Signal Processing Course
S2 Engineering Physics ITS

Week 9: Sampling & Sparse Sampling

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Course Reference

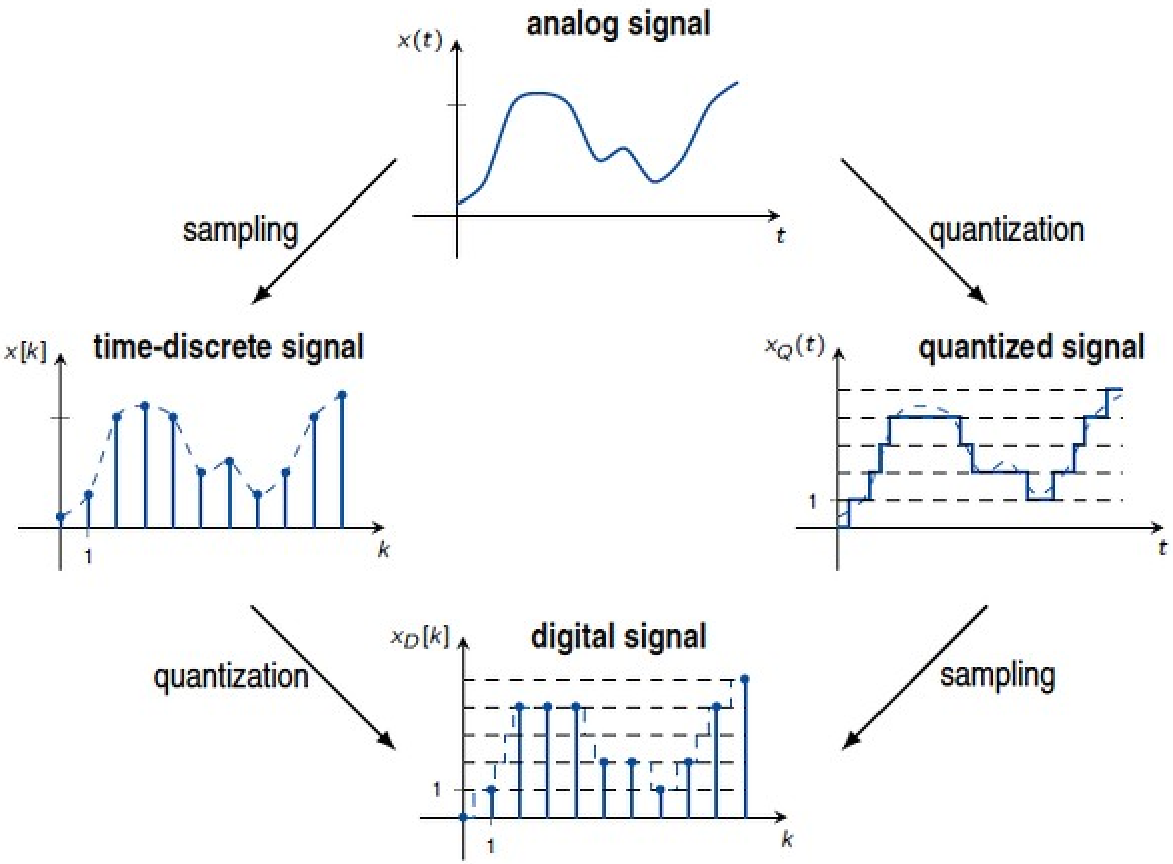
[1] Alan Oppenheim, Alan Willsky, Hamid Nawab, Signals and Systems, Prentice-Hall, 1996.

