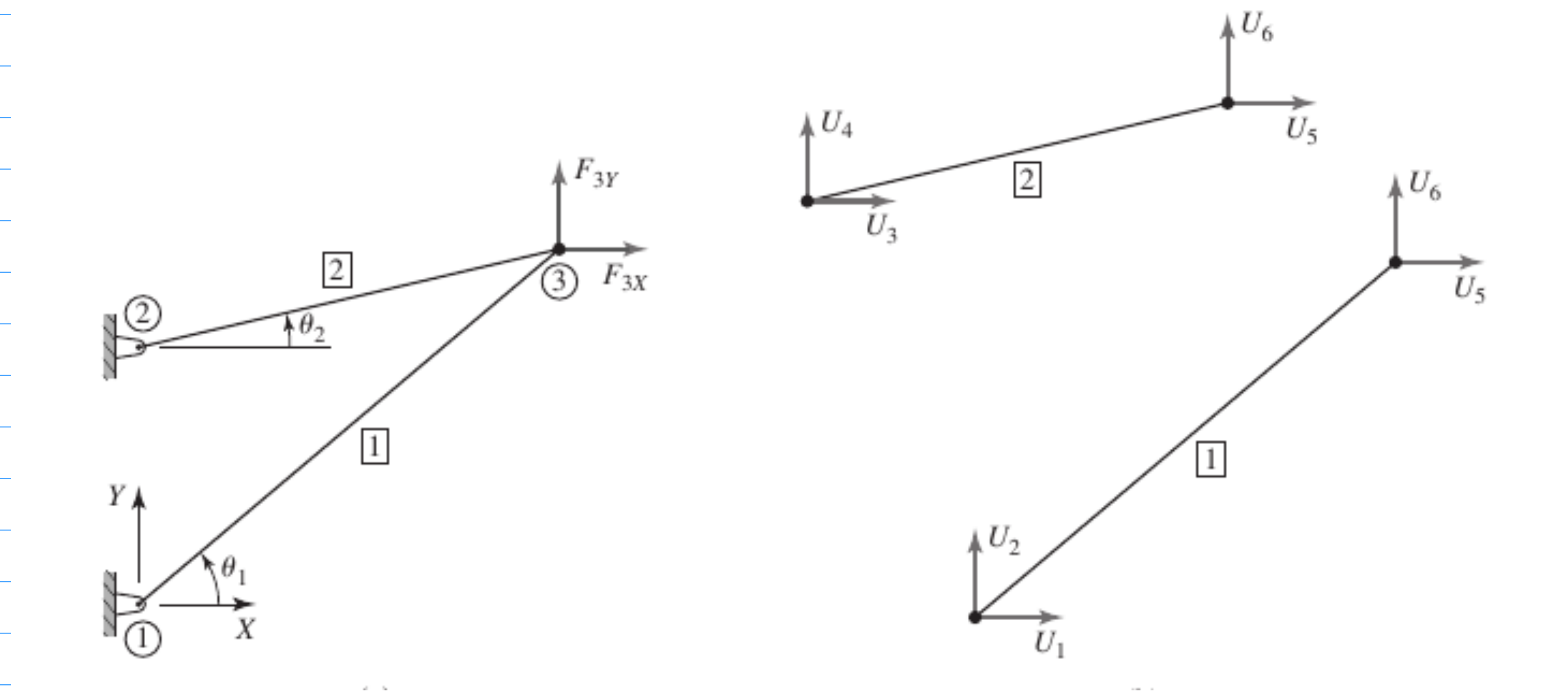
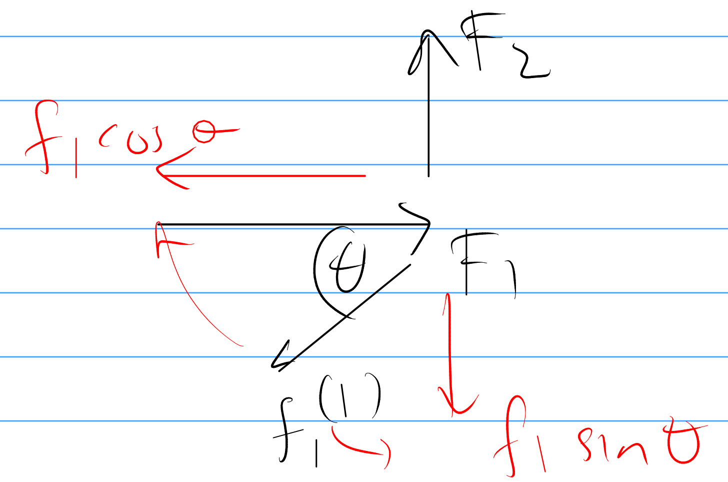
Global displacement (U)



