c_3(t_0 - t_3) + m_3l + m_3c_2(t - t_0) \qquad (16)$$

   However, only the solution satisfying the condition $t_0 < t < t_2$ does make physical sense.

2. $t_3 < t < t_0$, $m_2^{'} = m_2$, $m_3^{'} = 0$, $M_2 = 0$, $M_3 = m_2 + m_3$.
   Unknown final temperature t can be determined from the equation

$$m_1c_1(t_2 - t) + m_2c_2(t_2 - t_0) + m_2l + m_2c_3(t_0 - t) = m_3c_3(t - t_3) \qquad (17)$$

   However, only the solution satisfying the condition $t_3 < t < t_0$ does make physical sense.

3. $t = t_0$, $m_2^{'} = 0$, $0 \le m_3^{'} \le m_3$, $M_2 = m_2 + m_3^{'}$, $M_3 = m_3 - m_3^{'}$.
   Unknown mass $m_3^{'}$ can be calculated from the equation

$$(m_1c_1 + m_2c_2)(t_2 - t_0) = m_3c_3(t - t_3) + m_3^{'}l \qquad (18)$$

   However, only the solution satisfying the condition $0 < m_3^{'} < m_3$ does make physical sense.

4. $t = t_0$, $0 \le m_2^{'} \le m_2$, $m_3^{'} = 0$, $M_2 = m_2 - m_2^{'}$, $M_3 = m_3 + m_2^{'}$.
   Unknown mass $m_2^{'}$ can be calculated from the equation

$$(m_1c_1 + m_2c_2)(t_2 - t_0) - m_2^{'}l = m_3c_3(t_0 - t_3) \qquad (19)$$

   However, only the solution satisfying the condition $0 < m_2^{'} < m_2$ does make physical sense.

Substituting the particular values of m_1, m_2, m_3, t_2 and t_3 to Equations 16, 17 and 18, one obtains solutions not making the physical sense (not satisfying the above conditions for t, respectively $m_3^{'}$). The real physical process under given conditions is given by the Equation (19) which yields

$$m_2^{'} = \frac{m_3c_3(t_0 - t_3) - (m_1c_1 + m_2c_2)(t_2 - t_0)}{l}$$

Substituting given numerical values, one gets $m_2^{'} = 0.11\text{kg}$. Hence, $t = 0°\text{C}$, $M_2 = m_2 - m_2^{'} = 0.89\text{kg}$, $M_3 = m_3 - m_2^{'} = 2.11\text{kg}$.

Problem 3

A small charged ball of mass m and charge q is suspended from the highest point of a ring of radius R by means of an insulating cord of negligible mass. The ring is made of a rigid wire of negligible cross section and lies in a vertical plane. One the ring, there is uniformly distributed charge Q of the same sign as q. Determine the length l of the cord so as the equilibrium position of the ball lies on the symmetry axis perpendicular to the plane of the ring. Find first the general solution and then for particular values $Q = q = 9.0 \times 10^{-8}\text{C}$, $R = 5\text{cm}$, $m = 1.0\text{g}$, $\epsilon_0 = 8.9 \times 10^{-12}\frac{\text{F}}{\text{m}}$.

Solution

In equilibrium, the cord is stretched in the direction of resultant force of $\vec{G} = m\vec{g}$ and $\vec{F} = q\vec{E}$, where $\vec{E}$ stands for the electric field strength of the ring on the axis in distance x from the plane of the ring, see Figure 14. Using the triangle similarity, one can write

$$\frac{x}{R} = \frac{Eq}{mg}$$

For the calculation of the electric field strength, let us divide the ring to n identical parts, so as every part carries the charge $\frac{Q}{n}$. The electric field strength magnitude of one part of the ring is given by

$$\Delta E = \frac{Q}{4\pi\epsilon_0 l^2 n}$$

This electric field strength can be decomposed into the component in the direction of the x-axis and y-axis, see Figure 15. Magnitudes of both components obey

$$\begin{aligned} \Delta E_x &= \Delta E \cos\alpha = \frac{\Delta E x}{l} \\ \Delta E_y &= \Delta E \sin\alpha \end{aligned}$$

