

# Aljabar Vektor

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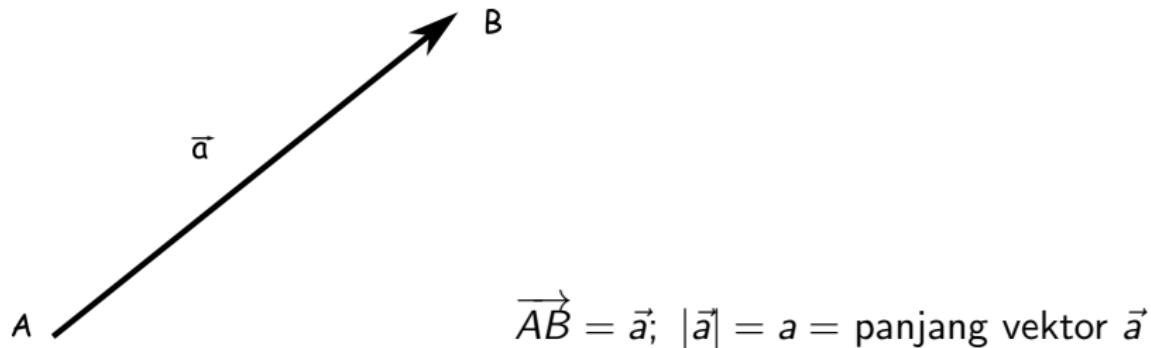
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# Besaran Vektor

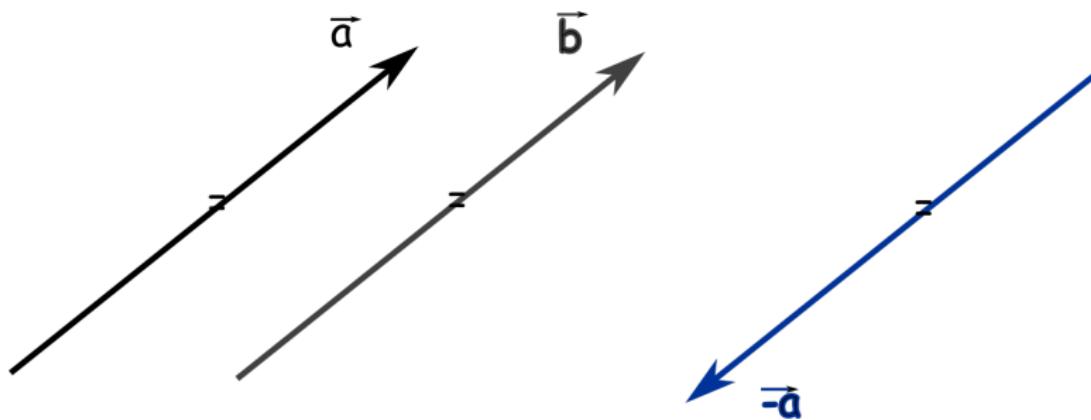
## Besaran

- ▶ Vektor: Kuantiti yang memiliki besar dan arah  
Contoh: kecepatan, percepatan, gaya.
- ▶ Skalar: Kuantiti yang hanya memiliki besar saja.  
Contoh: waktu, temperatur, massa, panjang.



## Dua vektor sama

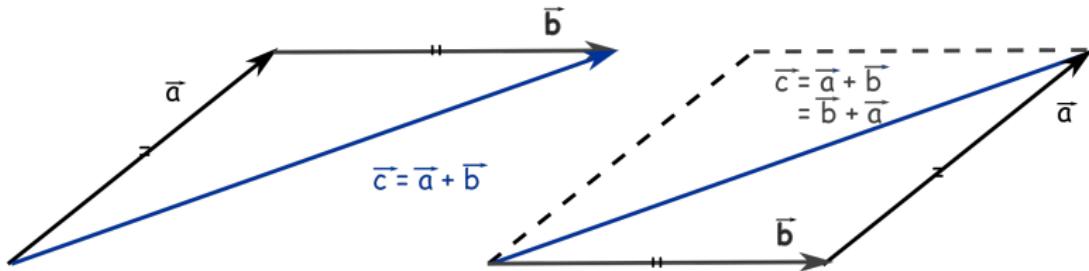
$\vec{a} = \vec{b} \Rightarrow$  searah, sama panjang