Pers. kesetimbangan

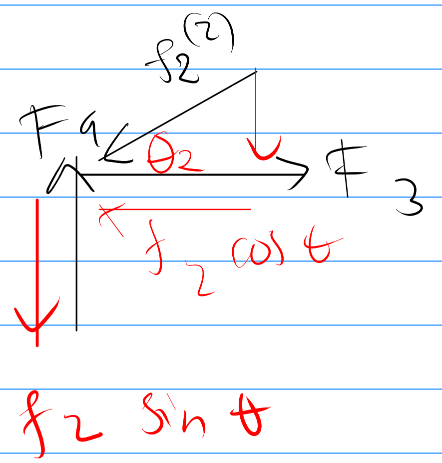
Pd node 1



$F_1 - f_1 \cos\theta_1 = 0$ 3.1a

$F_2 - f_1 \sin\theta_1 = 0$ 3.1b

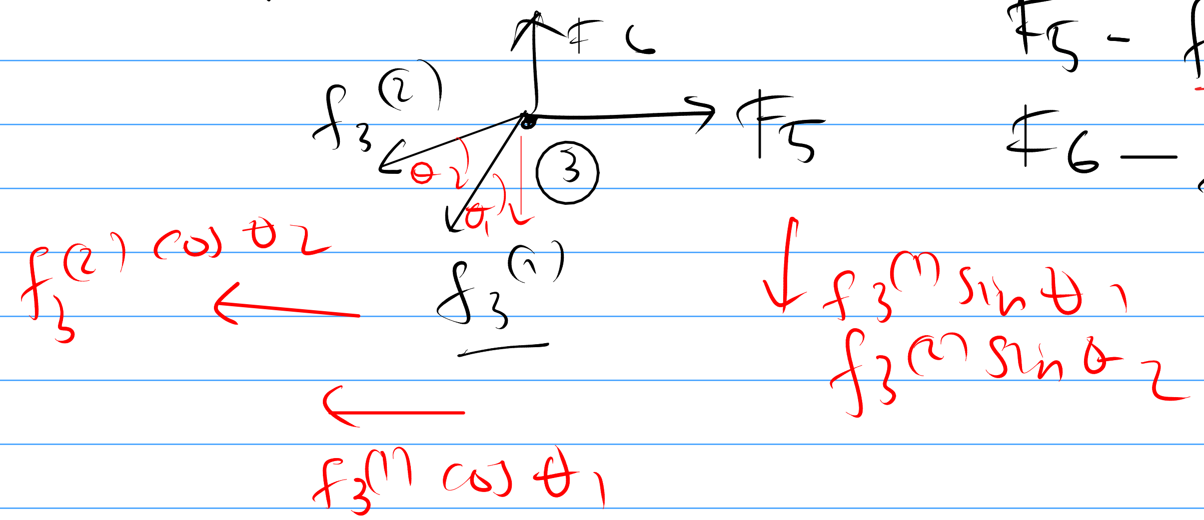
Pd node 2:



$F_3 - f_2^{(2)} \cos\theta_2 = 0$

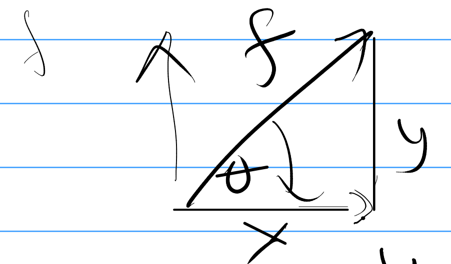
$F_4 - f_2^{(2)} \sin\theta = 0$

Pd node 3:



$F_5 - f_3^{(1)} \cos\theta_1 - f_3^{(2)} \cos\theta_2 = 0$

$F_6 - f_3^{(1)} \sin\theta_1 - f_3^{(2)} \sin\theta_2 = 0$

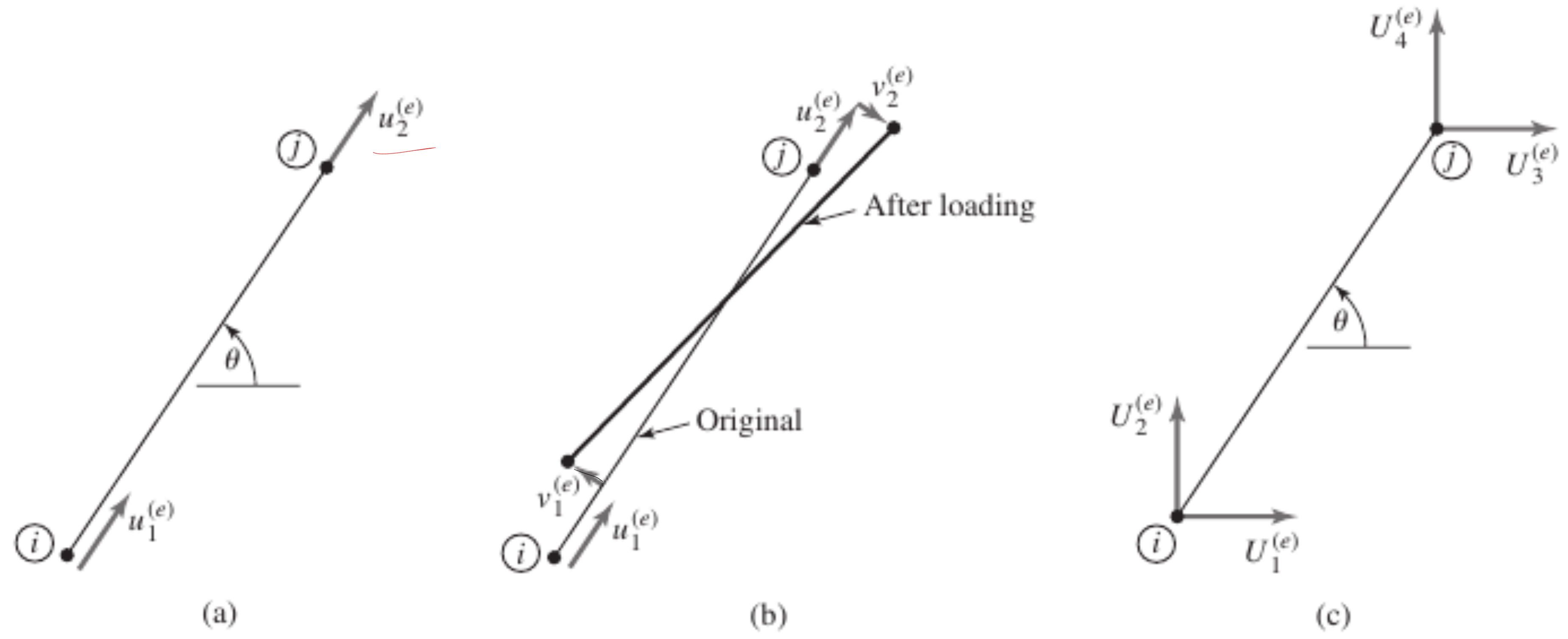


$\sin\theta = \frac{y}{f}$, $\cos\theta = \frac{x}{f}$

$y = f \sin\theta$ $x = f \cos\theta$

$u \to U \Rightarrow f \to F \Rightarrow [K]\{U\} = \{F\}$

Global displacement



$U_1^{(e)}$ = Global displacement of node 1 → X

$U_2^{(e)}$ = —— " —— 1 → Y

$U_3^{(e)}$ = —— " —— 2 → X

$U_4^{(e)}$ = —— " —— 2 → Y

$U_1^{(1)} = U_1$, $U_2^{(1)} = U_2$, $U_3^{(1)} = U_5$, $U_4^{(1)} = U_6$

$U_1^{(2)} = U_3$, $U_2^{(2)} = U_4$

node 3 ⇒ $f_3^{(1)} = -f_1^{(1)} = k^{(1)}[U_5 - U_1]\cos\theta_1 + k^{(1)}[U_6 - U_2]\sin\theta_1$ 3.7