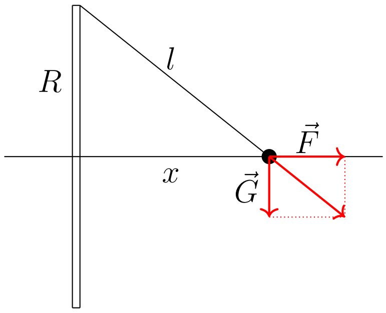


Figure 14: Sketch for Problem 3

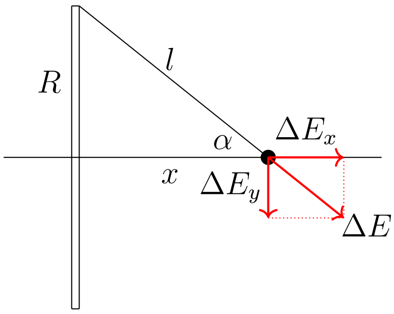


Figure 15: Auxiliary Sketch for Problem 3

It follows from the symmetry, that for every part of the ring, there exists another one having the component ΔE_y of the same magnitude, but oppositely oriented. Hence, components on y-axis cancel each other and resultant electric field strength has the magnitude

$$E = E_x = n\Delta E_x = \frac{Qx}{4\pi\epsilon_0 l^3}$$

Finally, we obtain the cord length

$$l = \sqrt[3]{\frac{QqR}{4\pi\epsilon_0 mg}} \tag{20}$$

Numerically,

$$l = \sqrt[3]{\frac{9.0\times10^{-8}\times9.0\times10^{-8}\times5.0\times10^{-2}}{4\pi\times8.9\times10^{-12}\times10^{-3}\times9.8}} = 7.2\times10^{-2}\text{m}$$

Problem 4

A glass plate is placed above a glass cube of 2cm edges in such a way that there remains a thin air layer between them, see Figure 16. Electromagnetic radiation of wavelength between 400nm and 1150nm (for which the plate is penetrable) incident perpendicular to the plate from above is reflected from both air surfaces and interferes. In this range, only two wavelengths give maximum reinforcements, one of them is $\lambda = 400$nm. Find the second wavelength. Determine how it is necessary to warm up the cube so as it would touch the plate. The coefficient of linear thermal expansion is $\alpha = 8.0 \times 10^{-6}{}^{\circ}\mathrm{C}^{-1}$, the refractive index of the air $n = 1$. The distance of the bottom of the cube from the plate does not change during warming up.

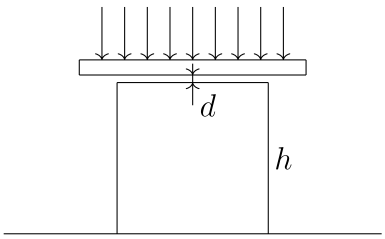


Figure 16: Sketch for Problem 4

Solution
Condition for the maximum reinforcement can be written as

$$2dn - \frac{\lambda_k}{2} = k\lambda_k \text{ for } k = 0, 1, 2, \ldots$$

i.e.

$$2dn = (2k+1)\frac{\lambda_k}{2} \tag{21}$$

with d being thickness of the layer, n the refractive index and k maximum order. Let us denote $\lambda' = 1150$nm. Since for $\lambda = 400$nm, the condition for maximum is satisfied by the assumption, let us denote $\lambda_p = 400$nm, where p is an unknown integer identifying the maximum order, for which

$$\lambda_p(2p+1) = 4dn \tag{22}$$

holds true. The Equation (21) yields that for fixed d, the wavelength λ_k increases with decreasing maximum order k and vice versa. According to the assumption

$$\lambda_{p-1} < \lambda' < \lambda_{p-2}$$

i.e.

