Sistem Linier

Week 5 - 6

B.T. Atmaja & D. Prananto
1) Teknik Fisika, ITS Surabaya
2) Prodi. Teknik Elektro, Universitas Panca Marga Probolinggo

Sistem Linear Vs Non-Linear

Contoh Sistem Linear:

$$y(n) = \frac{-x(n)}{2}$$

Contoh Sistem Non-Linear:

$$y(n) = -x(n)^2$$

Tentukan Sistem Berikut Linear/Tdk

- $y(n)=T[x(n)]=3x^2(n)$
- $y(n)=2x(n-2)+5$
- $y(n)=x(n+1)-x(n-1)$

Sistem Waktu-invarian Linier

- **Linier** → memiliki sifat **superposisi**:

  1. **Aditivitas** $$x_1(t) + x_2(t) \to y_1(t) + y_2(t)$$

  2. **Homogenitas** $$ax_1(t) \to ay_1(t)$$

  Gabungan kedua sifat menghasilkan **kombinasi linier**:

$$ax_1(t) + bx_2(t) \to ay_1(t) + by_2(t) \tag{1}$$

$$ax_1[n] + bx_2[n] \to ay_1[n] + by_2[n] \tag{2}$$

- **Waktu-invarian** (tak-ubah waktu):

$$x(t) \to y(t) \implies x(t - t_0) \to y(t - t_0)$$

Sistem Waktu-invarian Linier

Jika masukan,

$$x[n] = \sum_k a_k x_k[n]$$
$$= a_1 x_1[n] + a_2 x_2[n] + a_3 x_3[n] + ...$$

keluaran:

$$y[n] = \sum_k a_k y_k[n]$$
$$= a_1 y_1[n] + a_2 y_2[n] + a_3 y_3[n] + ...$$

Sehingga, jika dituliskan:

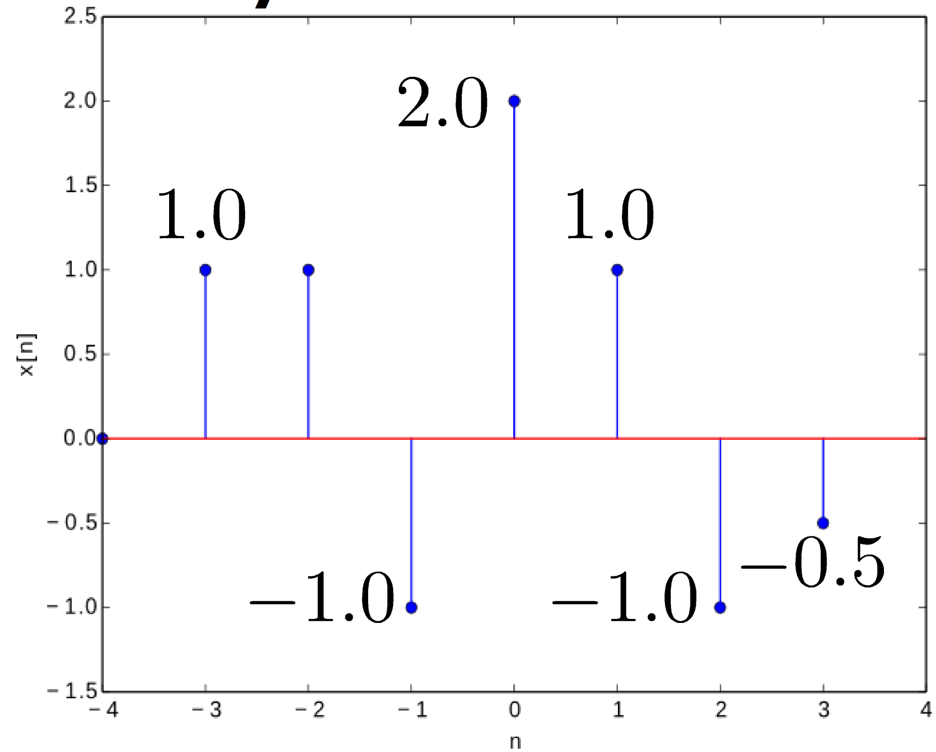
$$x[n] = \sum_k a_k x_k[n] \rightarrow y[n] = \sum_k a_k y_k[n]$$

Tentukan Sistem Berikut LTI/Tidak

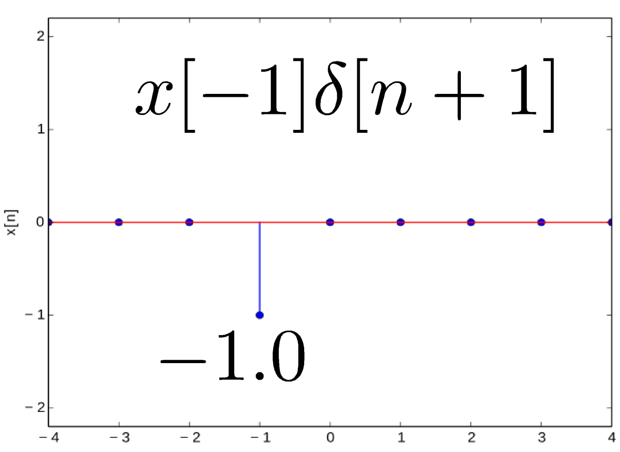
- $y(n) = L[x(n)] = 10\sin(0.1\pi n)$
- $y(n) = L[x(n)] = x(n+1) - x(1-n)$
- $y(n) = L[x(n)] = \frac{1}{4}x(n) + \frac{1}{2}x(n-1) + \frac{1}{4}x(n-2)$

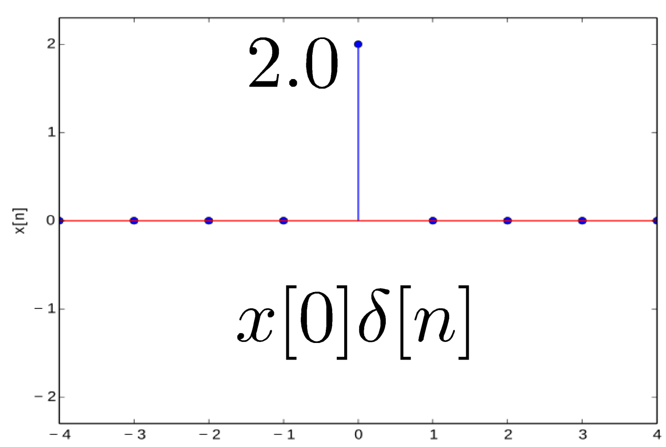
Sistem Waktu-invarian Linier

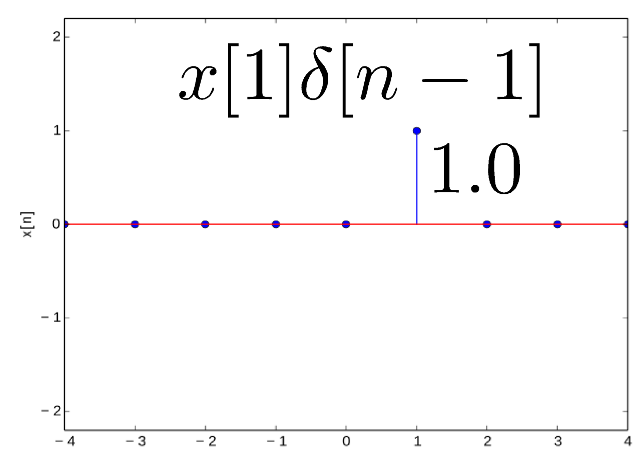
(1) Representasi sinyal waktu-diskrit sebagai impuls