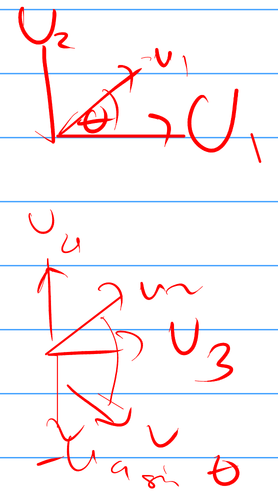
node 3 ⇒ $f_3^{(2)} = -f_2^{(2)} = k^{(2)}[U_5 - U_3]\cos\theta_2 + k^{(2)}[U_6 - U_4]\sin\theta_2$ 3.8

$u_1^{(e)} = U_1\cos\theta + U_2\sin\theta$

$v_1 = U_2\cos\theta - U_1\cos\theta$

$u_2^{(e)} = U_3\cos\theta + U_4\sin\theta$

$v_2 = -v_1 = U_3\cos\theta - U_4\sin\theta$



$\delta = u_2^{(e)} - u_1^{(e)} = (U_3\cos\theta + U_4\sin\theta) - (U_1\cos\theta + U_2\sin\theta)$

$= [U_3 - U_1]\cos\theta + [U_4 - U_2]\sin\theta$ (3.5)

$f = k.x \Rightarrow f^{(e)} = k^{(e)}\delta^{(e)} = k^{(e)}\left([U_3^{(e)} - U_1^{(e)}]\cos\theta + [U_4^{(e)} - U_2^{(e)}]\sin\theta\right)$ (3.6)

Masukkan pers 3.7 & 3.8 ke 3.1 ~ 3.3 $\qquad [k]\{u\} = \{F\}$

$$F_1 - f_1\cos\theta = 0 \rightarrow \underline{f_1\cos\theta_1} = (k^{(1)}[U_5 - U_1]\cos\theta_1 + k^{(1)}[U_6 - U_2]\sin\theta_1)\cos\theta_1 = F_1$$

$$F_2 - f_1\sin\theta_1 = 0 \rightarrow -(k^{(2)}\ U_5 - U_3]\cos\theta_2 + k^{(2)}[U_6 - U_4]\sin\theta_2)\sin\theta_1 = F_2$$

$$F_3 - f_2\cos\theta_2 = 0 \rightarrow -(k^{(2)}[U_5 - U_3]\cos\theta_2 + k^{(2)}[U_6 - U_4]\sin\theta_2)\cos\theta_2 = F_3$$

$$F_4 - f_2\sin\theta_2 = 0 \rightarrow -(k^{(2)}[U_5 - U_3]\cos\theta_2 + k^{(2)}[U_6 - U_4]\sin\theta_2)\sin\theta_2 = F_4$$

$$F_5 - f_3^{(1)}\cos\theta_1 - f_3^{(2)}\cos\theta_2 = 0 \rightarrow (k^{(2)}\ U_5 - U_3\ \cos\theta_2 + k^{(2)}[U_6 - U_4\ \sin\theta_2)\cos\theta_1$$
$$+ (k^{(2)}\ U_5 - U_3\ \cos\theta_2 + k^{(2)}[U_6 - U_4\ \sin\theta_2)\cos\theta_2 = F_5$$

$$F_6 - f_3^{(1)}\sin\theta_1 - f_3^{(2)}\sin\theta_2 = 0 \rightarrow (k^{(1)}[U_5 - U_1]\cos\theta_1 + k^{(1)}[U_6 - U_2]\sin\theta_1)\sin\theta_1 +$$
$$(k^{(1)}[U_5 - U_1]\cos\theta_1 + k^{(1)}[U_6 - U_2]\overset{3.7}{\sin\theta_1})\sin\theta_2 = F_6$$

