

Bilangan Kompleks

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Bilangan Kompleks

$$Z = a + bi$$

- ▶ $i = \sqrt{-1}$ (satuan imajiner)
- ▶ $i^2 = -1$
- ▶ a bagian real dari z , ditulis $\operatorname{Re} z = a$
- ▶ b bagian imajiner dari z , ditulis, $\operatorname{Im} z = b$

Bilangan Kompleks

Diberikan dua bilangan kompleks: $Z_1 = a + bi$
 $Z_2 = c + di$, maka:

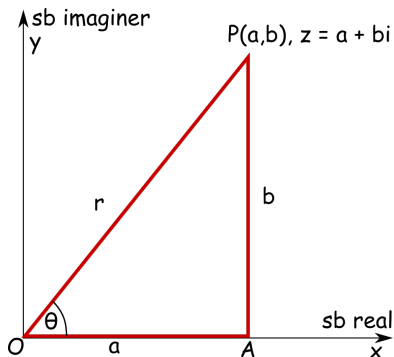
1. $z_1 + z_2 = (a + bi) + (c + di) = (a + c) + (b + d)i$

2. $z_1 - z_2 = (a + bi) - (c + di) = (a - c) + (b - d)i$

3. $z_1 z_2 = (a + bi)(c + di) = (ac - bd) + (ad + bc)i$

4. $\frac{z_1}{z_2} = \frac{a + bi}{c + di} = \frac{ac + bd}{c^2 + d^2} + \frac{(bc - ad)i}{c^2 + d^2}$

Bentuk kutub dari bilangan kompleks



Bidang XOY = bidang kompleks

$$z = a + bi \rightarrow r = \sqrt{x^2 + y^2}$$

r disebut modulus dari nilai z
atau

Nilai mutlak dari z , ditulis $|z|$

$$\sin \theta = \frac{b}{r} \rightarrow \theta \text{ disebut}$$

$$\cos \theta = \frac{a}{r} \quad \text{argumen dari } z$$

$$z = a + bi \rightarrow \boxed{z = r(\cos \theta + i \sin \theta)}$$

Soal:

Nyatakan $z = 1 + \sqrt{3}i$ ke dalam bentuk kutub

Conjugate

Conjugate dari $z = a + bi$ ialah $\bar{z} = a - bi$

► $z = a + bi$

► $\bar{z} = \overline{a + bi}$

► $z_1 = a + bi$

► $z_2 = c + di$

1. $\bar{\bar{z}} = z$

2. $z \cdot \bar{z} = |z|^2 = |\bar{z}|^2$

3. $\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$

4. $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$

5. $\frac{\bar{z}_1}{z_2} = \frac{\bar{\bar{z}_1}}{\bar{z}_2}$

Jika:

$$z_1 = r_1(\cos \theta_1 + i \sin \theta_1) \text{ dan } z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$$

maka:

1. $z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$

2. $\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)]$

Teorema De Moivre

Abraham De Moivre (1667-1754) menyatakan untuk setiap bilangan rasional n berlaku:

$$[r(\cos \theta + i \sin \theta)]^n = r^n(\cos(n\theta) + i \sin(n\theta))$$

Jika $r=1$ maka,

$$(\cos \theta + i \sin \theta)^n = \cos(n\theta) + i \sin(n\theta)$$

Contoh

Dapatkan nilai dari $(\sqrt{3} + i)^6$

Penarikan akar

► $a + bi = r (\cos \theta + i \sin \theta)$

Karena

$$\sin \theta = \sin(\theta + k.360^\circ) \rightarrow k = \text{bil.bulat} \quad (1)$$

$$\cos \theta = \cos(\theta + k.360^\circ) \quad (2)$$

► maka: $a + bi = r[\cos(\theta + k.360^\circ) + i \sin(\theta + k.360^\circ)]$

► Jika $z^n = a + bi \rightarrow z_{1,2,3,...,n} = \sqrt[n]{a + bi} =?$

► Penyelesaiannya:

$$z_{1,2,3,...,n} = r^{\frac{1}{n}} \left[\cos \frac{\theta + k.360^\circ}{n} + i \sin \frac{\theta + k.360^\circ}{n} \right]$$