Sinyal di samping dapat dipisahkan menjadi sinyal-sinyal impuls seperti 3 sinyal impuls di bawah

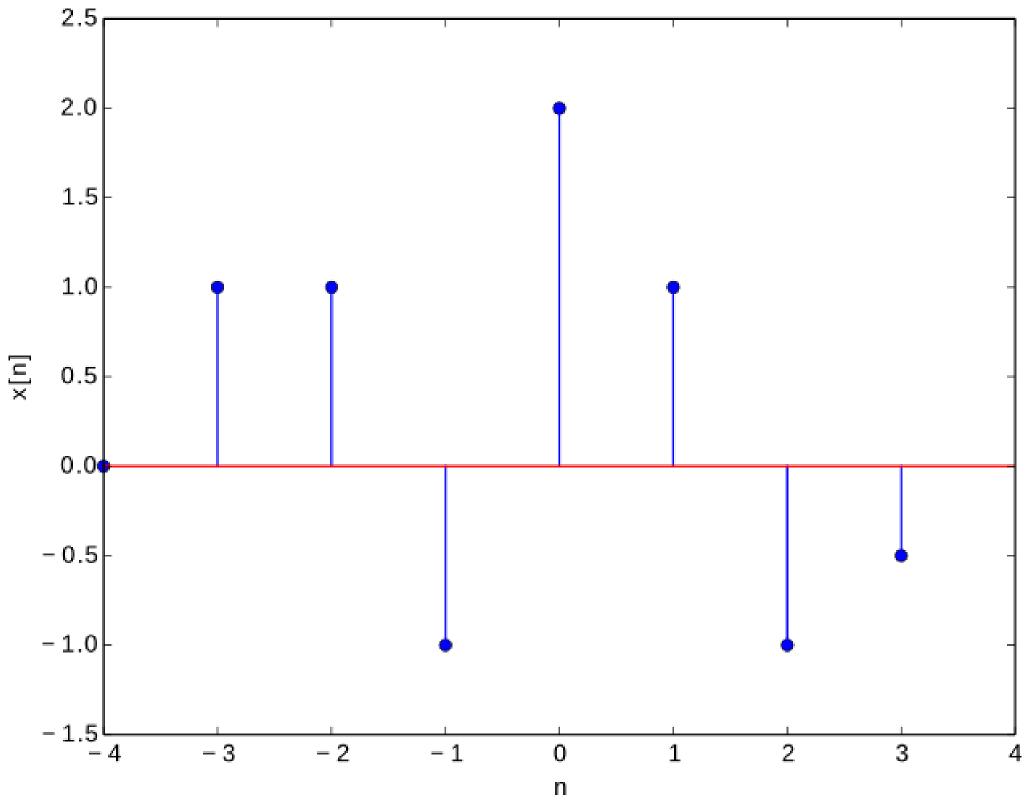






Sistem Waktu-invarian Linier

(1) Representasi sinyal waktu-diskrit sebagai impuls



Sifting property (sifat memisahkan)

$$x[n] = \sum_{k=-\infty}^{+\infty} \underbrace{x[k]}_{\text{Pembobotan}}\underbrace{\delta[n-k]}_{\text{Sinyal dasar}}$$

Sistem Waktu-invarian Linier

(1.1) Jumlahan Konvolusi Waktu-diskrit

$$x[n] \longrightarrow \boxed{\text{Sistem waktu-diskrit}} \longrightarrow y[n]$$

Jika ada sebuah sistem **linier** dan kita definisikan $h_k[n]$ sebagai tanggapan (*output*) dari $\delta[n-k]$

$$\delta[n-k] \rightarrow h_k[n],$$

hubungan masukan-keluaran;

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k] \rightarrow y[n] = \sum_{k=-\infty}^{+\infty} x[k]h_k[n]$$

Sistem Waktu-invarian Linier

(1.1) Jumlahan Konvolusi Waktu-diskrit

Jika sistem juga bersifat **waktu-invarian** dan $h_0[n]$ adalah tanggapan dari $\delta[n]$

$$\delta[n] \rightarrow h_0[n],$$

pergeseran waktu akan menghasilkan:

$$\delta[n-k] \rightarrow h[n-k]$$

Sehingga,

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k] \rightarrow y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

Sistem Waktu-invarian Linier

(1.1) Jumlahan Konvolusi Waktu-diskrit

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k] \rightarrow \boxed{y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]}$$

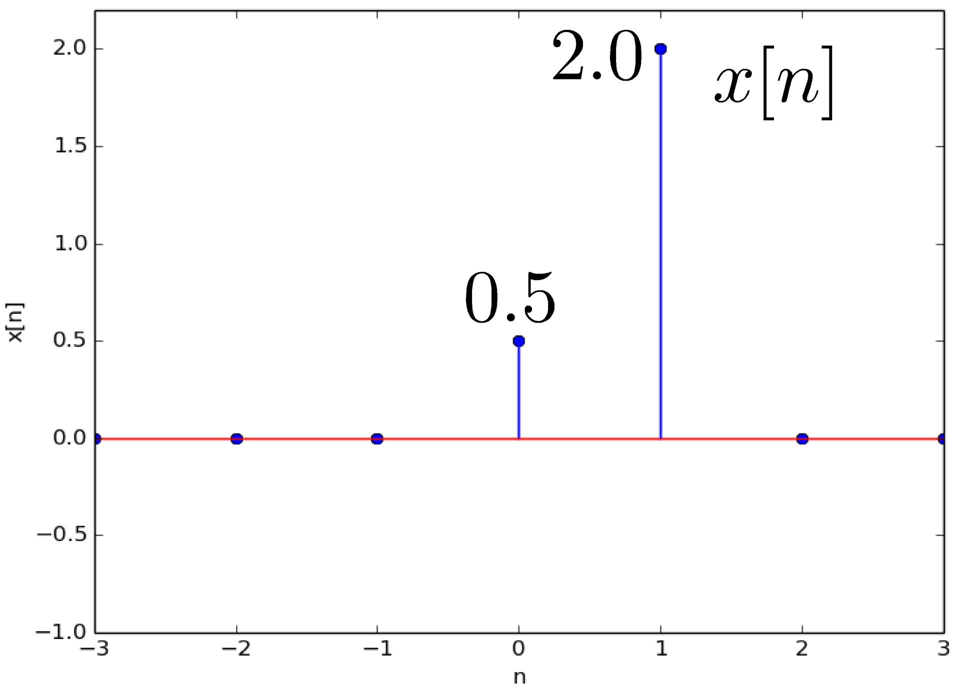
$$\boxed{y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n] * h[n]}$$