$$\frac{4dn}{2(p-1)+1} < \lambda^{'} < \frac{4dn}{2(p-2)+1}$$

Substituting to the last inequalities for $4dn$ using Equation (22), one gets

$$\frac{\lambda_p(2p+1)}{2(p-1)+1} < \lambda^{'} < \frac{\lambda_p(2p+1)}{2(p-2)+1}$$

Let us first investigate the first inequality, straightforward calculations give us gradually

$$\lambda_p(2p+1) < \lambda^{'}(2p-1) \text{ and } 2p(\lambda^{'}-\lambda_p) > \lambda^{'}+\lambda_p$$

i.e.

$$p > \left\lfloor \frac{1}{2}\frac{\lambda^{'}+\lambda_p}{\lambda^{'}-\lambda_p} \right\rfloor = \left\lfloor \frac{1}{2}\frac{1150+400}{1150-400} \right\rfloor = 1 \tag{23}$$

Similarly, from the second inequality, we have

$$\lambda_p(2p+1) > \lambda^{'}(2p-3) \text{ and } 2p(\lambda^{'}-\lambda_p) < 3\lambda^{'}+\lambda_p$$

i.e.

$$p < \left\lceil \frac{1}{2}\frac{3\lambda^{'}+\lambda_p}{\lambda^{'}-\lambda_p} \right\rceil = \left\lceil \frac{1}{2}\frac{3\times 1150+400}{1150-400} \right\rceil = 3 \tag{24}$$

The only integer p satisfying both Equations (23) and (24) is $p = 2$.

Let us now find the thickness d of the air layer:

$$d = \frac{\lambda_p}{4}(2p+1) = \frac{400}{4}(2\times 2+1) = 500\text{nm}$$

Substituting d to the Equation 21, we can calculate λ_{p-1}, i.e. λ_1

$$\lambda_1 = \frac{4dn}{2(p-1)+1} = \frac{4dn}{2p-1}$$

Introducing the particular values, we obtain

$$\lambda_1 = \frac{4\times 500\times 1}{2\times 2-1} = 666.7\text{nm}$$

Finally, let us determine temperature growth Δt. Generally, $\Delta l = \alpha l \Delta t$ holds true. Denoting the cube edge by h, we arrive at $d = \alpha h \Delta t$. Hence

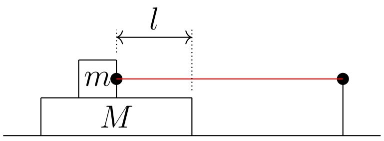
$$\Delta t = \frac{d}{\alpha h} = \frac{5\times 10^{-7}}{8\times 10^{-6}\times 5\times 2\times 10^{-2}} = 3.1^{\circ}\text{C}$$

IV IPhO (Moscow, 1970)

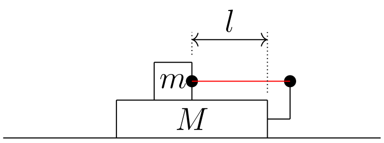
Theoretical problem

Problem 1

A long bar with mass $M = 1\text{kg}$ is placed on a smooth horizontal surface of a table where it can move frictionless. A carriage equipped with a motor can slide along the upper horizontal panel of the bar, the mass of the carriage is $m = 0.1\text{kg}$. The friction coefficient of the carriage is $\mu = 0.02$. The motor is winding a thread around a shaft at a constant speed $v_0 = 0.1\text{ms}^{-1}$. The other end of the thread is tied up to a rather distant stationary support in one case (Figure 17a), whereas in other case, it is attached to a picket at the edge of the bar (Figure 17b). While holding the bar fixed, one allows the carriage to start moving at the velocity V_0 then the bar is set loose.



(a)



(b)

Figure 17: Sketch for Problem 1

By the moment the bar is released, the front edge of the carriage is at distance $l = 0.5\text{m}$ from the front edge of the bar. For both cases, find the laws of movement of both the bar and the carriage and the time during which the carriage will reach the front edge of the bar.