[2] Jose Unpingco, Python for Signal Processing, Springer 2014

- Repository: https://github.com/bagustris/python-for-signal-processing

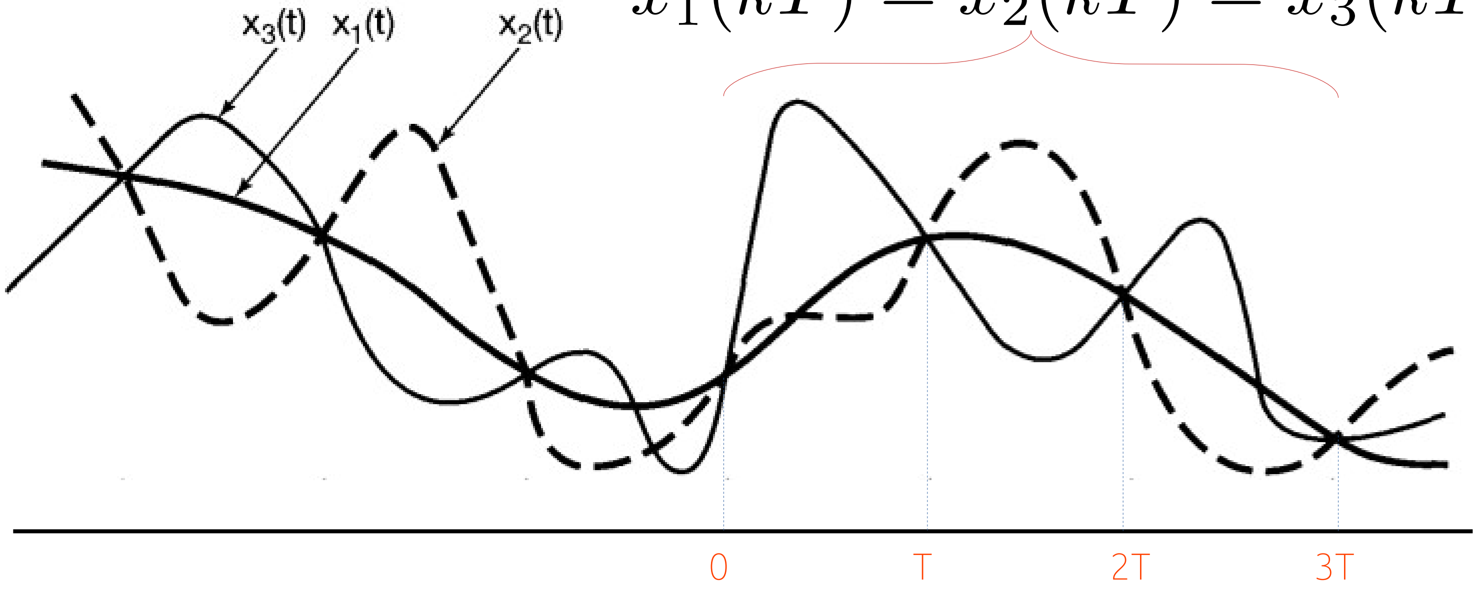
This slide is available in the repository above under "slides" directory

Sampling dan kuantifikasi



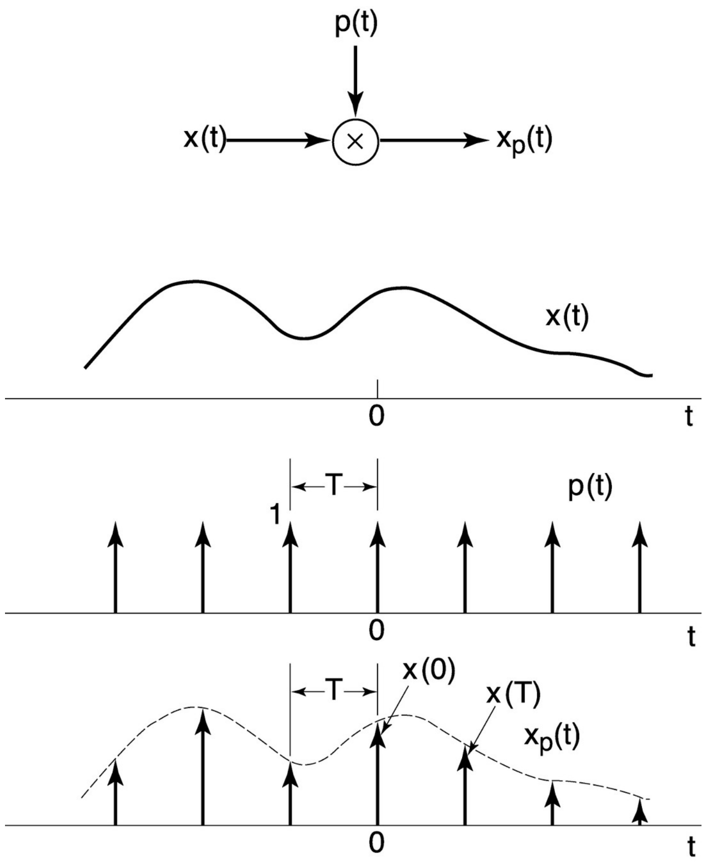
Teorema Sampling

$$x_1(kT) = x_2(kT) = x_3(kT)$$



Ketiga sinyal jika disampling dengan frekuensi T akan menghasilkan sinyal yang sama. Padahal ketiganya berbeda. *Berapa frekuensi sampling yang tepat?*