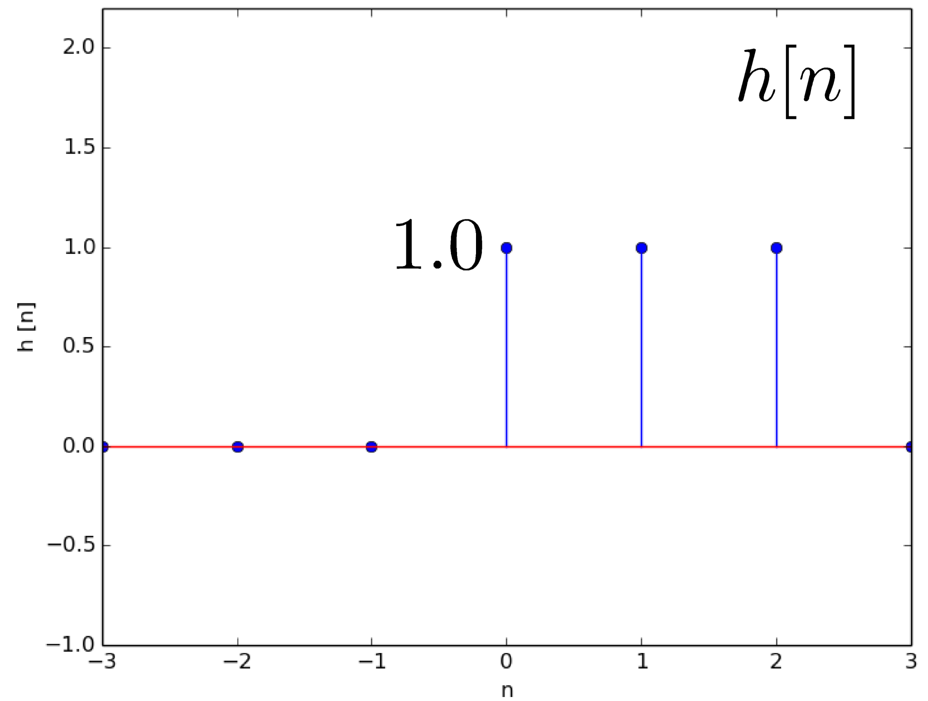
JUMLAHAN KONVOLUSI
(*CONVOLUTION SUM*)

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n] = x[n] * h[n]$$

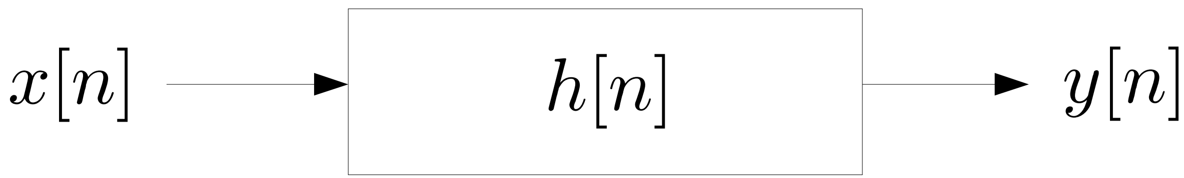
Sistem Waktu-invarian Linier

Contoh:





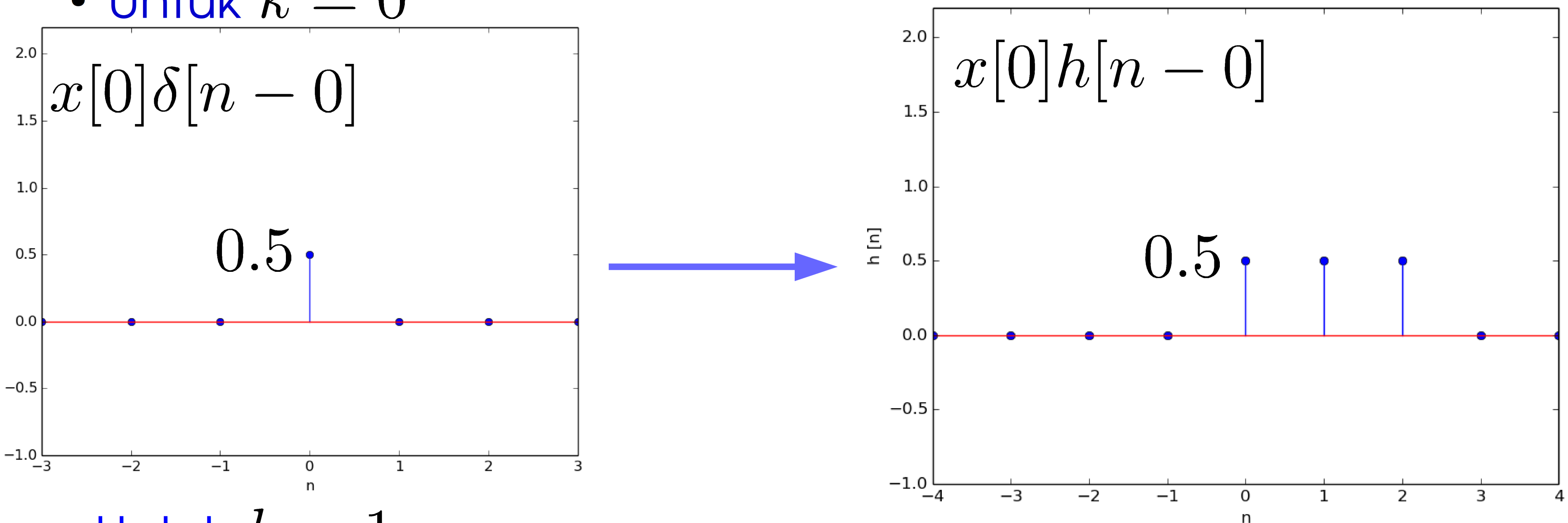
$$y[n] = x[n] * h[n]?$$



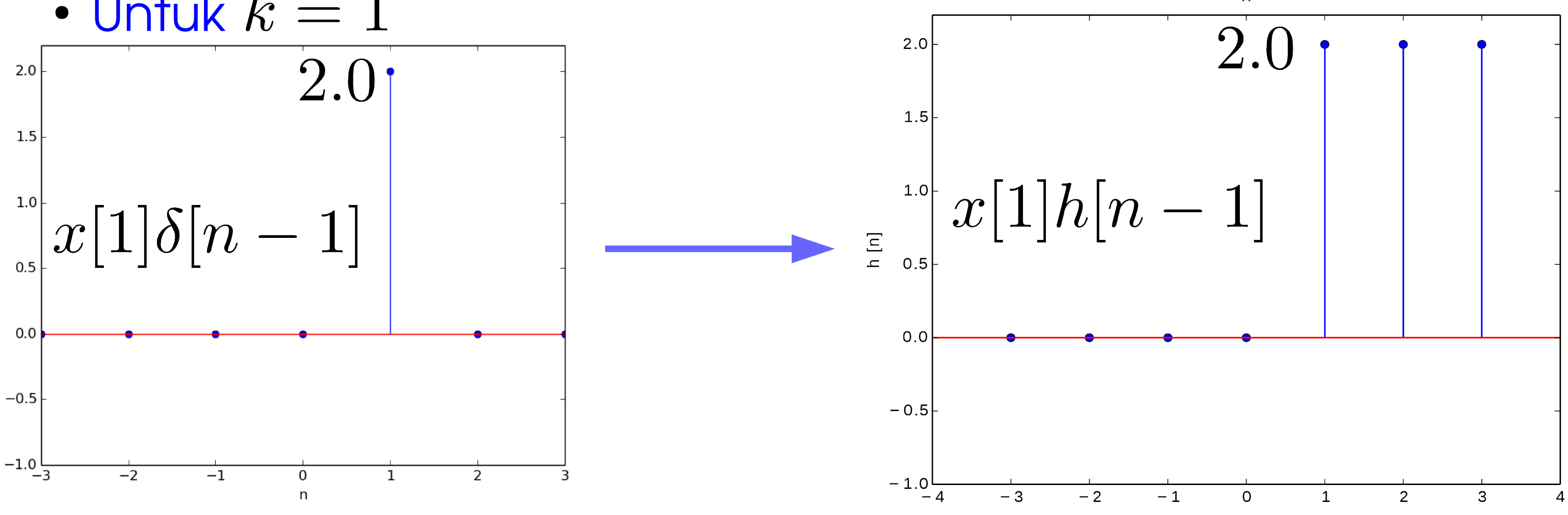
Sistem Waktu-invarian Linier

Solusi:

- Untuk $k = 0$



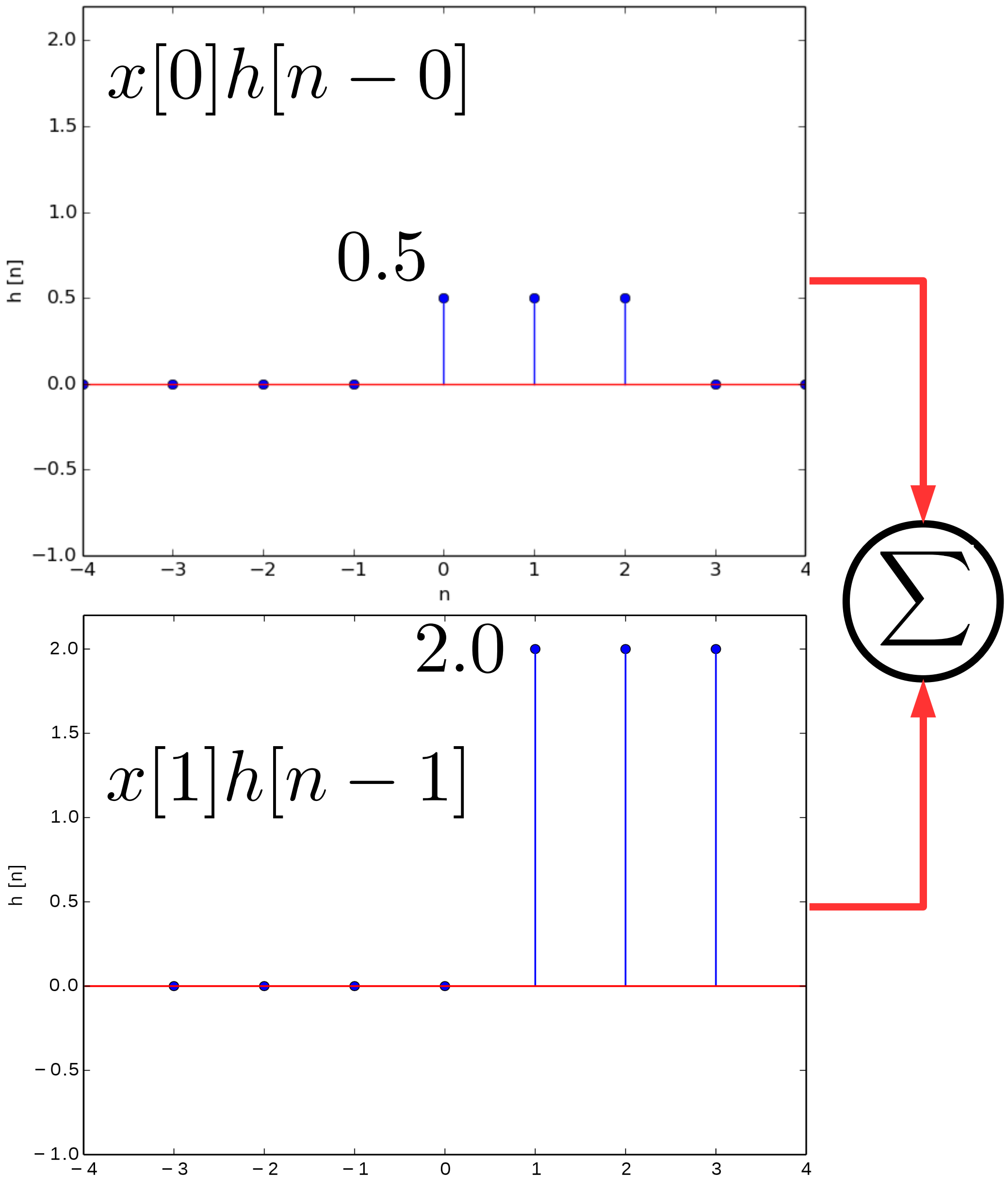
- Untuk $k = 1$

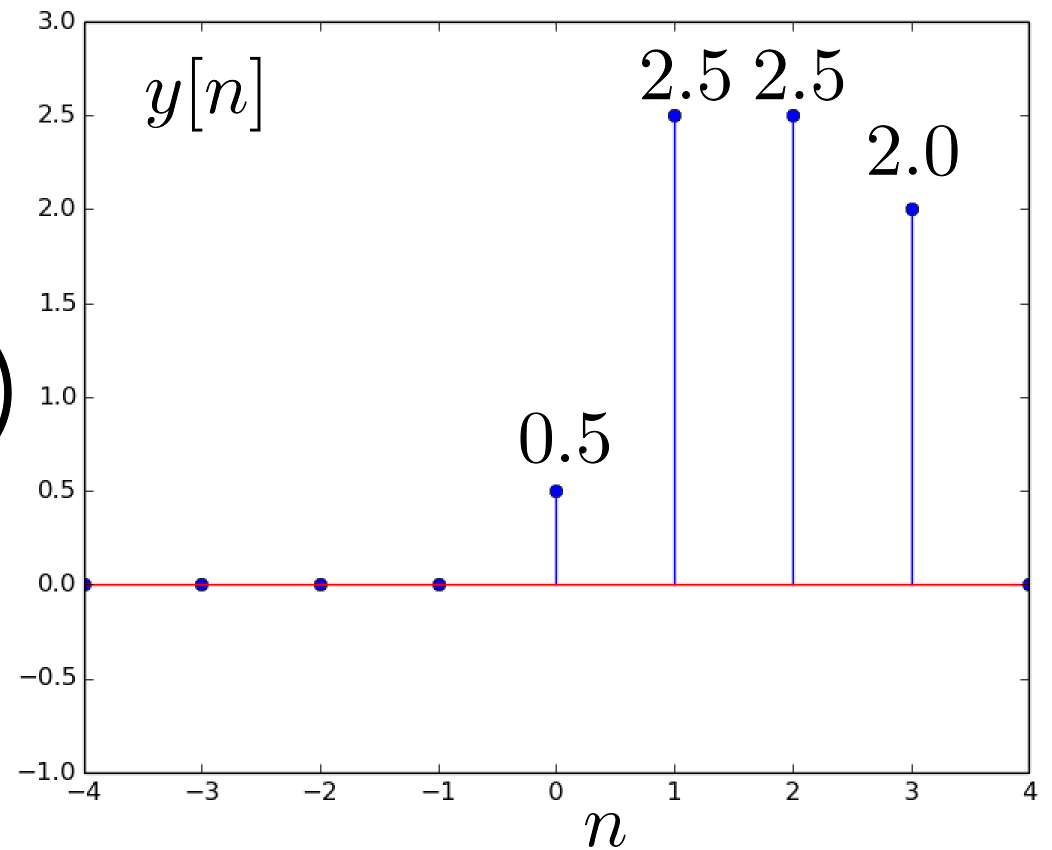


Sistem Waktu-invarian Linier

Solusi:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



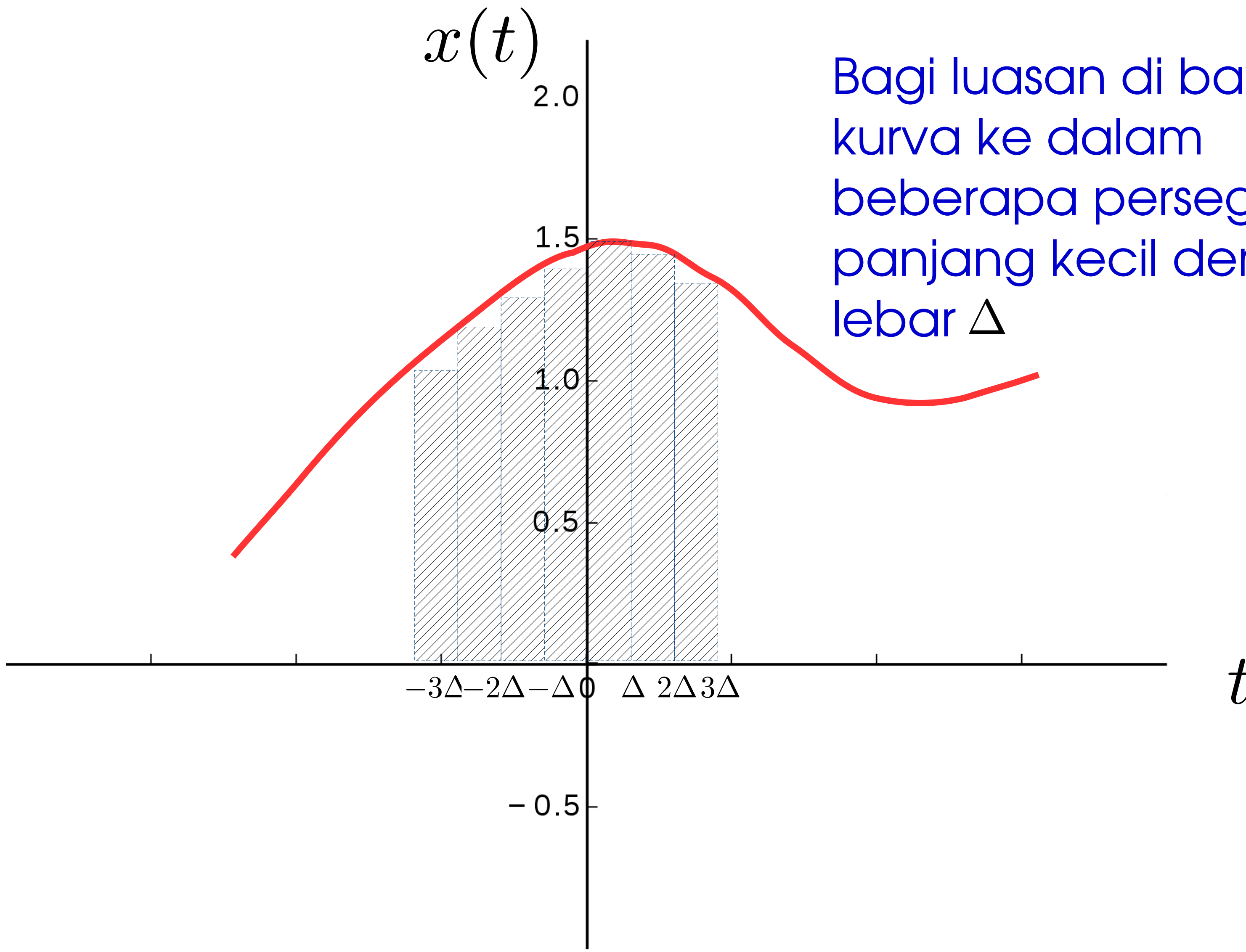


SOAL

- $x[n] = (-1)^n \{u[-n] - u[-n - 8]\}$
- $h[n] = u[n] - u[n - 8]$
- Cari $y[n] = x[n] * h[n]$