Solution

First Case

By the moment of releasing the bar, the carriage has a velocity v_0 relative to the table and continues to move at the same velocity.

The bar, influenced by the friction force $F_{friction} = \mu mg$ from the carriage, gets an acceleration $a = \frac{F_{friction}}{M} = \frac{\mu mg}{M}$ or $a = 0.02\text{ms}^{-2}$, while the velocity of the bar changes with the time according to the law $v_b = at$.

Since the bar cannot move faster than the carriage, then at a moment of time $t = t_0$, its sliding will stop, that is $v_b = v_0$. Let us determine this moment of time

$$t_0 = \frac{v_0}{a} = \frac{v_0 M}{\mu mg} = 5\text{s}$$

By that moment, the displacement of the S_b ar and the carriage S_c relative to the table will be equal to

$$\begin{aligned} S_c &= v_0 t_0 = \frac{v_0^2 M}{\mu mg} \\ S_b &= \frac{at_0^2}{2} = \frac{v_0^2 M}{2\mu mg} \end{aligned}$$

The displacement of the carriage relative to the bar is equal to

$$S = S_c - S_b = \frac{v_0^2 M}{2\mu mg} = 0.25\text{m}$$

Since $S < l$, the carriage will not reach the edge of the bar until the bar is stopped by an immovable support. The distance to the support is not indicated in the problem condition so we cannot calculate this time. Thus, the carriage is moving evenly at the velocity $v_0 = 0.1\text{ms}^{-1}$, whereas the bar is moving for the first 5s uniformly accelerated with an acceleration $a = 0.02\text{ms}^{-1}$ and then the bar is moving with constant velocity together with the carriage. *Second Case*

Since there is no friction between the bar and the table surface, the system of the bodies "bar-carriage" is a closed one. For this system, one can apply the law of conservation of momentum,

$$mv + Mu = mv_0$$

where v and u are projections of velocities of the carriage and the bar relative to the table onto the horizontal axis directed along the vector of the velocity

v_0. The velocity of the thread winding v_0 is equal to the velocity of the carriage relative to the bar $(v - u)$, that is

$$v_0 = v - u$$

Solving the above system of equations, we obtain

$$u = 0 \text{ and } v = v_0$$

Thus, being released, the bar remains fixed relative to the table, whereas the carriage will be moving with the same velocity v_0 and will reach the edge of the bar within the time t equal to

$$t = \frac{l}{v_0} = 5\text{s}$$

Problem 2

A unit cell of a crystal of natrium chloride (common salt, NaCl) is a cube with the edge length $a = 5.6 \times 10^{-10}$m (Figure 18). The black circles in the figure stand for the position of natrium atoms, whereas the white ones are chlorine atoms. The entire crystal of common salt turns out to be a repetition of such unit cells. The relative atomic mass of natrium is 23 and that of chlorine is 35.5. The density of the common salt $\rho = 2.22 \times 10^3$ kgm^{-3}. Find the mass of a hydrogen atom.

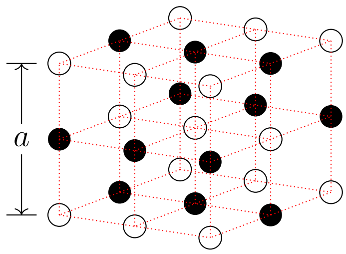


Figure 18: Sketch for Problem 2

Solution
Let us calculate the quantities of natrium atoms (n_1) and chlorine atoms (n_2) embedded in a single NaCl unit crystal cell (Figure 18).

