Truss

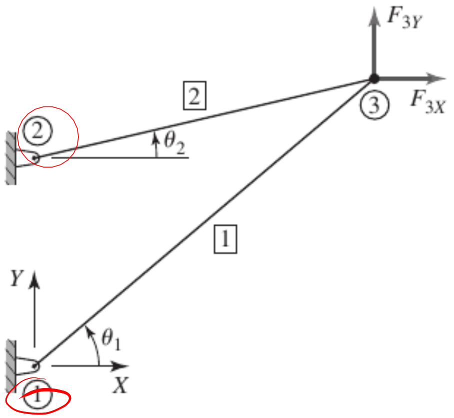
Diketahui truss spt Gb berikut :

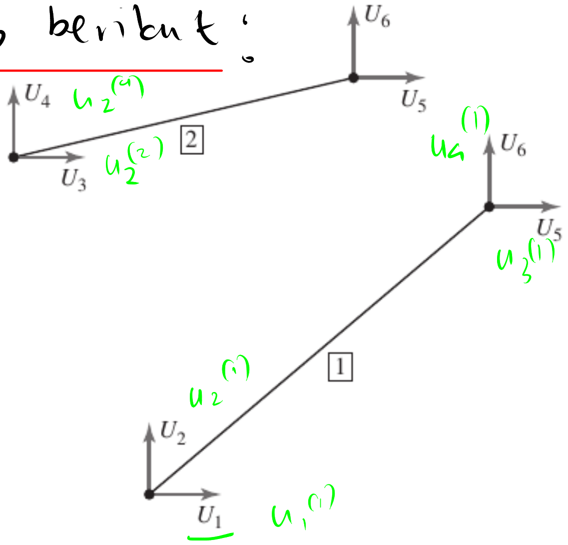
$F \rightarrow$ global

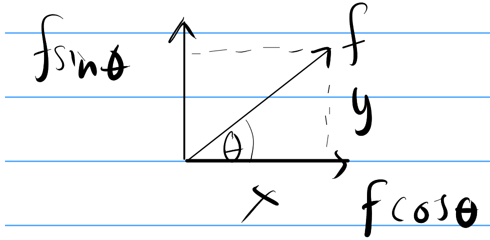
f_{3}^{2} $\rightarrow$ local

$^{2} \rightarrow$ element

$_{3} \rightarrow$ node



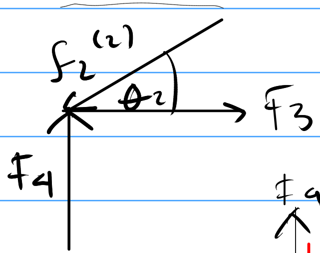




$\sin\theta = \dfrac{y}{f}$

$y = f\sin\theta$

Untuk node 2 :

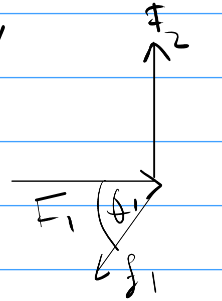


$$F_3 - f_2^{(2)}\cos\theta_2 = 0 \quad 3.2a$$
$$F_4 - f_2^{(2)}\sin\theta_2 = 0 \quad 3.2b$$

$f_2^{(2)}\cos\theta_2$ F_3

F_4 $f_2^{(2)}\sin\theta_2$

node 1 :

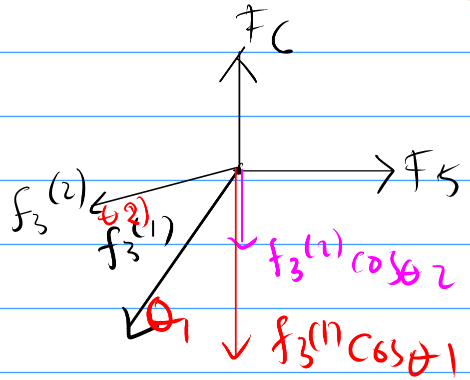


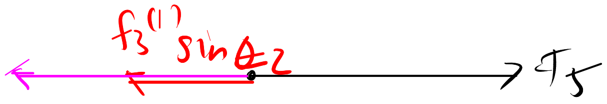
$$F_1 - f_1^{(1)}\cos\theta_1 = 0 \quad 3.1a$$
$$F_2 - f_1^{(1)}\sin\theta_1 = 0 \quad 3.1b$$

