Computational Vibration
S1, Engineering Physics, ITS
Week 9

Bagus Tris Atmaja, PhD

Contents

- Week 9: Review Matrix, Review Elasticity, Review Linear Algebra, Elemen segitiga node, Matriks kekakuan lokal dan global
- Week 10: Beam, matrik kekakuan elemen beam dan displacement node
- Week 11: Macam-macam pembebanan pada beam
- Week 12: Plane stress dan strain pada elemen segitiga, UTS Take Home (online)

Contents

- Week 13: Element axisymetris
- Week 14: Perpindahan panas konduksi dan konveksi
- Week 15: Komputasi FEM 1
- Week 16: Komputasi FEM 2, UAS Take Home

Homework, “Computation”, etc….

- Each class/week will have homework, the due date is the next class.
- Mid- and final-exam are take home (submission via email)
- Computation: Python 3.6, NumPy, Scipy, Sympy, Calfem
- Slide and lecture videos are available online
- Web: http://bagustris.github.io/compvib
- Contact: bagus@ep.its.ac.id

Academic Honesty

Adopted from CS50

The essence of all work that you submit to this course **must be your own**. Collaboration on assessments is permitted limited, i.e., to the extent that you may ask classmates and others for help so long as that help does not reduce to another doing your work for you. ***You may show your work to others, but you may not view theirs***, so long as you and they respect each other and this policy.

Review Matrix

Matrix

$$[A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Transpose

$$[A]^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$$

Identity

$$[A] = [I] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Symmetric

$$[A] = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 4 & -3 \\ 0 & -3 & 0 \end{bmatrix}$$

Review Matrix: skew matrix

$$[A] = \begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 3 \\ 0 & -3 & 0 \end{bmatrix}$$

$$a_{ij} = -a_{ji}$$

Transpose is obtained by changing the algebraic sign of each element

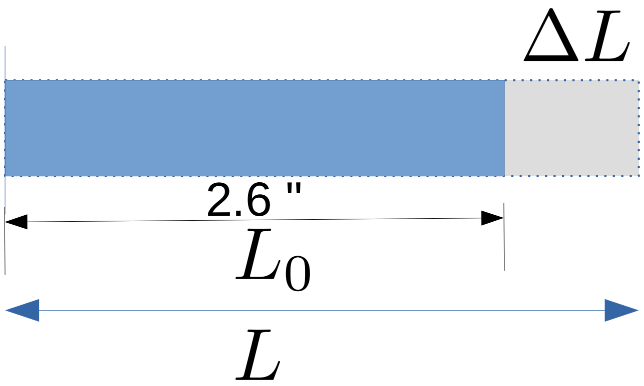
More detail about matrix, see **bab3_matriks.pdf**