One atom of natrium occupies the middle of the cell and it entirely belongs to the cell. Twelve atoms of natrium hold the edges of a large cube and they belong to three more celss so as $\frac{1}{4}$ part of each belongs to the first cell. Thus, we have

$$n_1 = 1 + 12 \times \frac{1}{4} = 4 \text{ atoms of natrium per unit cell}$$

In one cell, there are 6 atoms of chlorine placed on the side of the cube and 8 placed in the vertices. Each atom from a side belongs to another cell and the atom in the vertex (to seven others). Then, for one cell we have

$$n_2 = 6 \times \frac{1}{2} + 8 \times \frac{1}{8} = 4 \text{ atoms of chlorine}$$

Thus, 4 atoms of natrium and 4 atoms of chlorine belong to one unit cell of NaCl crystal.

The mass m of such a cell is equal

$$m = 4(m_{r,Na} + m_{r,Cl}) \text{ amu}$$

where $m_{r,Na}$ and $m_{r,Cl}$ are relative atomic masses of natrium and chlorine. Since the mass of hydrogen atom m_H is approximately equal to one atomic mass unit (amu),

$$m_H = 1.008 \approx 1 \text{ amu}$$

then the mass of an unit cell of NaCl is

$$m = 4(m_{r,Na} + m_{r,Cl})m_H$$

On the other hand, it is equal to

$$m = \rho a^3$$

Hence,

$$m_h = \frac{\rho a^3}{4(m_{r,Na} + m_{r,Cl})} \tag{25}$$

Numerically, $m_H = 1.67 \times 10^{-27}$kg.

Problem 3

Inside a thin-walled metal sphere with radius $R = 20$cm, there is a metal ball with the radius $r = 10$cm which has a common center with the sphere. The ball is connected with a very long wire to the Earth via an opening in the sphere (Figure 19). A charge $Q = 10^{-8}$C is placed onto the outside sphere. Calculate the potential of this sphere, electrical capacity of the obtained

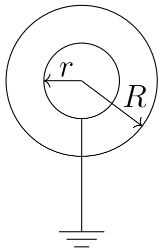


Figure 19: Sketch for Problem 3

system of conducting bodies and draw out an equivalent electric scheme.

Solution
Having no charge on the ball, the sphere has the potential

$$U_{0s} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

Numerically, $U_s = 450\text{V}$.

When connected with the Earth, the ball inside the sphere has the potential equal to zero, so there is an electric field between the ball and the sphere. This field moves a certain charge q from the Earth to the ball. Charge Q, uniformly distributed on the sphere, does not create any field inside, thus the electric field inside the sphere is defined by the ball's charge q. The potential difference between the balls and the sphere is equal

$$\Delta U = U_b - U_s = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r} - \frac{q}{R}\right)$$

Outside the sphere, the field is the same as in the case when all the charges were placed in its center. When the ball was connected with the Earth, the potential of the sphere U_s is equal

$$U_s = \frac{1}{4\pi\epsilon_0}\frac{q+Q}{R}$$

The potential of the ball

$$\begin{aligned} U_b &= U_s + \Delta U \\ &= \frac{1}{4\pi\epsilon_0}\left(\frac{q+Q}{R} + \frac{q}{r} - \frac{a}{R}\right) \\ &= \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right) = 0 \end{aligned}$$

which leads to

$$q = -Q\frac{r}{R}$$

Finally, we obtain for potential of the sphere

$$U_s = \frac{1}{4\pi\epsilon_0}\frac{Q(R-r)}{R^2} \tag{26}$$

Numerically, $U_s = 225$V.

The electric capacity of whole system of conductor is

$$C = \frac{Q}{U_s} = \frac{4\pi\epsilon_0 R^2}{R-r} \tag{27}$$

Numerically, $C = 44$pF.