$f_1^{(1)}\cos\theta_1$ F_1

F_2 $f_1^{(1)}\sin\theta_1$

Untuk node 3 :





$f_3^{(1)} \sin\theta_2$

$\therefore$ 3.3a

$$F_6 - f_3^{(1)} \sin\theta_1 - f_3^{(2)} \sin\theta_2 = 0 \quad 3.3b$$

$u_1^{(e)} = U_1^{(e)} \cos\theta + U_2^{(e)} \sin\theta$

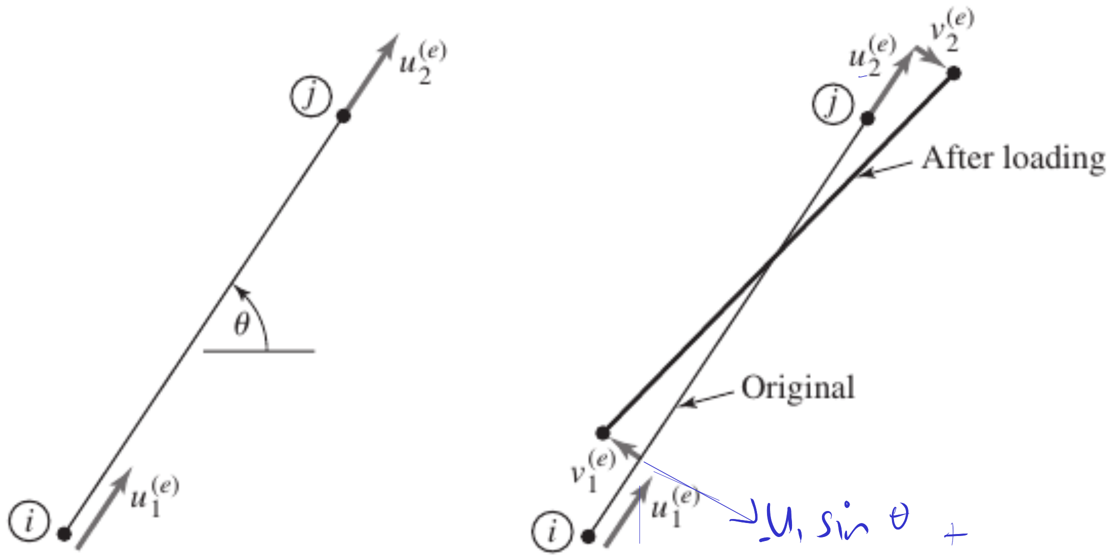
$v_1^{(e)} = -U_1^{(e)} \cos\theta + U_2^{(e)} \sin\theta$

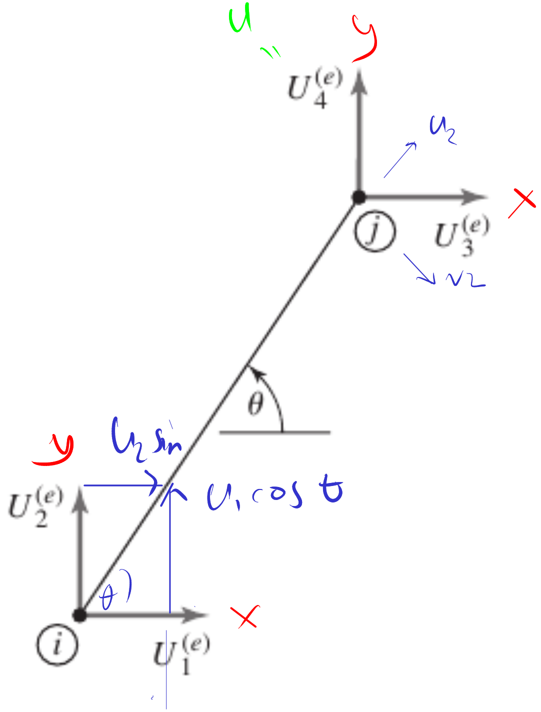
$u_2^{(e)} = U_3^{(e)} \cos\theta + U_4^{(e)} \sin\theta$