Sistem Waktu-invarian Linier

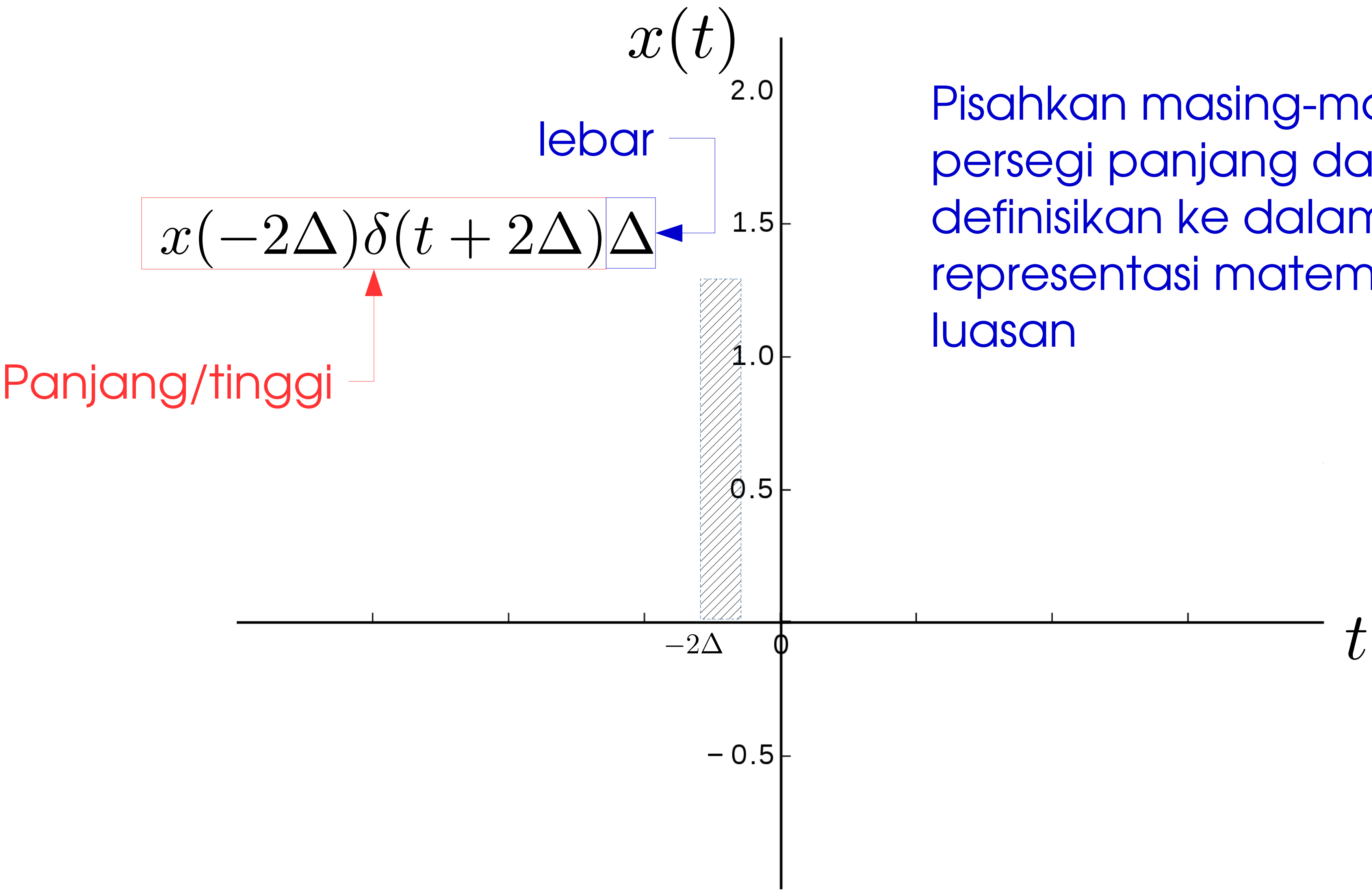
(2) Representasi sinyal waktu-kontinu sebagai impuls



Bagi luasan di bawah kurva ke dalam beberapa persegi panjang kecil dengan lebar Δ

Sistem Waktu-invarian Linier

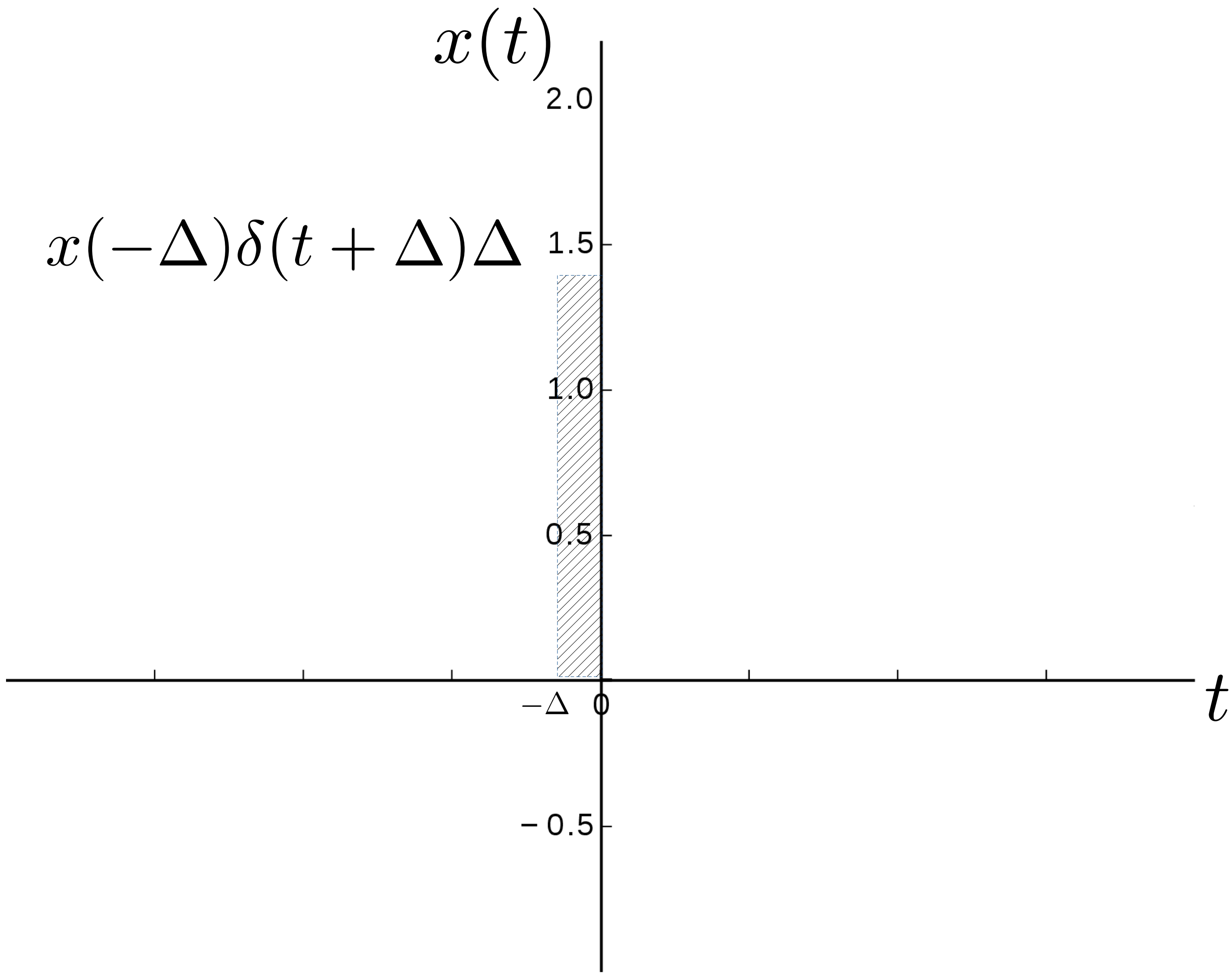
(2) Representasi sinyal waktu-kontinu sebagai impuls