The equivalent electric scheme consists of two parallel capacitors (Figure 20)

1. a spherical one with charges $+q$ and $-q$ at the plates

2. a capacitor "sphere-Earth" with charges $+(Q-q)$ and $-(Q-q)$ at the plates

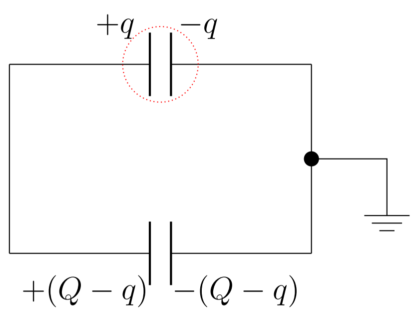


Figure 20: Auxiliary Sketch for Problem 3

Problem 4

A spherical mirror is installed into a telescope. Its lateral diameter is $D = 0.5$m and the radius of the curvature $R = 2$m. In the main focus of the mirror, there is an emission receiver in the form of a round disk. The disk is placed perpendicular to the optical axis of the mirror (Figure 21). What should the radius r of the receiver be so that it could receive the entire flux of the emission reflected by the mirror? How would the received flux of the emission decrease if the detector's dimensions decreased by 8 times? For this problem, when calculating small values α ($\alpha \ll 1$), one may perform a

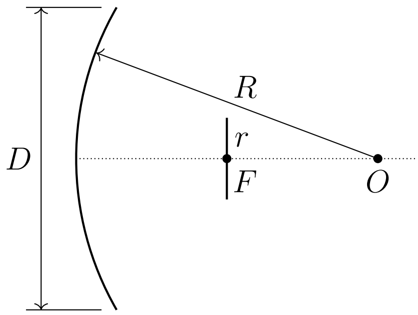


Figure 21: Sketch for Problem 4

substitution $\sqrt{1-\alpha} \approx 1 - \frac{\alpha}{2}$. Besides, diffraction should not be taken into account.

Solution

As known, rays parallel to the main optical axis of a spherical mirror, passing at little distance from it after having been reflected, join at the main focus of the mirror F, which is at the distance $\frac{R}{2}$ from the center O of the spherical surface. Let us consider now the movement of the ray reflected near the edge of the spherical mirror of large diamter D (Figure 22). The angle of incidence α of the ray onto the surface is equal to the angle of reflection. That is why the angle OAB within triangle, formed by the radius OA of the sphere, traced to the incidence point of the ray by the reflected ray AB and an intercept BO of the main optical axis, is equal to α. The angles BOA and MAO are equal, that is the angle BOA is equal to α.

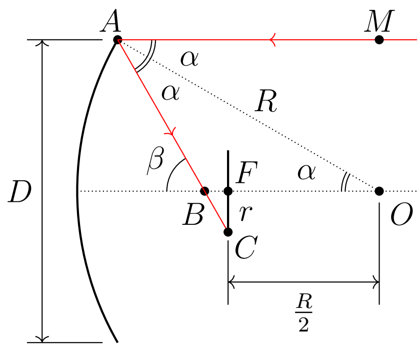


Figure 22: Auxiliary Sketch for Problem 4

;

Thus, the triangle AOB is isosceles with its side AB being equal to the side BO. Since the sum of the lengths of its two other sides exceeds the length of its third side, $AB + BO > OA = R$, hence $BO > \frac{R}{2}$. This means that a ray parallel to the main optical axis of the spherical mirror and passing not too close to it, after having been reflected, crosses the main optical axis at the point B lying between the focus F and the mirror. The focal surface is crossed by this ray at the point C which is at a certain distance $CF = r$ from the main focus.

Thus, when reflecting a parallel beam of rays by a spherical mirror finite in size, it does not join at the focus of the mirror, but forms a beam with radius r on the focal plane.

From ΔBFC, we can write

$$\begin{aligned} r &= BF \tan\beta \\ &= BF \tan 2\alpha \end{aligned}$$

where α is the maximum angle of incidence of the extreme ray onto the mirror, while $\sin\alpha = \frac{D}{2R}$.

$$\begin{aligned} BF &= BO - OF \\ &= \frac{R}{2\cos\alpha} - \frac{R}{2} \\ &= \frac{R}{2}\frac{1-\cos\alpha}{\cos\alpha} \end{aligned}$$

Thus,

$$r = \frac{R}{2}\frac{1-\cos\alpha}{\cos\alpha}\frac{\sin 2\alpha}{\cos 2\alpha}$$