$v_2 = -v_1 = U_3^{(e)} \cos\theta - U_4^{(e)} \sin\theta$

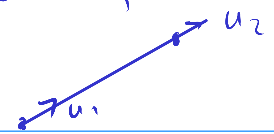
$v_1 = -U_3^{(e)} \cos\theta + U_4^{(e)} \sin\theta$

$v \rightarrow$ displacement dg arah $\perp$ u





axial force deformation $\delta = u_2 - u_1 = U_3^{(e)} \cos\theta + U_4^{(e)} \sin\theta - [U_1^{(e)} \cos\theta + U_2^{(e)} \sin\theta]$



$$= (U_3 - U_1)\cos\theta + (U_4 - U_2)\sin\theta \quad \text{---} (3.5)$$

axial force element :

$$f = kx$$

$$f^{(e)} = k^e \delta^{(e)} = k^{(e)} \left(U_3^{(e)} - U_1^{(e)} \right) \cos\theta + \left(U_4^{(e)} - U_2^{(e)} \right) \sin\theta \quad \text{----} (3.6)$$

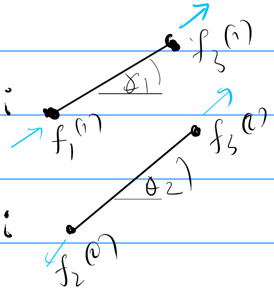
Karena

$$u_1^{(1)} = U_1, \quad u_2^{(1)} = U_2, \quad u_3^{(1)} = U_5, \quad u_4^{(1)} = U_6$$

Gaya pd elemen 1 :

(dr free body diagram)

elemen 2 :



$$f_3^{(1)} = -f_1^{(1)} = k^{(1)} [(U_5 - U_1)] \cos\theta_1 + [U_6 - U_2] \sin\theta_1 \quad \text{---} (3.7)$$

$$f_3^{(2)} = -f_2^{(2)} = k^{(2)} [U_5 - U_3] \cos\theta_2 + [U_6 - U_4] \sin\theta_2 \quad \text{---} (3.8)$$

Masukkan pers. 3.7 – 3.8 ke pers. 3.1 – 3.3

$$3.1.2 \rightarrow F_1 - f_1 \cos\theta_1 = 0 \Rightarrow F_1 = f_1^{(1)} \cos\theta_1 = \left(k^{(1)} [(U_5 - U_1)] \cos\theta_1 + [U_6 - U_2] \sin\theta_1 \right) \cos\theta_1$$

$$3.3\,b \longrightarrow F_6 - f_3^{(1)} \sin\theta_1 - f_3^{(2)} \sin\theta_2 = 0 \implies F_6 = f_3^{(1)} \sin\theta_1 + f_3^{(2)} \sin\theta_2 =$$

$$\left(k^{(2)} [U_5 - U_3] \cos\theta_2 + [U_6 - U_4] \sin\theta_2 \right) \sin\theta_1 + \left(k^{(2)} [U_5 - U_3] \cos\theta_2 + [U_6 - U_4] \sin\theta_2 \right) \sin\theta_2 = F_6$$