Spektrum sinyal tersampel



Sampling function/
Impulse train

$$x_p(t) = x(t)p(t)$$

Dirac
comb

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

Sampling
frequency

$$\omega_s = \frac{2\pi}{T}$$

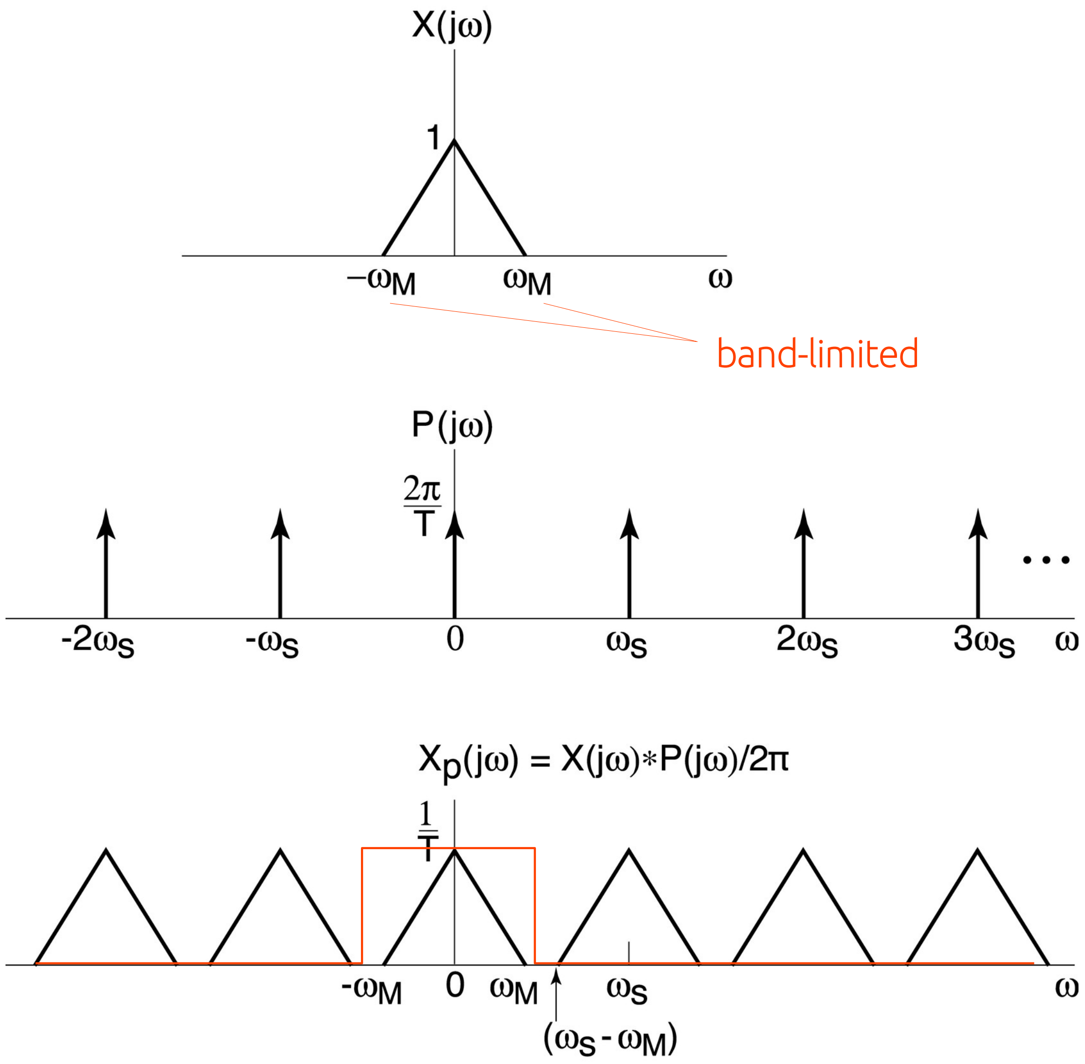
Perkalian dalam domain waktu sama dengan konvolusi dalam domain frekuensi:

$$X_p(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X(j(\omega - k\omega_s))$$