Let us express the values of $\cos\alpha$, $\sin 2\alpha$, $\cos 2\alpha$ via $\sin\alpha$ taking into account the small value of the angle α,

$$\begin{aligned} \cos\alpha &= \sqrt{1-\sin^2\alpha} \approx 1 - \frac{\sin^2\alpha}{2} \\ \sin^2\alpha &= 2\sin\alpha\cos\alpha \\ \cos 2\alpha &= \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha \end{aligned}$$

which leads to

$$r = \frac{R}{2}\frac{\sin^3\alpha}{1-2\sin^2\alpha} \approx \frac{R}{2}\sin^3\alpha$$

With $\sin\alpha = \frac{D}{2R}$,

$$r \approx \frac{D^3}{16R^2} \tag{28}$$

Substituting numerical data, we will obtain $r \approx 1.95 \times 10^{-3}\text{m} \approx 2\text{mm}$.

From the expression $D = \sqrt[3]{16R^2r}$, one can see that if the radius of the receiver is decreased 8 times, the transversal diameter D' of the mirror, from which the light comes to the receiver, will be decreased 2 times and thus the "effective" area of the mirror will be decreased 4 times.

The radiation flux Φ reflected by the mirror and received by the receiver will also be decreased twice since $\Phi \sim S$.

Experimental Problem

Determine the focal distances of lenses. List of instruments: three different lenses installed on posts, a screen bearing an image of a geometric figure, some vertical wiring also fixed on the posts and a ruler.

Solution

While looking at objects through lenses, it is easy to establish that there were given two converging lenses and a diverging one.

The peculiarity of the given problem is the absence of a white screen on the list of the equipment that is used to observe real images. The competitors were supposed to determine the position of the images by the parallax method observing the images with their eyes.

The focal distance of the converging lens may be determined by the following method.

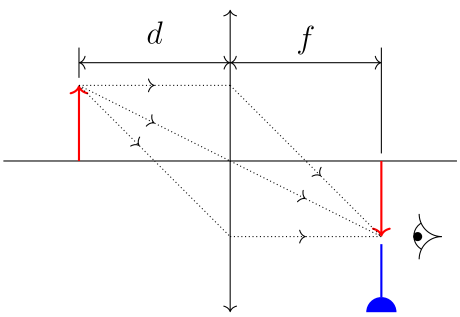


Figure 23: Sketch for Experimental Problem

Using a lens one can obtain a real image of a geometrical figure shown on the screen. The position of the real image is registered by the parallax method: if one places a vertical wire (Figure 23) to the point, in which the image is located, then at small displacements of the eye from the main optical axis of the lens, the image of this object and the wire will not diverge.

We obtain the value of focal distance F from the formula of thin lens by the measured distances d and f,

$$\frac{1}{F_{1,2}} = \frac{1}{d} + \frac{1}{f}$$

which leads to

$$F_{1,2} = \frac{df}{d+f}$$

In this method, the best accuracy is achieved in the case of $f = d$.

The competitors were not asked to make a conclusion.

The error of measuring the focal distance for each of the two converging lenses can be determined by multiple repeated measurements. The total number of points was given to those competitors who carried out not less fewer than $n = 5$ measurements of the focal distance and estimated the mean value of the focal distance,

$$F_{av} = \frac{1}{n}\sum_{i=1}^{n} F_i$$

and the absolute error,

$$\Delta F = \frac{1}{n}\sum_{i=1}^{n} \Delta F_i,\ \Delta F_i = |F_i - F_{av}|$$

or root mean square error,

$$\Delta F_{rms} = \frac{1}{n}\sqrt{\sum_{i=1}^{n}(\Delta F_i)^2}$$

One could calculate the error by graphic method.

Determination of the focal distance of the diverging lens can be carried out by the method of compensation. With this goal, one has to obtain a real image S' of the object S using a converging lens. The position of the image can be registered using the parallax method.