Dalam bentuk matriks

$$
\begin{bmatrix}
k^{(1)}c^2\theta_1 & k^{(1)}s\theta_1 c\theta_1 & 0 & 0 & -k^{(1)}c^2\theta_1 & -k^{(1)}s\theta_1 c\theta_1 \\
k^{(1)}s\theta_1 c\theta_1 & k^{(1)}s^2\theta_1 & 0 & 0 & -k^{(1)}s\theta_1 c\theta_1 & -k^{(1)}s^2\theta_1 \\
0 & 0 & k^{(2)}c^2\theta_2 & k^{(2)}s\theta_2 c\theta_2 & -k^{(2)}c^2\theta_2 & -k^{(2)}s\theta_2 c\theta_2 \\
0 & 0 & k^{(2)}s\theta_2 c\theta_2 & k^{(2)}s^2\theta_2 & -k^{(2)}s\theta_2 c\theta_2 & -k^{(2)}s^2\theta_2 \\
-k^{(1)}c^2\theta_{12} & -k_1 s\theta_1 c\theta_1 & -k^{(2)}c^2\theta_2 & -k^{(2)}s\theta_2 c\theta_2 & \begin{matrix} k^{(1)}c^2\theta_1 + \\ k^{(2)}c^2\theta_2 \end{matrix} & \begin{matrix} k^{(1)}s\theta_1 c\theta_1 + \\ k^{(2)}s\theta_2 c\theta_2 \end{matrix} \\
-k_1 s\theta_1 c\theta_1 & -k^{(1)}s^2\theta_1 & -k^{(2)}s\theta_2 c\theta_2 & -k^{(2)}s^2\theta_2 & \begin{matrix} k^{(1)}s\theta_1 c\theta_1 + \\ k^{(2)}s\theta_2 c\theta_2 \end{matrix} & \begin{matrix} k^{(1)}s^2\theta_1 + \\ k^{(2)}s^2\theta_2 \end{matrix}
\end{bmatrix}
\begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix}
=
\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix}
$$

$$[K]\{U\} = \{F\}$$

Global stiffness matrix

nodal displacement

Nodal forces

Transformasi Element

modulus young / elastisitas

Pers. bar :

$$\frac{AE}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_1^{(e)}\\ u_2^{(e)}\end{Bmatrix}=\begin{bmatrix}k_e & -k_e\\ -k_e & k_e\end{bmatrix}\begin{Bmatrix}u_1^{(e)}\\ u_2^{(e)}\end{Bmatrix}=\begin{Bmatrix}f_1^{(e)}\\ f_2^{(e)}\end{Bmatrix} \qquad (3.17)$$

L (u4)

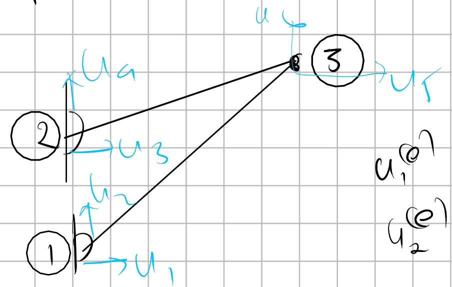
Panjang

dlm bentuk matriks

$$[K^{(e)}]\{u\}=F \Rightarrow [K^{e}]\begin{Bmatrix}u_1\\ u_2\\ u_3\\ u_4\end{Bmatrix}=\begin{Bmatrix}F_1\\ F_2\\ F_3\\ F_4\end{Bmatrix} \qquad (3.18)$$

x

y



$$u_1^{(e)}=U_1^{(e)}\cos\theta+U_2^{(e)}\sin\theta$$

$$u_2^{(e)}=U_3^{(e)}\cos\theta+U_4^{(e)}\sin\theta$$ ..... dlm bentuk matrik

$$\begin{Bmatrix}u_1\\ u_2\end{Bmatrix}=\begin{bmatrix}\cos\theta & \sin\theta & 0 & 0\\ 0 & 0 & \cos\theta & \sin\theta\end{bmatrix}\begin{Bmatrix}U_1\\ U_2\\ U_3\\ U_4\end{Bmatrix}=[R]\begin{Bmatrix}U_1\\ U_2\\ U_3\\ U_4\end{Bmatrix}$$

