# Bilangan Kompleks

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  $z = a + bi$

# Bilangan Kompleks

$$\boxed{Z = a + bi}$$

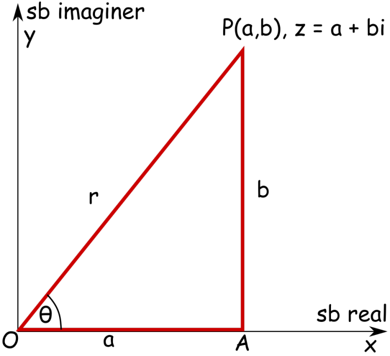
- $i = \sqrt{-1}$ (satuan imaginer)
- $i^2 = -1$
- a bagian real dari z, ditulis Re z $=$a
- b bagian imaginer dari z, ditulis, Im z $=$b

# Bilangan Kompleks

Diberikan dua bilangan kompleks: $Z_1 = a + bi$
$Z_2 = c + di$, maka:

1. $z_1 + z_2 = (a + bi) + (c + di) = (a + c) + (b + d)i$
2. $z_1 - z_2 = (a + bi) - (c + di) = (a - c) + (b - d)i$
3. $z_1 z_2 = (a + bi)(c + di) = (ac - bd) + (ad + bc)i$
4. $\dfrac{z_1}{z_2} = \dfrac{a + bi}{c + di} = \dfrac{ac + bd}{c^2 + d^2} + \dfrac{(bc - ad)i}{c^2 + d^2}$

# Bentuk kutub dari bilangan kompleks




Bidang XOY = bidang kompleks

$z = a + bi \rightarrow r = \sqrt{x^2 + y^2}$

r disebut modulus dari nilai z

atau

Nilai mutlak dari z, ditulis $|z|$

$\sin\theta = \dfrac{b}{r} \longrightarrow$ $\theta$ disebut

$\cos\theta = \dfrac{a}{r}$ argumen dari z

$$z = a + bi \rightarrow \boxed{z = r(\cos\theta + i\sin\theta)}$$

Soal:

Nyatakan $z = 1 + \sqrt{3}i$ ke dalam bentuk kutub

# Conjugate

Conjugate dari $z = a + bi$ ialah $\underline{\bar{z} = a - bi}$

- $z = a + bi$
- $\bar{z} = \overline{a + bi}$
- $z_1 = a + bi$
- $z_2 = c + di$

1. $\bar{\bar{z}} = z$
2. $z.\bar{z} = |z|^2 = |\bar{z}|^2$
3. $\overline{z_1 \pm z_2} = \bar{z_1} \pm \bar{z_2}$
4. $\overline{z_1 z_2} = \bar{z_1}.\bar{z_2}$
5. $\overline{\frac{z_1}{z_2}} = \frac{\bar{z_1}}{\bar{z_2}}$

Jika:

$$\boxed{z_1 = r_1(\cos\theta_1 + i\sin\theta_1) \text{ dan } z_2 = r_2(\cos\theta_2 + i\sin\theta_2\ )}$$

maka:

1. $z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$
2. $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$

# Teorema De Moivre

Abraham De Moivre (1667-1754) menyatakan untuk setiap bilangan rasional n berlaku:

$$\boxed{[r(\cos\theta + i\sin\theta)]^n = r^n(\cos(n\theta) + i\sin(n\theta))}$$

Jika r=1 maka,

$$\boxed{(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)}$$

## Contoh

Dapatkan nilai dari $(\sqrt{3} + i)^6$

# Penarikan akar

- a + bi = r $(\cos\theta + i\sin\theta)$
  Karena

$$\sin\theta = \sin(\theta + k.360^o) \rightarrow k = bil.bulat \tag{1}$$
$$\cos\theta = \cos(\theta + k.360^o) \tag{2}$$

- maka: a + bi = $r[\cos(\theta + k.360^o) + i\sin(\theta + k.360^o)]$
- Jika $z^n = a + bi \rightarrow z_{1,2,3,..,n} = \sqrt[n]{a+bi} = ....?$
- Penyelesaiannya:

$$\boxed{z_{1,2,3,\ldots,n} = r^{\frac{1}{n}}\left[\cos\frac{\theta + k.360^o}{n} + i\sin\frac{\theta + k.360^o}{n}\right]}$$