Review equation of elasticity

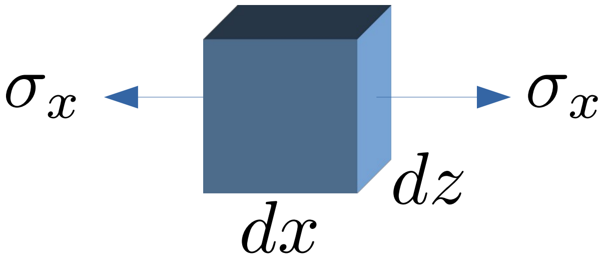
- Strain-displacement relation

$$\varepsilon = \frac{L - L_0}{L_0} = \frac{\Delta L}{L_0}$$

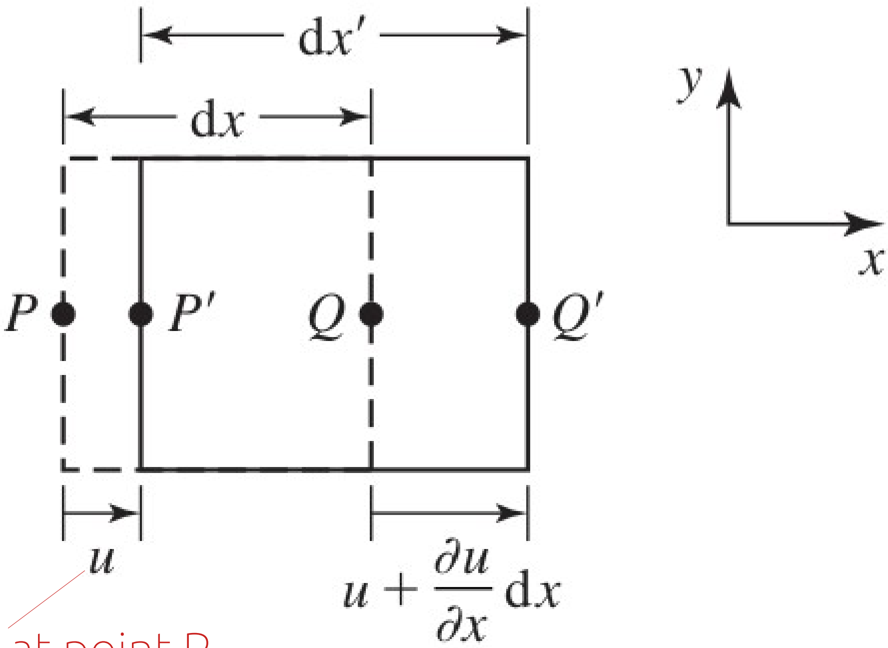
Normal strain



dy

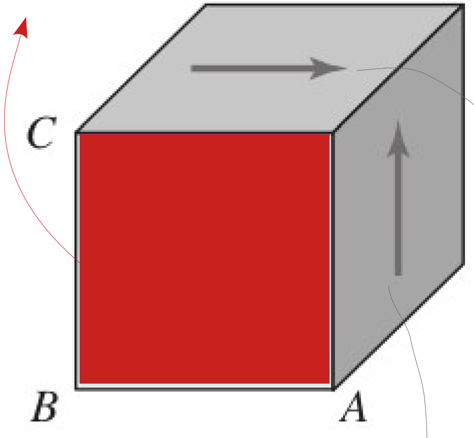


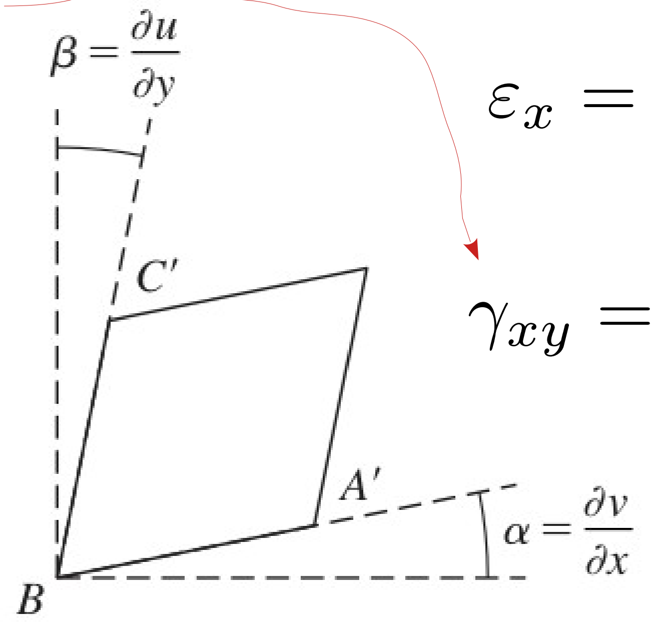
displacement



Strain at point P

Shear strain: change in the angle of an angle that was originally a right angle





$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \varepsilon_y = \frac{\partial v}{\partial y} \qquad \varepsilon_z = \frac{\partial w}{\partial z}$$

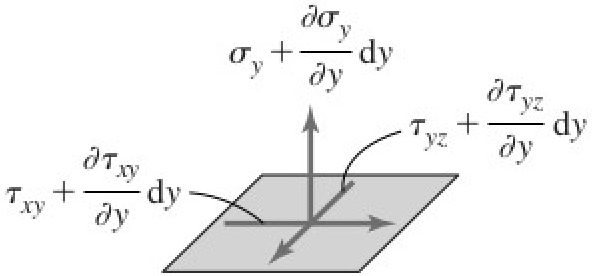
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad \varepsilon = [L]\delta$$