$$\begin{bmatrix} k^{(1)}c^2\theta_1 & k^{(1)}s\theta_1 c\theta_1 & 0 & 0 & -k^{(1)}c^2\theta_1 & -k^{(1)}s\theta_1 c\theta_1 \\ k^{(1)}s\theta_1 c\theta_1 & k^{(1)}s^2\theta_1 & 0 & 0 & -k^{(1)}s\theta_1 c\theta_1 & -k^{(1)}s^2\theta_1 \\ 0 & 0 & k^{(2)}c^2\theta_2 & k^{(2)}s\theta_2 c\theta_2 & -k^{(2)}c^2\theta_2 & -k^{(2)}s\theta_2 c\theta_2 \\ 0 & 0 & k^{(2)}s\theta_2 c\theta_2 & k^{(2)}s^2\theta_2 & -k^{(2)}s\theta_2 c\theta_2 & -k^{(2)}s^2\theta_2 \\ -k^{(1)}c^2\theta_{12} & -k_1 s\theta_1 c\theta_1 & -k^{(2)}c^2\theta_2 & -k^{(2)}s\theta_2 c\theta_2 & \begin{matrix} k^{(1)}c^2\theta_1 + \\ k^{(2)}c^2\theta_2 \end{matrix} & \begin{matrix} k^{(1)}s\theta_1 c\theta_1 + \\ k^{(2)}s\theta_2 c\theta_2 \end{matrix} \\ -k_1 s\theta_1 c\theta_1 & -k^{(1)}s^2\theta_1 & -k^{(2)}s\theta_2 c\theta_2 & -k^{(2)}s^2\theta_2 & \begin{matrix} k^{(1)}s\theta_1 c\theta_1 + \\ k^{(2)}s\theta_2 c\theta_2 \end{matrix} & \begin{matrix} k^{(1)}s^2\theta_1 + \\ k^{(2)}s^2\theta_2 \end{matrix} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix}$$

Transformasi Elemen

$$u_1^{(e)} = U_1^{(e)} \cos\theta + U_2^{(e)} \sin\theta$$
$$u_2^{(e)} = U_3^{(e)} \cos\theta + U_4^{(e)} \sin\theta$$

$$\begin{Bmatrix} u_1^{(e)} \\ u_2^{(e)} \end{Bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix}}_{R} \begin{Bmatrix} U_1^{(e)} \\ U_2^{(e)} \\ U_3^{(e)} \\ U_4^{(e)} \end{Bmatrix} = R \begin{Bmatrix} U_1^{(e)} \\ U_2^{(e)} \\ U_3^{(e)} \\ U_4^{(e)} \end{Bmatrix}$$

$$[K]\{U\} = F$$

$$\underbrace{\frac{AE}{L}}_{k_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1^{(e)} \\ u_2^{(e)} \end{Bmatrix} = \begin{Bmatrix} f_1^{(e)} \\ f_2^{e} \end{Bmatrix}$$

$$\begin{bmatrix} k_e & -k_e \\ -k_e & k_e \end{bmatrix} \underbrace{\begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix}}_{R} \begin{Bmatrix} U_1^{(e)} \\ U_2^{(e)} \\ U_3^{(e)} \\ U_4^{(e)} \end{Bmatrix} = \begin{Bmatrix} f_1^{(e)} \\ f_2^{(e)} \end{Bmatrix}$$

$$\begin{bmatrix} k_e & -k_e \\ -k_e & k_e \end{bmatrix} R \begin{Bmatrix} U_1^{e} \\ U_2^{e} \\ U_3^{e} \\ U_4^{e} \end{Bmatrix} = \begin{Bmatrix} f_1^{e} \\ f_2^{e} \end{Bmatrix}$$

$$R^T$$

$$R^T \begin{bmatrix} k_e & -k_e \\ k_e & -k_e \end{bmatrix} R \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = R^T \begin{Bmatrix} f_1^e \\ f_2^e \end{Bmatrix}$$

$$F_1 - f_1 \cos\theta = 0$$

$$F_1 = f_1 \cos\theta$$

$$[K]\{U\} = \{F\}$$

$$u = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & \cos\theta \\ 0 & \sin\theta \end{bmatrix} \begin{Bmatrix} f_1^e \\ f_2^e \end{Bmatrix} = \begin{Bmatrix} f_1 \cos\theta \\ f_1 \sin\theta \\ f_2 \cos\theta \\ f_2 \sin\theta \end{Bmatrix} = \begin{Bmatrix} F_1^e \\ F_2^e \\ F_3^e \\ F_4^e \end{Bmatrix}$$