Nyquist Rate

$$\omega_S - \omega_M > \omega_M$$
$$\omega_S > 2\omega_M$$

Dengan mematuhi teorema ini, Sinyal yang dicuplik merepresentasikan sinyal asli secara sempurna untuk kasus *bandlimited signal* ($-\omega_M$ - ω_M)



Check!

1) Human can only hears audio signal from 20 Hz to 20 kHz.
   1) What is sampling interval in micro seconds?
   2) What is sampling interval in Hertz?
2) If we consider voice frequency is in the range 300-3400 Hz (narrowband), what is the minimum sampling frequency?

1

$$\omega_S > 2\omega_M$$

$$\frac{2\pi}{T} > 2 \cdot 2\pi f_m$$

$$T < \frac{1}{2f_m}$$

$$T < \frac{1}{2 \times 20000\frac{1}{s}}$$

$$T < 25 \times 10^{-6} = 25\ \mu s$$

$$\omega_S > 2\omega_M$$

$$2\pi f_s > 2 \cdot 2\pi f_m$$

$$f_s > 2 \cdot 20000$$

$$fs_s > 40000\ Hz$$

2

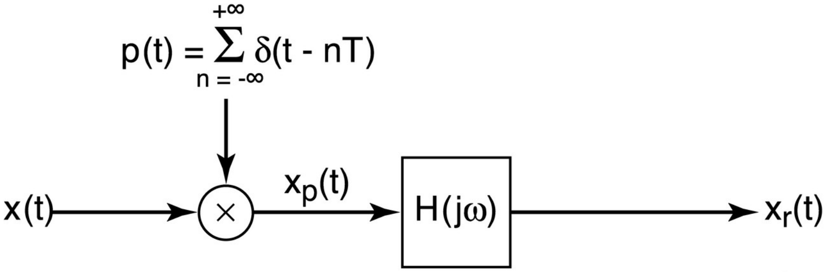
$$\omega_S > 2\omega_M$$

$$2\pi f_s > 2 \cdot 2\pi f_m$$

$$f_s > 2 \cdot 3400$$

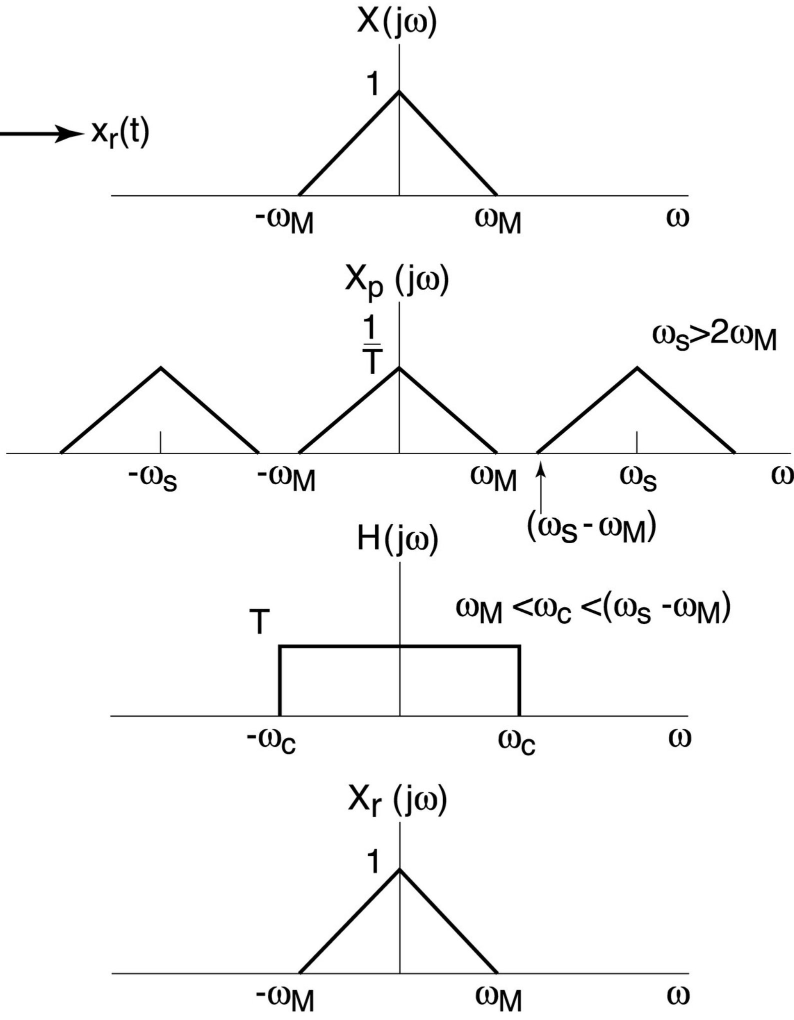
$$fs_s > 7800\ Hz$$

Rekonstruksi



$$x_r(t) = x_p(t) * h(t)$$

$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT)h(t-nT)$$



Jika tidak ada overlap antar spektra, maka sinyal dapat direkontsruksi secara sempurna

Bagaimana jika ada overlap (undersampling)?

Undersampling & Aliasing

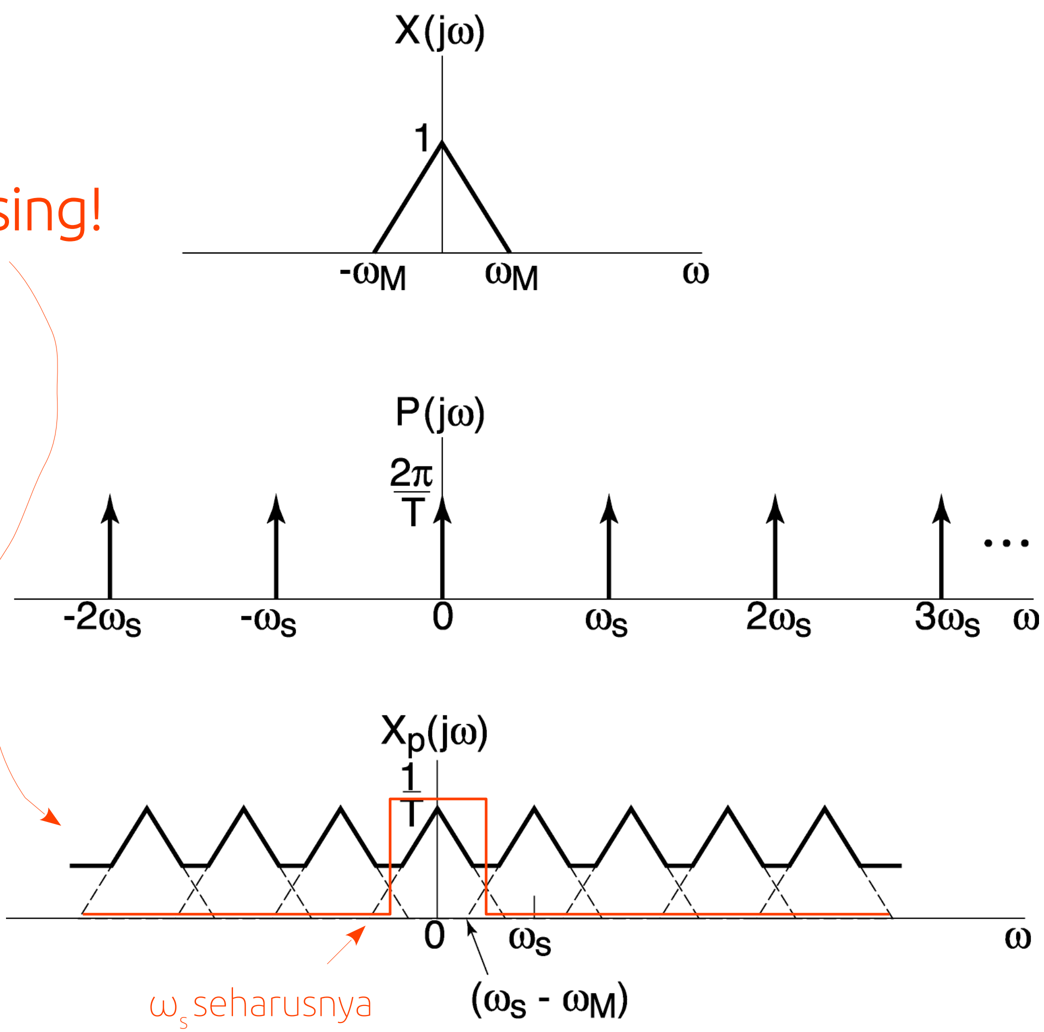
Jika undersampling, maka terjadi aliasing!