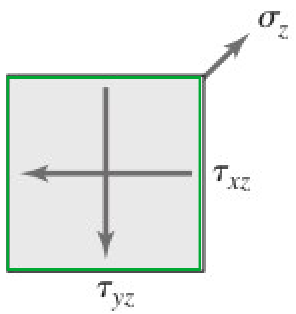
$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \qquad \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

$$[L] = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \end{bmatrix}$$

Equilibrium Equation

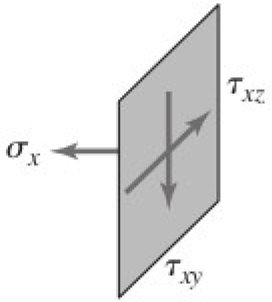
$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + B_x = 0$$

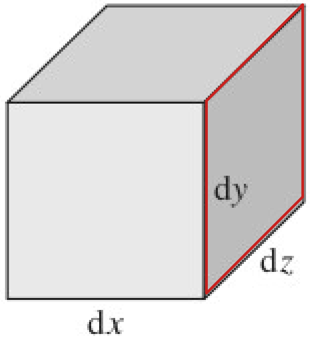


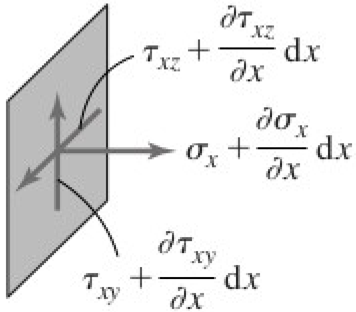


$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + B_y = 0$$

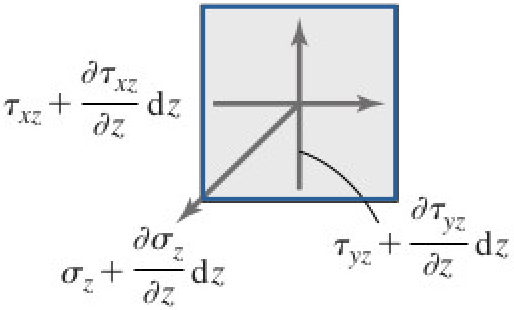
body force

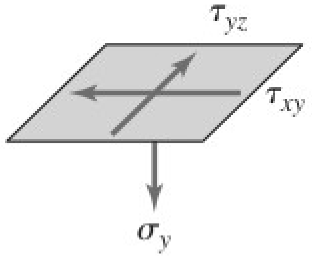






$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + B_z = 0$$

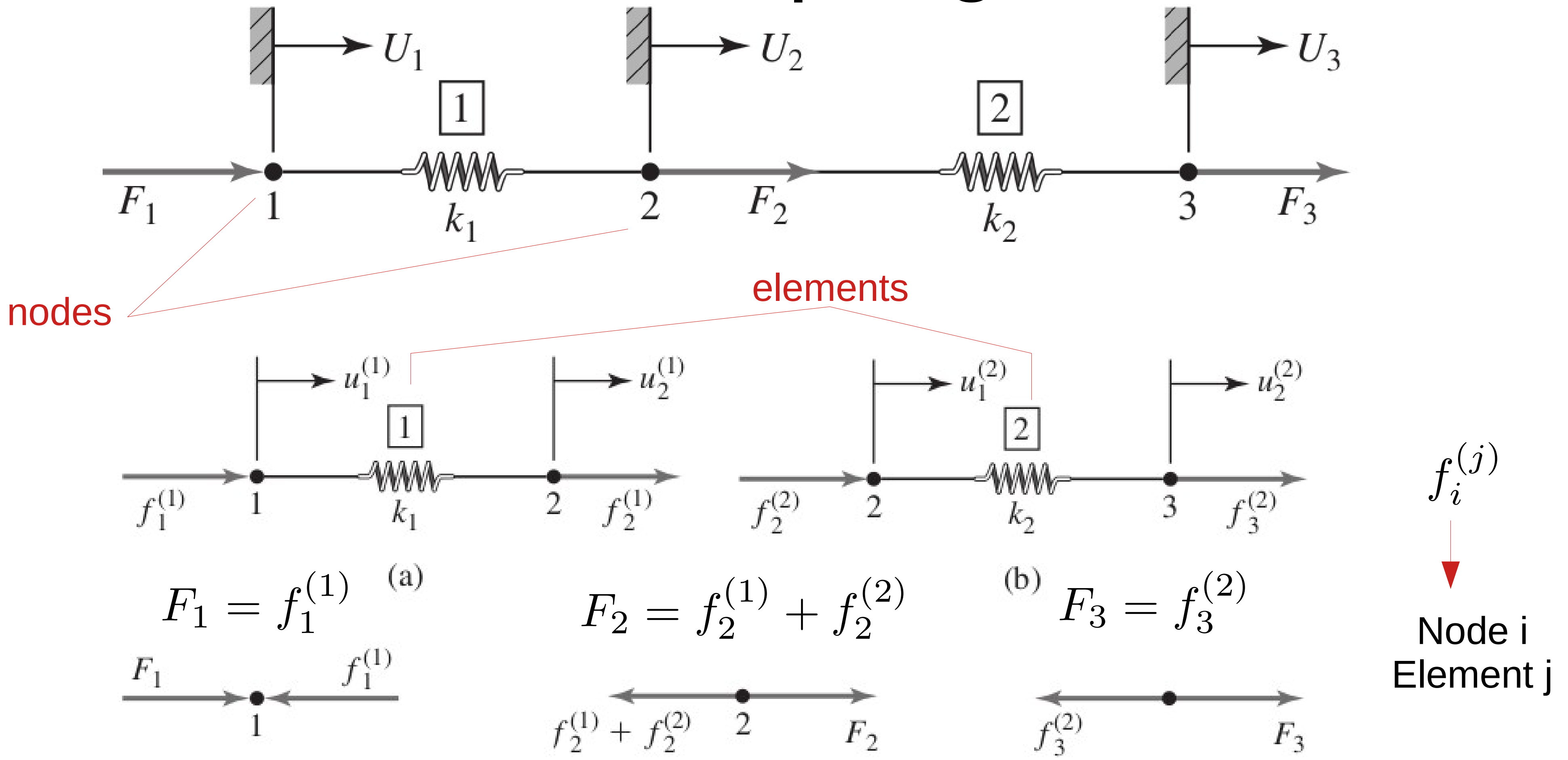