$-\vec{a}$ , sama panjang tetapi berlawanan arah dengan  $\vec{a}$

## Jumlah dua vektor: $\vec{a} + \vec{b}$

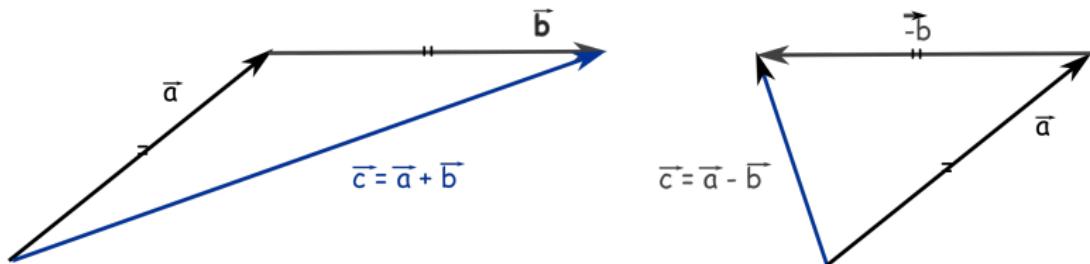
→  $\vec{b}$  dimulai dari ujung vektor  $\vec{a}$ , vektor  $\vec{b} \neq \vec{a}$ , lalu hubungkan pangkal  $\vec{a}$  dengan ujung  $\vec{b}$  tsb.



$$\boxed{\vec{a} + \vec{b} = \vec{b} + \vec{a}}$$

## Selisih dua vektor: $\vec{a} - \vec{b}$

$$\vec{a} - \vec{b} = \vec{a} + (-\vec{b}) = \vec{a} \text{ ditambah } -\vec{b}$$



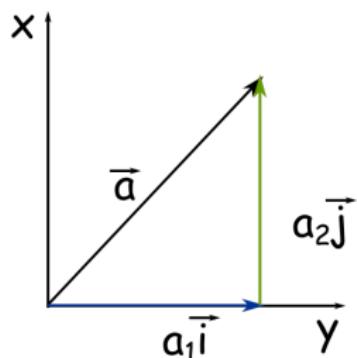
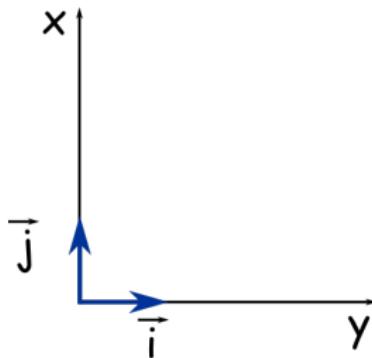
$$\text{Jika } \vec{a} = \vec{b} \implies \vec{a} - \vec{b} = \vec{0}$$

Vektor nol, panjang nol, arah tak didefinisikan

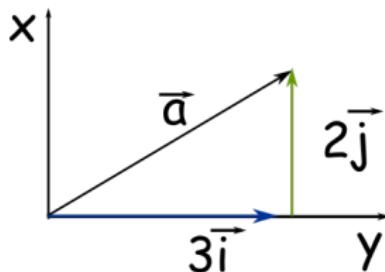
## Vektor Satuan

Vektor satuan  $\vec{i}$ : Vektor dari titik  $(0,0)$  sampai titik  $(1,0)$ .

Vektor satuan  $\vec{j}$ : Vektor dari titik  $(0,0)$  sampai titik  $(0,1)$ .

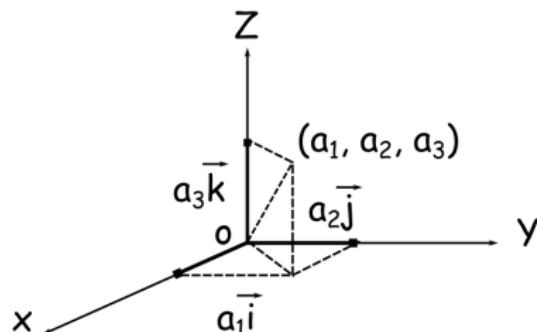
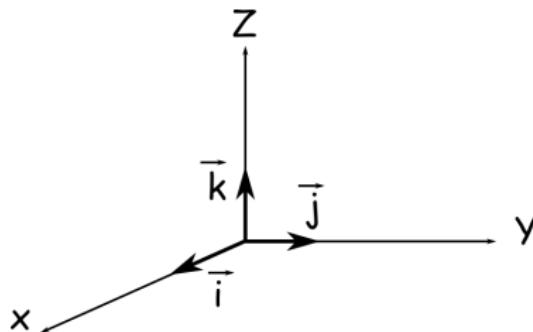


$$\vec{a} = a_1 \vec{i} + a_2 \vec{j}, \text{ Contoh: } \vec{a} = 3\vec{i} + 2\vec{j}$$



## Unit vektor siku-siku

$$|\vec{i}| = |\vec{j}| = |\vec{k}| = 1$$



$$\vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$$

Vektor posisi  $\vec{r}$  dari O ke  $P(x, y, z)$  adalah:

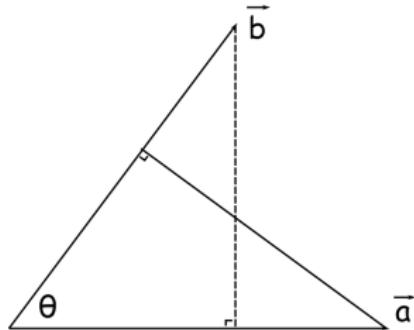
$$\vec{r} = x \vec{i} + y \vec{j} + z \vec{k}$$

dengan panjang  $r = \sqrt{x^2 + y^2 + z^2}$ ;  $a = |\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