Oversampling $\omega_s > \omega_M$

Critical sampling $\omega_s = \omega_M$

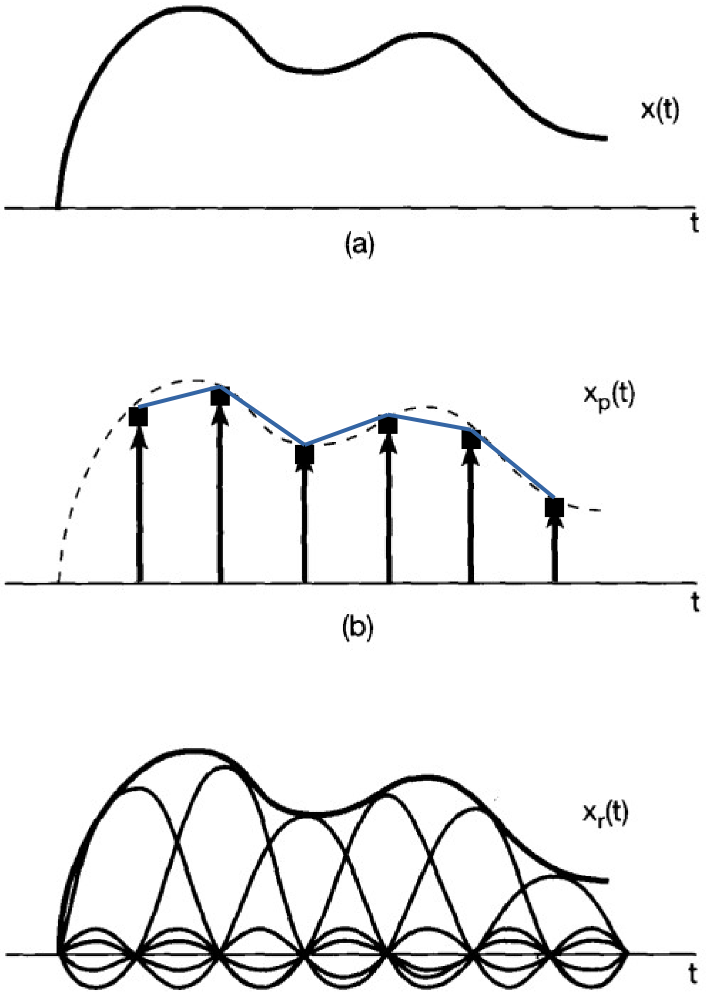
Undersampling $\omega_s < \omega_M$

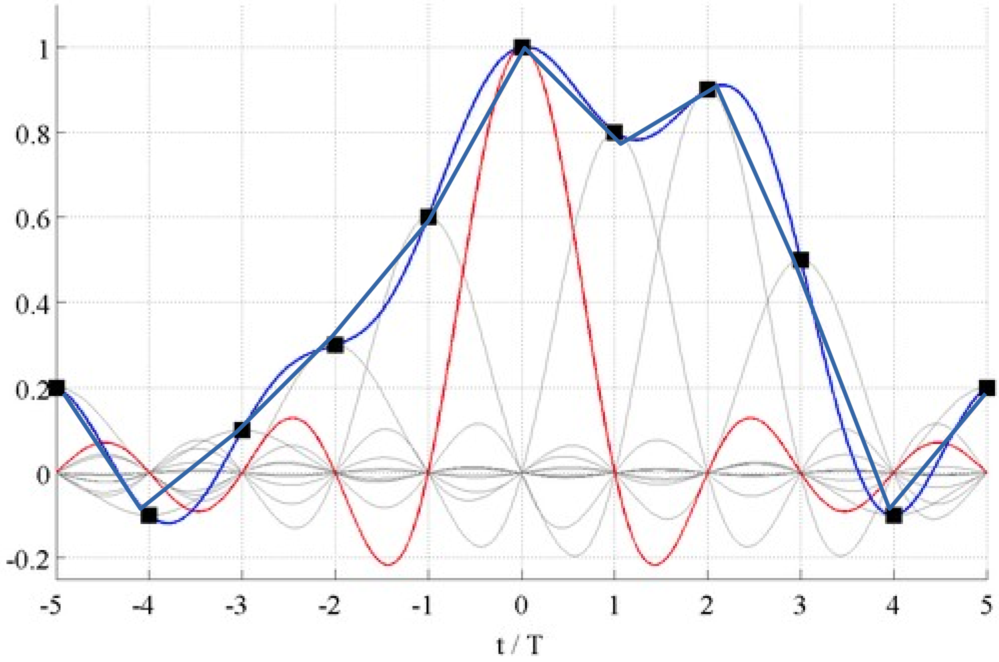
Bagaimana jika over sampling?



Interpolasi

- Linear
- Sinc





(a) / blue line: band-limited signal x(t); (b) / black square impulse train of samples of x(t) and linear interpolation; (c) / grey line ideal band-limited interpolation in which the impulse train is replaced by a superposition of sine functions

Practice Session

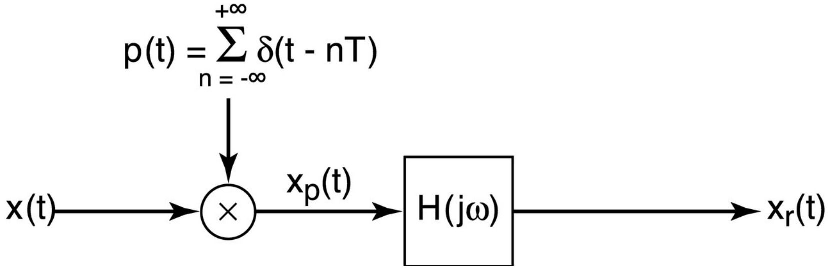
- https://nbviewer.jupyter.org/github/bagustris/Python-for-Signal-Processing/blob/master/book-version/Chapter_1_Intro.ipynb