Finite Elements Calculation

- define the model
- generate element matrices
- assemble element matrices into the global system of equations
- solve the global system of equations
- evaluate element forces

Linear Spring



$$F_1 = f_1^{(1)}$$

$$F_2 = f_2^{(1)} + f_2^{(2)}$$

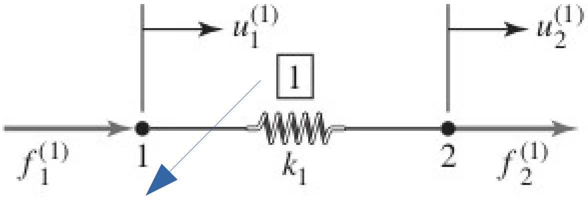
$$F_3 = f_3^{(2)}$$

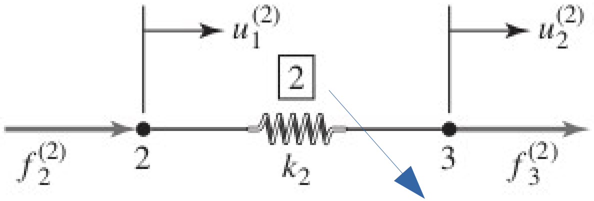
$$[K]\{U\} = \{F\} \rightarrow \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k1 + k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\begin{bmatrix} 50 & -50 & 0 \\ -50 & 125 & -75 \\ 0 & -75 & 75 \end{bmatrix} \begin{Bmatrix} \overset{=0}{U_1} \\ U_2 \\ U_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ 75 \\ 75 \end{Bmatrix}$$

