

Bilangan Kompleks

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Bilangan

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 - ▶ Bil. Asli, 1, 2, 3, ...
 - ▶ Bil. Bulat, ..., $-2, -1, 0, 1, 2, \dots$
 - ▶ Bil. Rasional, $\frac{1}{2}, \frac{1}{3}, \frac{2}{5}, \dots$
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- ▶ Bilangan Kompleks
$$z = a + bi$$

Bilangan Kompleks

$$Z = a + bi$$

- ▶ $i = \sqrt{-1}$ (satuan imaginer)
- ▶ $i^2 = -1$
- ▶ a bagian real dari z, ditulis $\operatorname{Re} z = a$
- ▶ b bagian imaginer dari z, ditulis, $\operatorname{Im} z = b$

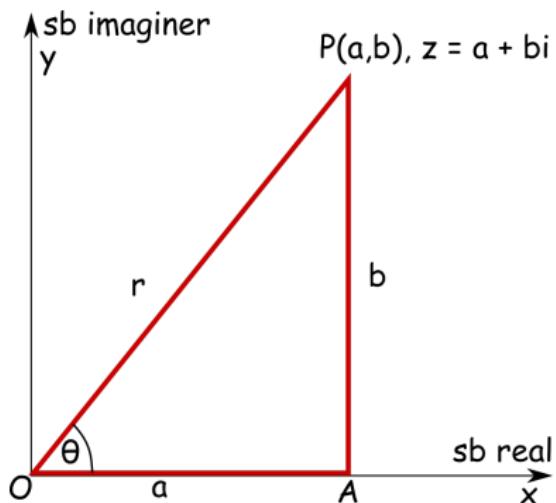
Bilangan Kompleks

Diberikan dua bilangan kompleks: $Z_1 = a + bi$

$Z_2 = c + di$, maka:

1. $z_1 + z_2 = (a + bi) + (c + di) = (a + c) + (b + d)i$
2. $z_1 - z_2 = (a + bi) - (c + di) = (a - c) + (b - d)i$
3. $z_1 z_2 = (a + bi)(c + di) = (ac - bd) + (ad + bc)i$
4. $\frac{z_1}{z_2} = \frac{a + bi}{c + di} = \frac{ac + bd}{c^2 + d^2} + \frac{(bc - ad)i}{c^2 + d^2}$

Bentuk kutub dari bilangan kompleks



$$z = a + bi \rightarrow z = r(\cos \theta + i \sin \theta)$$

Bidang XOY = bidang kompleks
 $z = a + bi \rightarrow r = \sqrt{x^2 + y^2}$

r disebut modulus dari nilai z
atau

Nilai mutlak dari z, ditulis $|z|$

$\sin \theta = \frac{b}{r} \longrightarrow \theta$ disebut

$\cos \theta = \frac{a}{r}$ argumen dari z

Soal:

Nyatakan $z = 1 + \sqrt{3}i$ ke dalam bentuk kutub

Conjugate

Conjugate dari $z = a + bi$ ialah $\bar{z} = a - bi$

- $z = a + bi$
- $\bar{z} = \overline{a + bi}$
- $z_1 = a + bi$
- $z_2 = c + di$

1. $\bar{\bar{z}} = z$
2. $z \cdot \bar{z} = |z|^2 = |\bar{z}|^2$
3. $\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$
4. $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$
5. $\frac{\bar{z}_1}{z_2} = \frac{\bar{z}_1}{\bar{z}_2}$

Jika:

$$z_1 = r_1(\cos \theta_1 + i \sin \theta_1) \text{ dan } z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$$

maka:

$$1. z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$$

$$2. \frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)]$$

Teorema De Moivre

Abraham De Moivre (1667-1754) menyatakan untuk setiap bilangan rasional n berlaku:

$$[r(\cos \theta + i \sin \theta)]^n = r^n(\cos(n\theta) + i \sin(n\theta))$$

Jika $r=1$ maka,

$$(\cos \theta + i \sin \theta)^n = \cos(n\theta) + i \sin(n\theta)$$

Contoh

Dapatkan nilai dari $(\sqrt{3} + i)^6$

Penarikan akar

► $a + bi = r (\cos \theta + i \sin \theta)$

Karena

$$\sin \theta = \sin(\theta + k \cdot 360^\circ) \rightarrow k = \text{bil. bulat} \quad (1)$$

$$\cos \theta = \cos(\theta + k \cdot 360^\circ) \quad (2)$$

- maka: $a + bi = r[\cos(\theta + k \cdot 360^\circ) + i \sin(\theta + k \cdot 360^\circ)]$
- Jika $z^n = a + bi \rightarrow z_{1,2,3,\dots,n} = \sqrt[n]{a + bi} = \dots?$
- Penyelesaiannya:

$$z_{1,2,3,\dots,n} = r^{\frac{1}{n}} \left[\cos \frac{\theta + k \cdot 360^\circ}{n} + i \sin \frac{\theta + k \cdot 360^\circ}{n} \right]$$