, dimana

$$[R]=\begin{bmatrix}\cos\theta & \sin\theta & 0 & 0\\ 0 & 0 & \cos\theta & \sin\theta\end{bmatrix} \qquad (3.22)$$

Matrik Transformasi

masuk ke persamaan 3.22 ke 3.17 :

$$\begin{bmatrix}k_e & -k_e\\ -k_e & k_e\end{bmatrix}\underbrace{\begin{bmatrix}\cos\theta & \sin\theta & 0 & 0\\ 0 & 0 & \cos\theta & \sin\theta\end{bmatrix}}_{[R]}\cdot\begin{Bmatrix}U_1\\ U_2\\ U_3\\ U_4\end{Bmatrix}=\begin{Bmatrix}f_1^{(e)}\\ f_2^{(e)}\end{Bmatrix}$$

$$\times [R]^T$$

$$\overbrace{R^T \begin{bmatrix} k_e & -k_e \\ -k_e & k_e \end{bmatrix} \cdot R}^{k^{(e)}} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \overset{R^T}{\begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & \cos\theta \\ 0 & \sin\theta \end{bmatrix}} \begin{Bmatrix} f_1^{(e)} \\ f_2^{(e)} \end{Bmatrix} = \underbrace{\begin{Bmatrix} f_1 \cos\theta \\ f_1 \sin\theta \\ f_2 \cos\theta \\ f_2 \sin\theta \end{Bmatrix}}_{\text{Global Coordinate}} \quad (3.28)$$

Matrik kekakuan global:

$$[k^{(e)}] = [R]^T \begin{bmatrix} k_e & -k_e \\ -k_e & k_e \end{bmatrix} [R] \longrightarrow c = \cos\theta, \quad s = \sin\theta$$

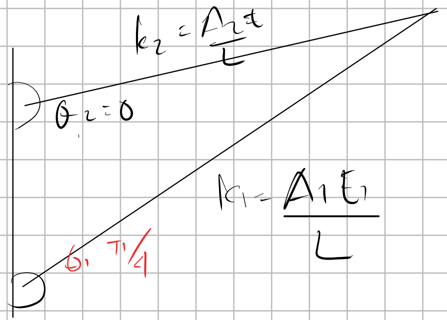
$$(k^{(e)}) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & \cos\theta \\ 0 & \sin\theta \end{bmatrix} \begin{bmatrix} k_e & -k_e \\ -k_e & k_e \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} k_e \cos\theta + 0 & -k_e \cos\theta + 0 \\ k_e \sin\theta + 0 & -k_e \sin\theta + 0 \\ 0 - k_e \cos\theta & k_e \cos\theta \\ 0 - k_e \sin\theta & 0 + k_e \sin\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} k_e \cos^2\theta + 0 & -k_e \cos\theta \sin\theta + 0 & -k_e \cos^2\theta & -k_e \cos\theta \sin\theta \\ k_e \sin\theta \cos\theta & k_e \sin^2\theta & -k_e \sin\theta \cos\theta & -k_e \sin^2\theta \\ -k_e \cos^2\theta & -k_e \cos\theta \sin\theta & k_e \cos^2\theta & k_e \sin\theta \cos\theta \\ -k_e \sin\theta \cos\theta & -k_e \sin^2\theta & k_e \sin\theta \cos\theta & k_e \sin^2\theta \end{bmatrix}$$

$$[K^{(e)}] = k_e \begin{bmatrix} c^2 & -cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & s^2 & cs & s^2 \end{bmatrix}$$

Ex 3.1



$\cos\theta_1 = \cos\frac{\pi}{4} = \frac{1}{2}\sqrt{2}$ $\rightarrow c^2 = \frac{1}{2}$

$\sin\theta_1 = \frac{1}{2}\sqrt{2}$ $\rightarrow s^2 = \frac{1}{2}$

$k^{(1)} = k_1 \Big[$