$$U_2 = 3 \quad U_3 = 4 \quad F_1 = -150$$

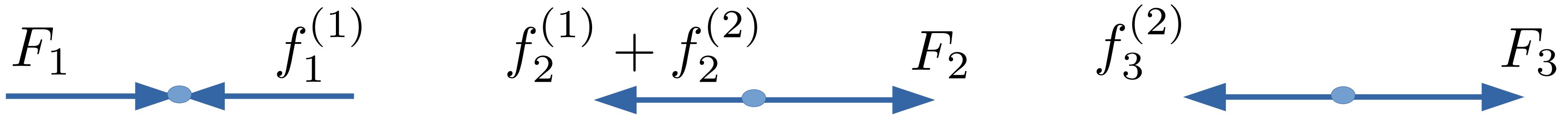
Cek dengan:





$$\begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} f_1^{(1)} \\ f_2^{(1)} \end{Bmatrix} \qquad \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} U_2 \\ U_3 \end{Bmatrix} = \begin{Bmatrix} f_2^{(2)} \\ f_3^{(2)} \end{Bmatrix}$$

Equilibrium equation (at 1 & 2)



$$f_1^{(1)} = F_1 = -150 \qquad f_2^{(2)} = F_2 - f_2^{(1)} = 75 - 150 = -75$$

$$f_2^{(1)} = -F_1 = 150 \qquad f_3^{(2)} = F_3 = 75$$

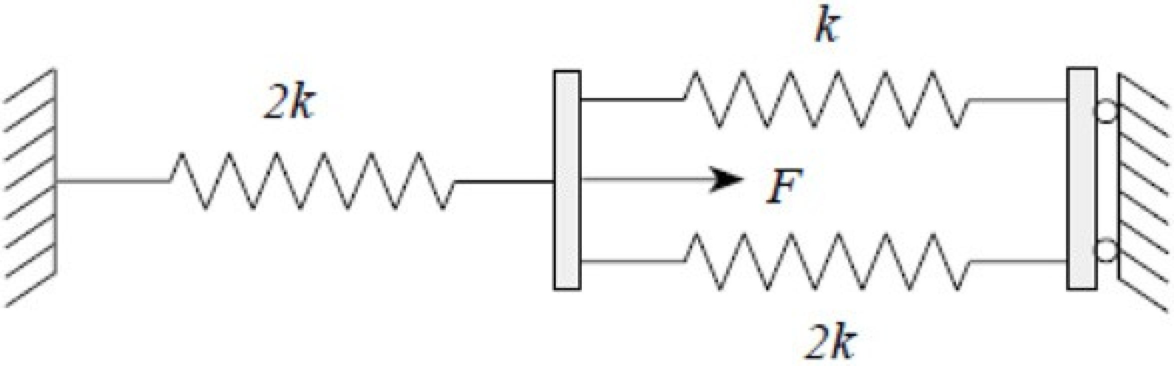
$$\begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} f_1^{(1)} \\ f_2^{(1)} \end{Bmatrix} \qquad \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} U_2 \\ U_3 \end{Bmatrix} = \begin{Bmatrix} f_2^{(2)} \\ f_3^{(2)} \end{Bmatrix}$$

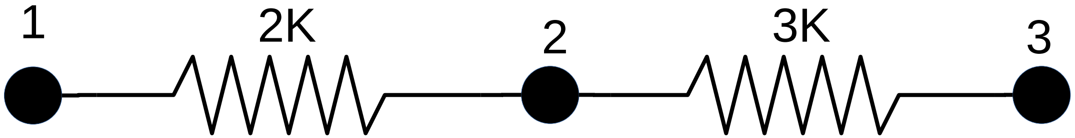
$$\underbrace{\begin{bmatrix} 50 & -50 \\ -50 & 50 \end{bmatrix} \begin{Bmatrix} 0 \\ 3 \end{Bmatrix} = \begin{Bmatrix} -150 \\ 150 \end{Bmatrix}}_{\text{Elemen \#1}} \qquad \underbrace{\begin{bmatrix} 75 & -75 \\ -75 & 75 \end{bmatrix} \begin{Bmatrix} 3 \\ 4 \end{Bmatrix} = \begin{Bmatrix} -75 \\ 75 \end{Bmatrix}}_{\text{Elemen \#2}}$$