## Perkalian titik (Dot product)

Definisi:  $\vec{a} \cdot \vec{b} = ab \cos \theta$ ;  $(0 \leq \theta \leq \pi)$

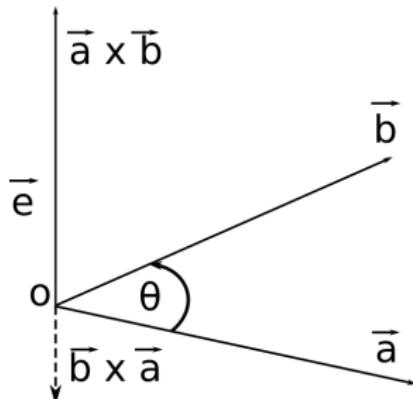
$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} \implies \vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1$$
$$\vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{k} = \vec{k} \cdot \vec{i} = 0$$



$$\vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k} \implies \vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$
$$\vec{b} = b_1 \vec{i} + b_2 \vec{j} + b_3 \vec{k}$$
 skalar

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{ab} = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \cdot \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

## Perkalian silang (Cross product)



Definisi:

$$\vec{a} \times \vec{b} = (ab \sin \theta) \vec{e}; \quad 0 \leq \theta \leq \pi$$

$\theta = \angle(\vec{a}, \vec{b})$  diukur dari  $\vec{a}$  ke  $\vec{b}$

$\vec{e}$  = vektor satuan  $\perp$  bidangnya  $\vec{a}$  dan  $\vec{b}$ .

$$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$$

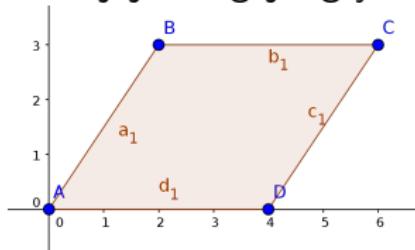
$$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$$

$$\vec{i} \times \vec{j} = \vec{k}; \quad \vec{j} \times \vec{k} = \vec{i}; \quad \vec{k} \times \vec{i} = \vec{j};$$

$$\vec{j} \times \vec{i} = -\vec{k}; \quad \vec{k} \times \vec{j} = -\vec{i}; \quad \vec{i} \times \vec{k} = -\vec{j};$$

## Beberapa rumus[1]

- ▶ Luas jajaran gejjang yang dibentuk  $\vec{a}$  dan  $\vec{b}$



- ▶ Penulisan disingkat

## Beberapa rumus[2]

- ▶ Tiga vektor  $\vec{a}$   $\vec{b}$   $\vec{c}$  membentuk paralelepipedum (balok miring)
- ▶ Rumus

$$\boxed{\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c}) - (\vec{a} \cdot \vec{b}) \vec{c}}$$

## Contoh Soal

- ▶ Dapatkan sudut antara 2 vektor:

$$\vec{a} = 2\vec{i} + 2\vec{j} - \vec{k} \quad \text{dan} \quad \vec{b} = 6\vec{i} - 3\vec{j} - 2\vec{k}$$

- ▶ Jawab:

$$\vec{a} = \sqrt{(2)^2 + (2)^2 + (-1)^2} = 1;$$

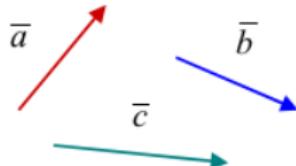
$$\vec{b} = \sqrt{(6)^2 + (-3)^2 + (2)^2} = 1;$$

$$\vec{a} \cdot \vec{b} = (2)(6) + (2)(-3) + (-1)(2) = 4$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{ab} = \frac{4}{(3)(7)} = 0.1905 \Rightarrow \theta = 79^\circ$$

## Soal

1. Diketahui vektor  $\vec{a} = 3i + 4j$  dan vektor  $\vec{b} = 2i + j$ . Hitunglah harga-harga :
  - ▶  $\vec{a} + \vec{b}$
  - ▶  $\vec{b} + \vec{a}$
  - ▶  $\vec{a} - \vec{b}$
  - ▶  $\vec{b} - \vec{a}$
  - ▶  $|\vec{a}|$  dan  $|\vec{b}|$
  - ▶ sudut  $\vec{a}$
  - ▶ sudut  $\vec{b}$
  - ▶  $\vec{a} \cdot \vec{b}$
  - ▶  $\vec{a} \times \vec{b}$
2. Diketahui vektor-vektor  $\vec{a}$ ,  $\vec{b}$  dan  $\vec{c}$  seperti di bawah ini.  
Lukislah secara grafis operasi vektor :  $\vec{a} - \vec{b} + 2\vec{c}$  dan  $3\vec{c} - 0,5(2\vec{a} - \vec{b})$ .



# Cartesian System

- ▶ a
- ▶ b