If one places a diverging lens between the image and the converging lens, the image will be displaced. Let us find a new position of the image S". Using the reversibility property of the light rays, one can admit that the light rays leave the point S". Then point S' is a virtual image of the pint S", whereas the distances from the optical center of the concave lens to the points S' and S" are, respectively, the distances f to the image and d to the object (Figure 24). Using the formula of a thin lens, we obtain

$$\frac{1}{F_3} = -\frac{1}{d} + \frac{1}{d},\ F_3 = -\frac{fd}{d-f} < 0$$

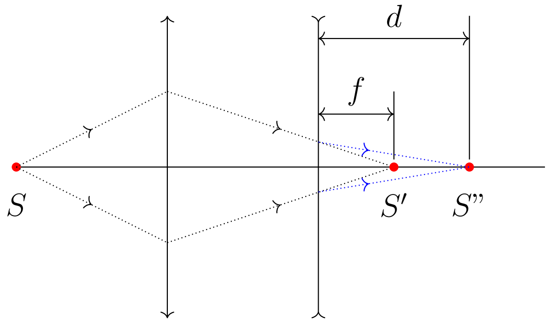


Figure 24: Auxiliary Sketch for Experimental Problem

Here, $F < 0$ is the focal distance of the diverging lens. In this case, the error of measuring the focal distance can also be estimated by the method of repeated measurements similar to the case of the converging lens.

Typical results are $F_1 = (22.0 \pm 0.4)$cm, $F_2 = (12.3 \pm 0.3)$cm, and $F_3 = (-8.4 \pm 0.4)$cm.

V IPhO (Sofia, 1971)

Theoretical Problem

Problem 1

A triangular prism of mass M is placed one side on a frictionless horizontal plane as shown in Figure . The other two sides are inclined with respect to the plane at angles α_1 and α_2, respectively. Two blocks of masses m_1 and m_2, connected by an inextensible thread, can slide without friction on the surface of the prism. The mass of the pulley, which supports the thread, is negligible. Express the acceleration a of the blocks relative to the prism in terms of the acceleration a_0 of the prism. Find the acceleration a_0 of the prism in terms of quantities given and the acceleration g due to gravity. At what ratio $\frac{m_1}{m_2}$, the prism will be in equilibrium?

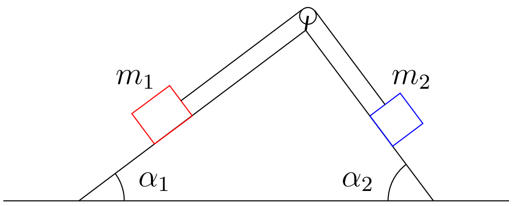


Figure 25: Sketch for Problem 1

Solution
The blocks slide relative to the prism with accelerations $\vec{a_1}$ and $\vec{a_2}$, which are parallel to its sides and have the same magnitude a.

Problem 2

A vertical glass tube of cross section $S = 1.0\text{cm}^2$ contains unknown amount of hydrogen. The upper end of the tube is closed.

VI IPhO (Bucharest, 1972)

VII IPhO (Warsaw, 1974)

VIII IPhO (Güstrow, 1975)

IX IPhO (Budapest, 1976)

X IPhO (Hadrec Králové, 1977)

XI IPhO (Moscow, 1979)

XII IPhO (Varna, 1981)

XIII IPhO (Malente, 1982)

XIV IPhO (Bucharest, 1983)

XV IPhO (Sigtuna, 1984)

XVI IPhO (Portorož, 1985)

XVII IPhO (London-Harrow, 1986)

XVIII IPhO (Jena, 1987)

XIX IPhO (Bad Ischl, 1988)

XX IPhO (Warsaw, 1989)

XXI IPhO (Groningen, 1990)

XXII IPhO (Havana, 1991)

XXIII IPhO (Helsinki-Espoo, 1992)

XXIV IPhO (Williamsburg, 1993)

XXV IPhO (Beijing, 1994)

XXVI IPhO (Canberra, 1995)