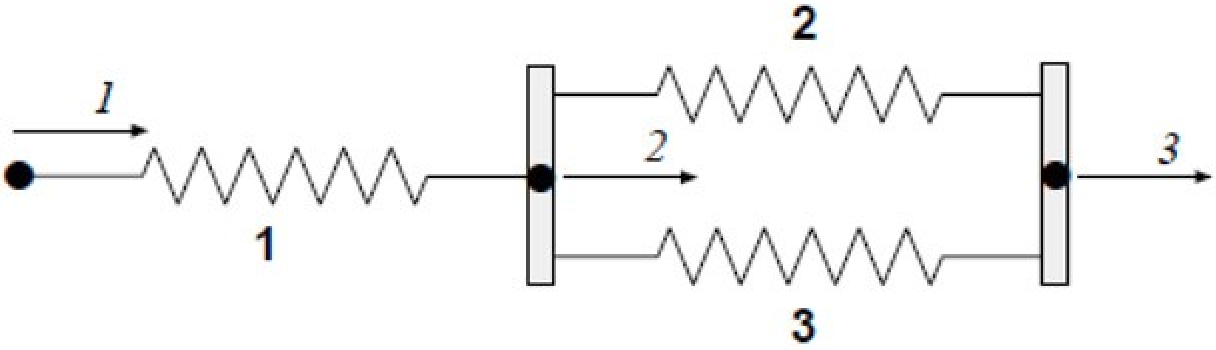
Contoh

$$K = 1500$$

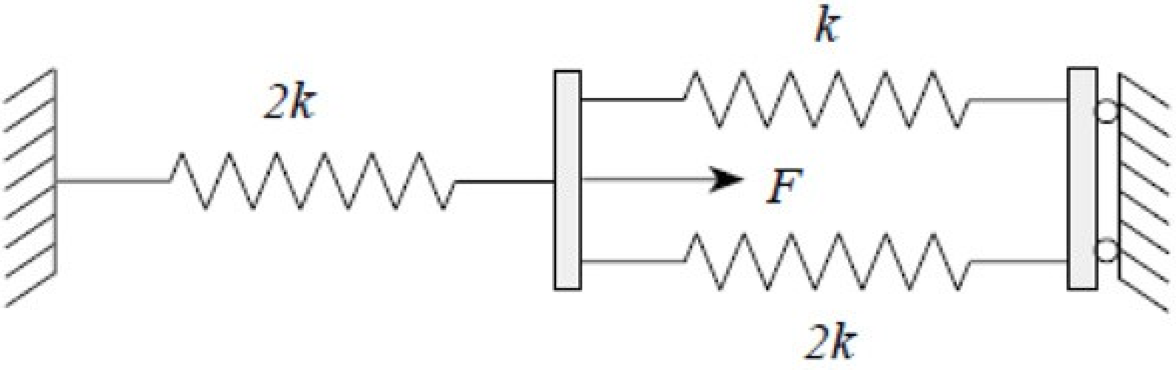
$$F = 100$$

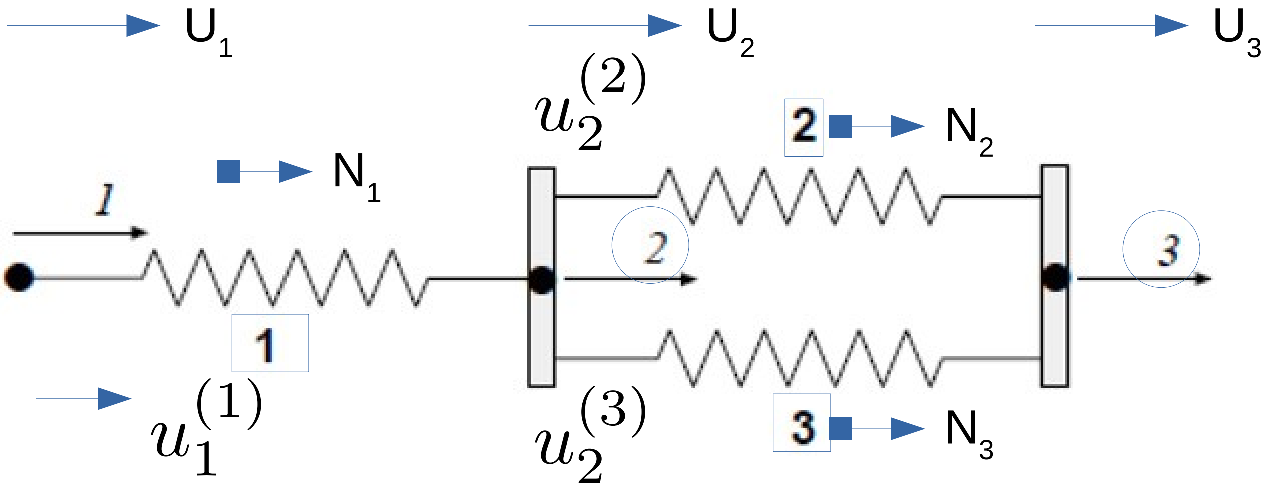






Notasi:
F → Global
f → local
○ → node
□ → elemen



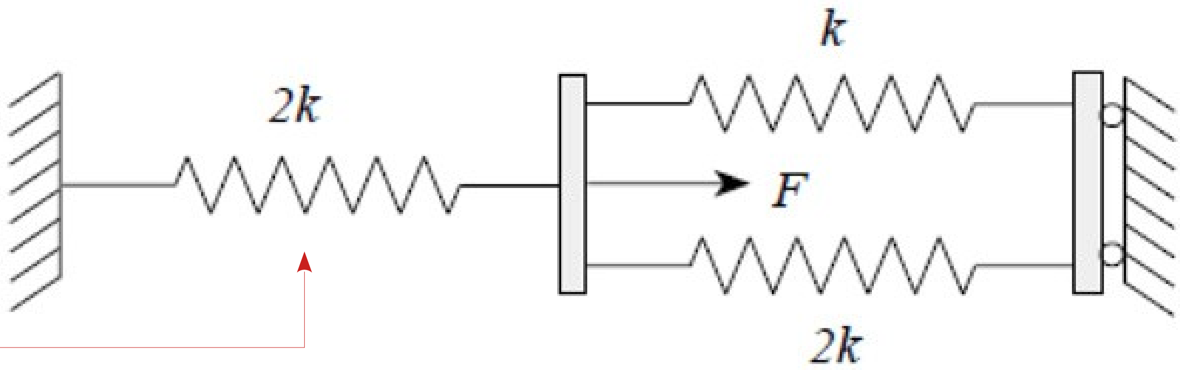


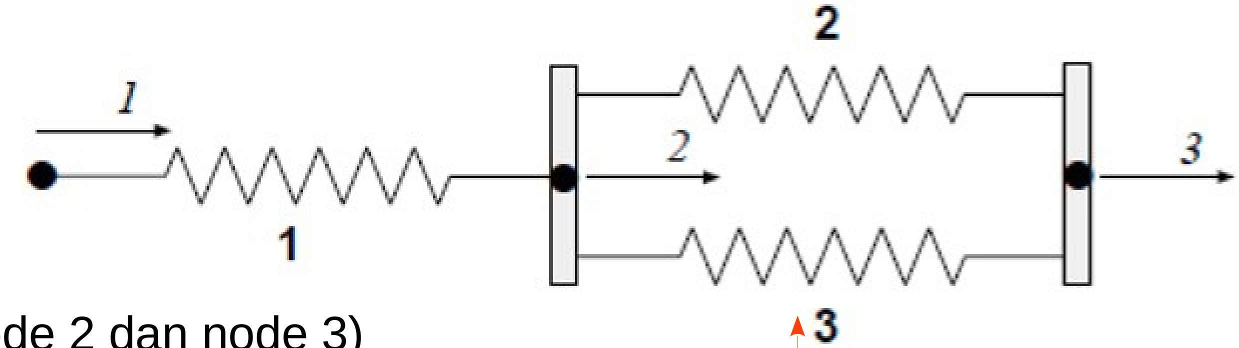
Calfem-Python

- Instalasi
- Running example
  - Jupyter notebook
  - Python file

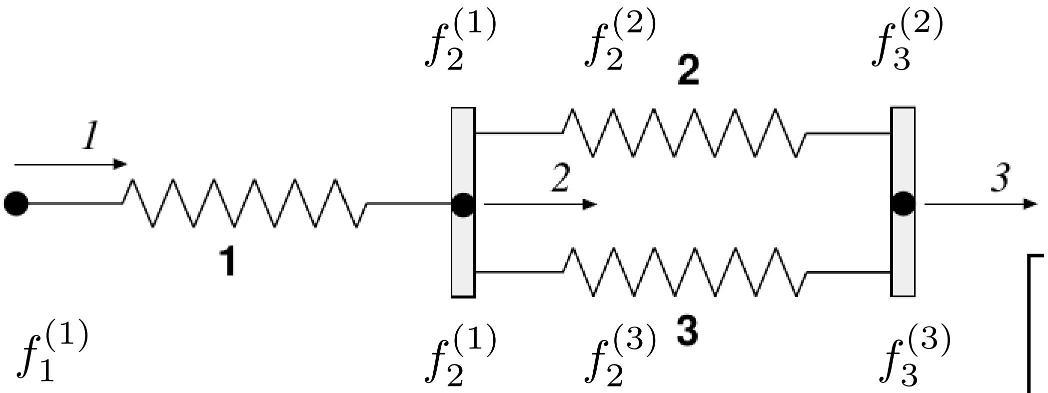
exs1.m → DEMO

```
In [3]: Edof = np.array([
   ...: [1,2],
   ...: [2,3],
   ...: [2,3]
   ...: ])
```





Elemen 2 (antara node 2 dan node 3)



$$f_2^{(3)} = -N_3 = 40 \quad f_3^{(3)} = -f_3^{(2)} = -40$$

$$f_1^{(1)} = -N1 = -40 \qquad f_2^{(2)} = -N_2 = 20$$

$$f_2^{(1)} = -f_1^{(1)} = 40 \qquad f_3^{(2)} = -f_2^{(2)} = -20$$

Elemen #2:

$$\begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} U_2 \\ U_3 \end{Bmatrix} = \begin{Bmatrix} f_2^{(2)} \\ f_3^{(2)} \end{Bmatrix}$$

$$\begin{bmatrix} 1500 & -1500 \\ -1500 & 1500 \end{bmatrix} \begin{Bmatrix} 0.013 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 20 \\ -20 \end{Bmatrix}$$

Elemen #3:

Elemen #1:

$$\begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} f_1^{(1)} \\ f_2^{(1)} \end{Bmatrix}$$

$$\begin{bmatrix} 3000 & -3000 \\ -3000 & 3000 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.013 \end{Bmatrix} = \begin{Bmatrix} -40 \\ 40 \end{Bmatrix}$$

$$\begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} U_2 \\ U_3 \end{Bmatrix} = \begin{Bmatrix} f_2^{(3)} \\ f_3^{(3)} \end{Bmatrix}$$

$$\begin{bmatrix} 3000 & -3000 \\ -3000 & 3000 \end{bmatrix} \begin{Bmatrix} 0.013 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 40 \\ -40 \end{Bmatrix}$$

Homework #1

2311840000041	Problem 2.1 manual
2311840000029	Problem 2.2 manual
2311840000083	Problem 2.3 manual
2311740000013	Problem 2.4 manual
2311740000078	Problem 2.5 manual
2311840000124	Problem 2.9 manual
2311840000086	Problem 2.1 komputasi
2311640000109	Problem 2.2 komputasi
2311840000117	Problem 2.3 komputasi
2311840000097	Problem 2.4 komputasi
2311840000099	Problem 2.5 komputasi

Manual: boleh tulis tangan atau ketik komputer, dikirim dalam bentuk pdf
Komputasi: python files (.py), matlab files (.m), etc., (lengkapi dengan komen)

Homework #1

- URL submisi: https://itsacid-my.sharepoint.com/:f:/g/personal/198608102_staff_integra_its_ac_id/EnxgNnBvdShNrz05104hlgEBnWIY5RztLIARCQRFqFgXAg

- Deadline: 2021/05/25 23:59 WIB