$$[K^e] = R^T \begin{bmatrix} k_e & -k_e \\ -k_e & k_e \end{bmatrix} R$$

$$\cos = c$$

$$\sin = s$$

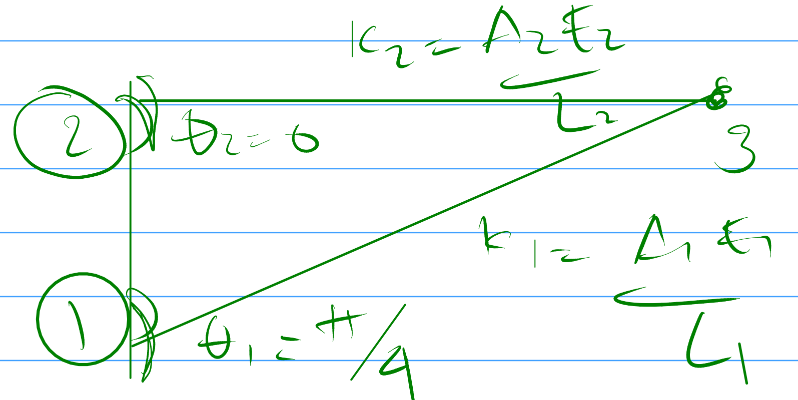
$$= \begin{bmatrix} \boxed{\cos\theta \quad 0} \\ \sin\theta & 0 \\ 0 & \cos\theta \\ 0 & \sin\theta \end{bmatrix} \begin{bmatrix} k_e & -k_e \\ -k_e & k_e \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} k_e\cos\theta + 0 & -k_e\cos\theta \\ k_e \sin\theta & -k_e \sin\theta \\ -k_e\cos\theta & k_e\cos\theta \\ -k_e\sin\theta & k_e\sin\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} k_e\cos^2\theta & k_e\cos\theta\sin\theta & -k_e\cos^2\theta & -k_e\cos\theta\sin\theta \\ k_e\cos\theta\sin\theta & k_e\sin^2\theta & -k_e\sin\theta\cos\theta & -k_e\sin^2\theta \\ -k_e\cos^2\theta & -k_e\cos\theta\sin\theta & k_e\cos^2\theta & k_e\cos\theta\sin\theta \\ -k_e\sin\theta\cos\theta & -k_e\sin^2\theta & k_e\sin\theta\cos\theta & k_e\sin^2\theta \end{bmatrix} = k_e \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

(4 × 4)

Contd Soln [Ex 3.1]



$\cos(\frac{\pi}{4}) = \frac{1}{2}\sqrt{2} \to c^2 = \frac{1}{2}$

$\sin(\frac{\pi}{4}) = \frac{1}{2}\sqrt{2} \to s^2 = \frac{1}{2}$

$c \cdot s = \frac{1}{2}$

$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ K^{(1)} = \frac{k_e^{(1)}}{2} & \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 5 \\ 6 \end{array} \end{array}$$

$\cos\theta = 1 \quad \sin\theta_2 = 0$

$$k_2 = k_e \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{c} 3 \\ 4 \\ 5 \\ 6 \end{array}$$

Direct assembly $e = 1$

$$K^{(1)} = \begin{bmatrix} k_{11}^e & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix}$$

$k^{(2)} =$

$e = 2$

$$U^{(1)} = \begin{Bmatrix} U_1^e \\ U_2^e \\ U_3^e \\ U_4^e \end{Bmatrix} = \begin{Bmatrix} U_1 \\ U_2 \\ U_5 \\ U_6 \end{Bmatrix}$$

$$U^{(2)} = \begin{Bmatrix} U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix}$$

$k_1^{(e)} \to k_{44}^{(e)} \Rightarrow$ element 1

$\Rightarrow$ element 2

$k_{11} = k_1/2$

$\downarrow$

k_{66}

k_{11}

$$[K] = \begin{bmatrix} k_1/2 & k_1/2 & 0 & 0 & -k_1/2 & -k_1/2 \\ k_1/2 & k_1/2 & 0 & 0 & -k_1/2 & -k_1/2 \\ 0 & 0 & k_2 & 0 & -k_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -k_1/2 & -k_1/2 & -k_2 & 0 & k_1/2+k_2 & k_1/2 \\ -k_1/2 & -k_1/2 & 0 & 0 & k_1/2 & k_1/2 \end{bmatrix}$$

k_{66}

Tugas :

2311840000033 Problem 3.12
2311840000041 Problem 3.11
2311840000029 Problem 3.10
2311840000083 Problem 3.9
2311740000013 Problem 3.8
2311740000078 Problem 3.7
2311840000124 Problem 3.6
2311840000086 Problem 3.5
2311640000109 Problem 3.4
2311840000117 Problem 3.3
2311840000097 Problem 3.2
2311840000099 Problem 3.1