- https://nbviewer.jupyter.org/github/spatialaudio/signals-and-systems-lecture/blob/master/sampling/ideal.ipynb
- https://nbviewer.jupyter.org/github/bagustris/Python-for-Signal-Processing/blob/master/notebook/Sampling_Theorem.ipynb
- http://nbviewer.ipython.org/github/bagustris/Python-for-Signal-Processing/blob/master/notebook/Sampling_Theorem_Part_2_v2.ipynb

Key points ideas

- Sampling

$$x(t) \to x[n] = x(nT)$$

- Bandlimited reconstruction



$$x_r(t) = x_p(t) * h(t)$$

- Sampling Theorem / Nyquist rate

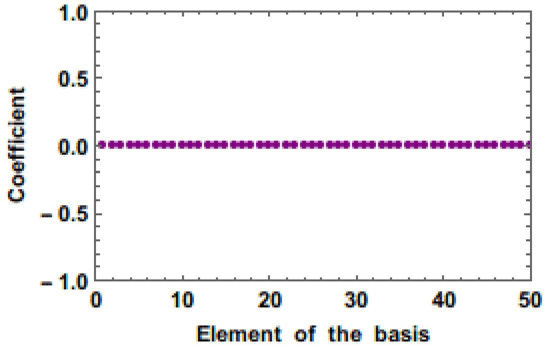
$$\omega_S > 2\omega_M$$

- Nyquist rate is sufficient condition for perfect reconstruction, not the only way for reconstruction.

Sparse Sampling

- Teknik yang dibahas sebelumnya, Teorema Nyquist Sampling, hanya mensyaratkan frekuensi sampling yang tetap (fixed-rate)
- Pada prakteknya, teori ini tidak wajib, hanya berlaku agar tidak terjadi aliasing (perfect reconstruction)
- Sparse sampling/compressive sampling menawarkan alternatifnya

Sparse Sampling



$$y = Dx$$

Data pengukuran → y
Tujuan: mendapatkan x

Aplikasi:
- Underdetermined problem
- Inverse Problem

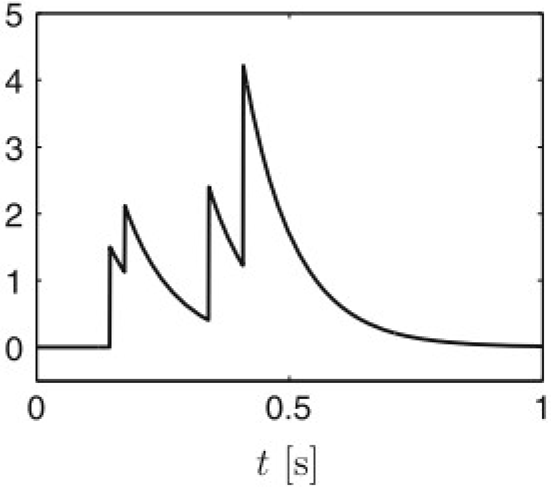
Tenik:
- Iterative

Eksploitasi sifat sparse domain/**sparse matriks:**

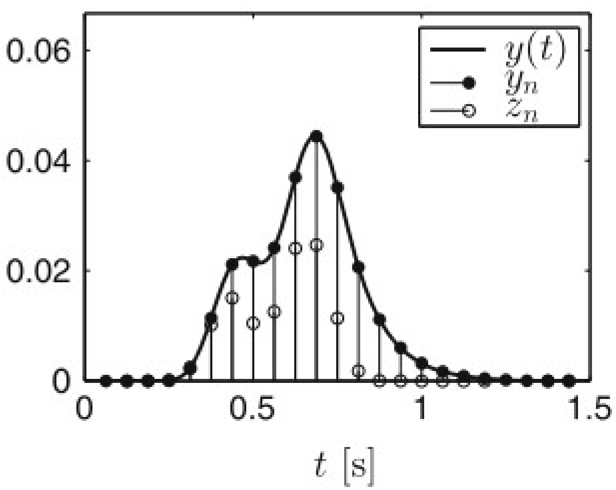
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 3 & 2 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

Hasil pengukuran diasumsikan memiliki nilai non-zero

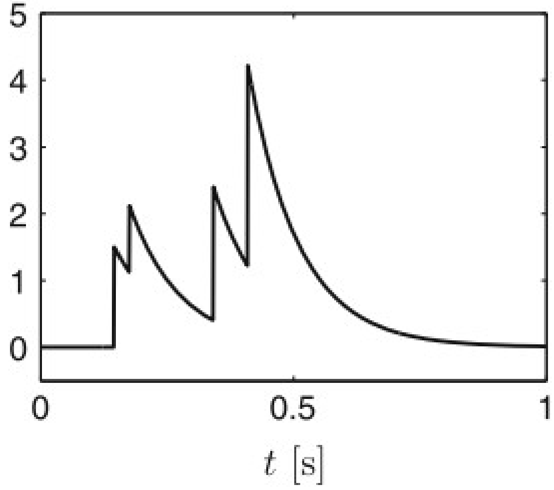
Contoh hasil rekonstruksi dengan Sparse



Sinyal Asli



Sinyal Sampling



Sinyal Rekonstruksi

Oñativia, J., & Dragotti, P. L. (2015). Sparse sampling: theory, methods and an application in neuroscience. Biological Cybernetics, 109(1), 125–139. https://doi.org/10.1007/s00422-014-0639-x

Practice Session

- http://nbviewer.ipython.org/github/bagustris/Python-for-Signal-Processing/blob/master/notebook/Compressive_Sampling.ipynb

Additional Resources

- https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-003-signals-and-systems-fall-2011/
- https://ocw.mit.edu/resources/res-6-007-signals-and-systems-spring-2011/index.htm

Homework

6009201001	7.2
6009201002	7.3
6009201003	7.4
6009201004	7.5
6009201006	7.6
6009201007	7.7
6009201008	7.8
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