Pisahkan masing-masing persegi panjang dan definisikan ke dalam representasi matematis luasan

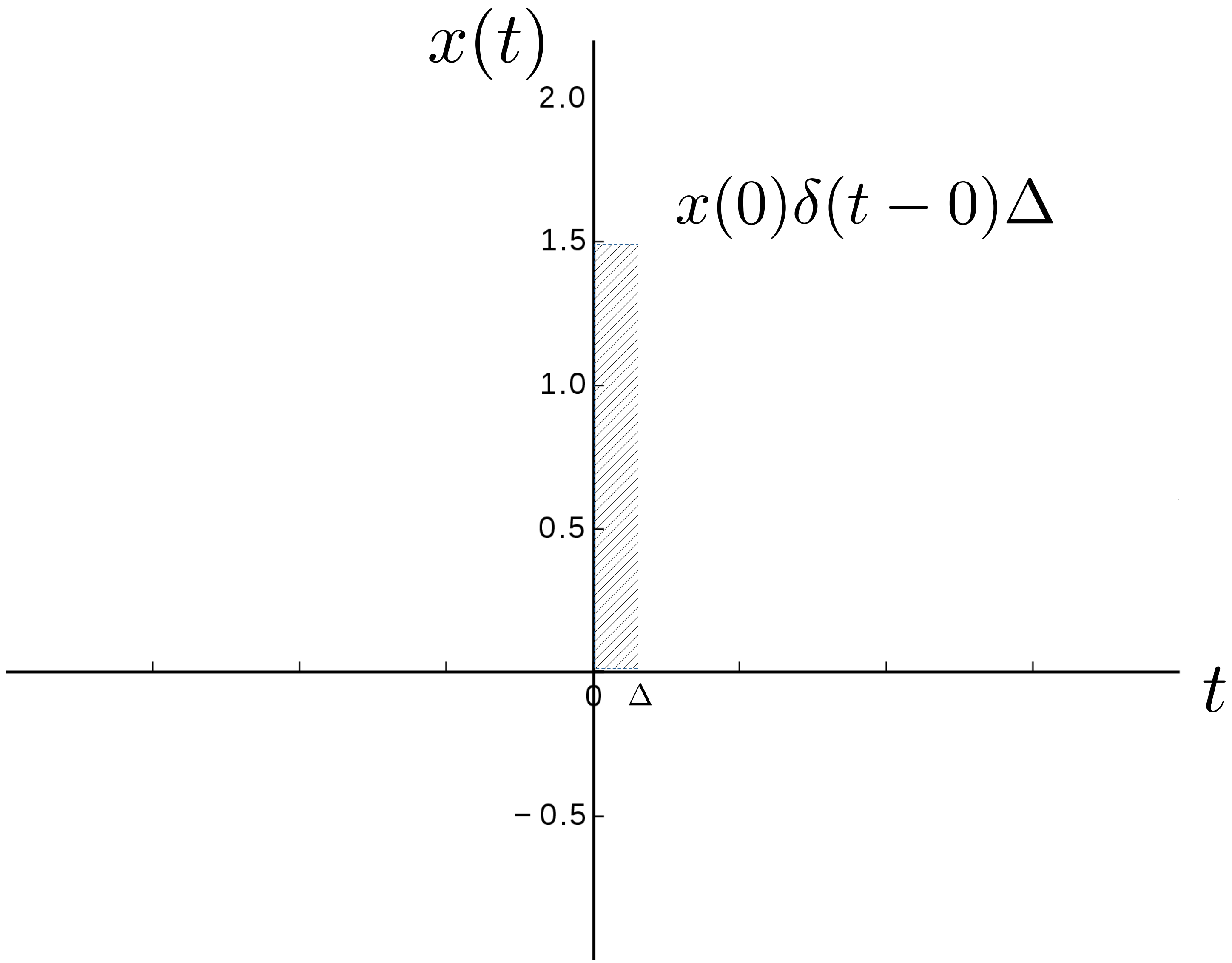
Sistem Waktu-invarian Linier

(2) Representasi sinyal waktu-kontinu sebagai impuls



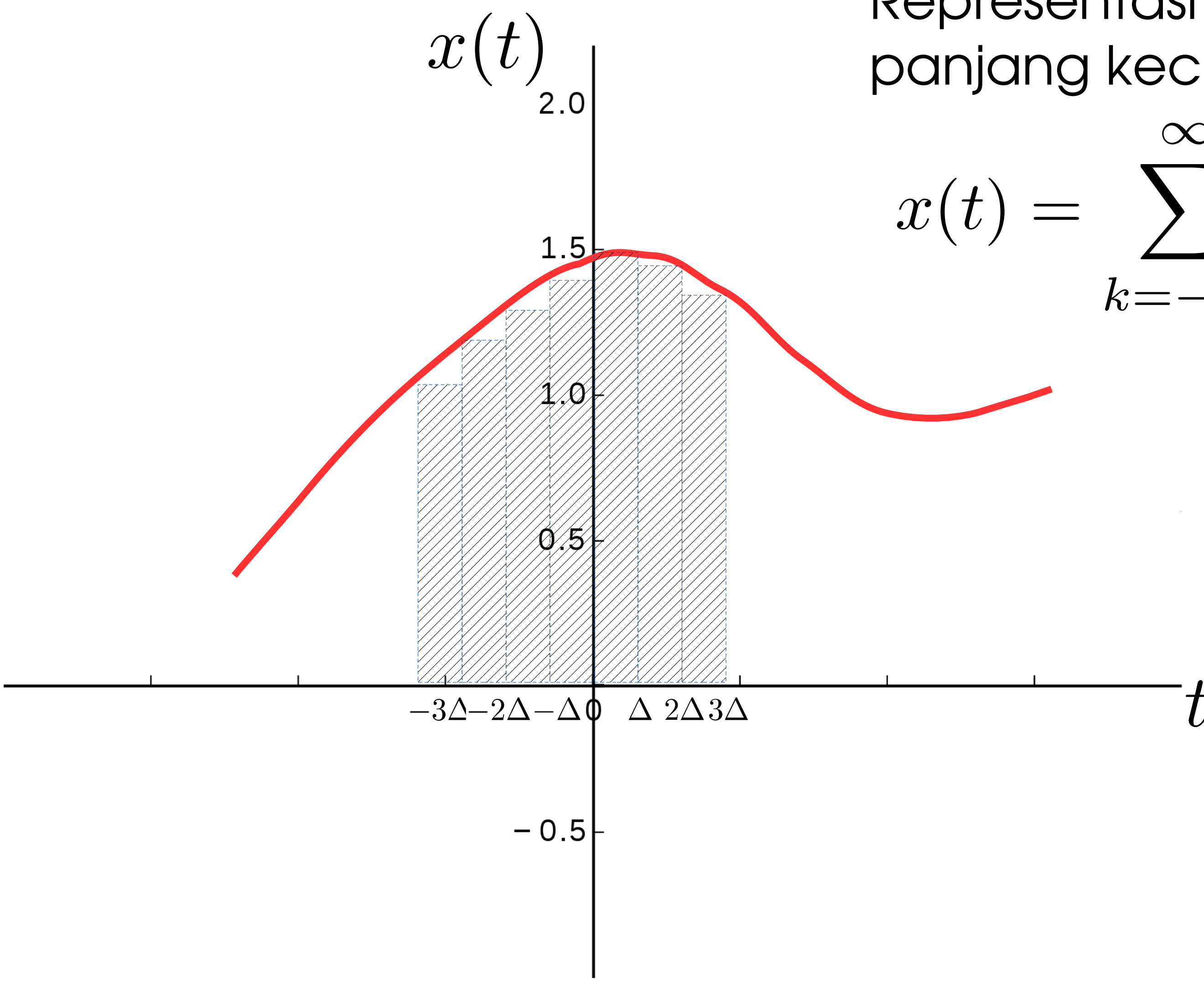
Sistem Waktu-invarian Linier

(2) Representasi sinyal waktu-kontinu sebagai impuls



Sistem Waktu-invarian Linier

(2) Representasi sinyal waktu-kontinu sebagai impuls

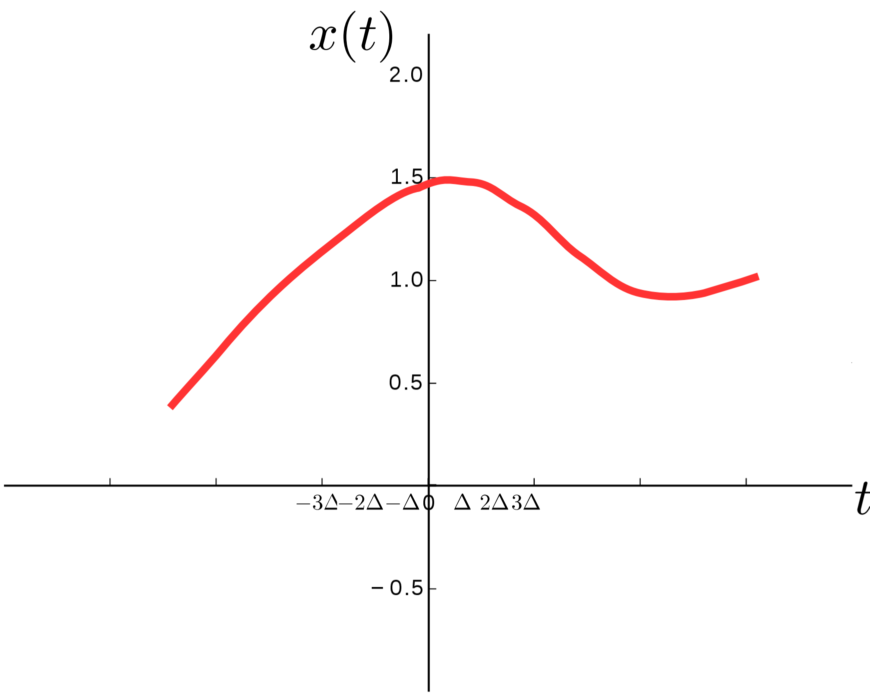


Representasi jumlahan persegi panjang kecil dalam kombinasi linier:

$$x(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\delta(t - k\Delta)\Delta$$

Sistem Waktu-invarian Linier

(2) Representasi sinyal waktu-kontinu sebagai impuls



Agar menghasilkan kurva yang halus, limitkan delta mendekati nol

$$x(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{\infty} x(k\Delta)\delta(t - k\Delta)\Delta$$

Sehingga dihasilkan:

$$\boxed{x(t) = \int_{-\infty}^{+\infty} x(\tau)\delta(t - \tau)d\tau}$$

Sifting property untuk Sinyal waktu-kontinu

Sistem Waktu-invarian Linier

(2.1) Integrasi konvolusi

Analog dengan LTI waktu-diskrit...

- **Linier**

$$\delta(t-\tau) \rightarrow h_\tau(t)$$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau \rightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)h_\tau(t)d\tau$$

- **Waktu-invarian**

$$\delta(t) \rightarrow h_0(t) \Rightarrow \delta(t-\tau) \rightarrow h(t-\tau)$$

$$x(t) = \int_{-\infty}^{+\infty} x(\tau)\delta(t-\tau)d\tau \rightarrow \boxed{y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau}$$

INTEGRAL KONVOLUSI

Sistem Waktu-invarian Linier

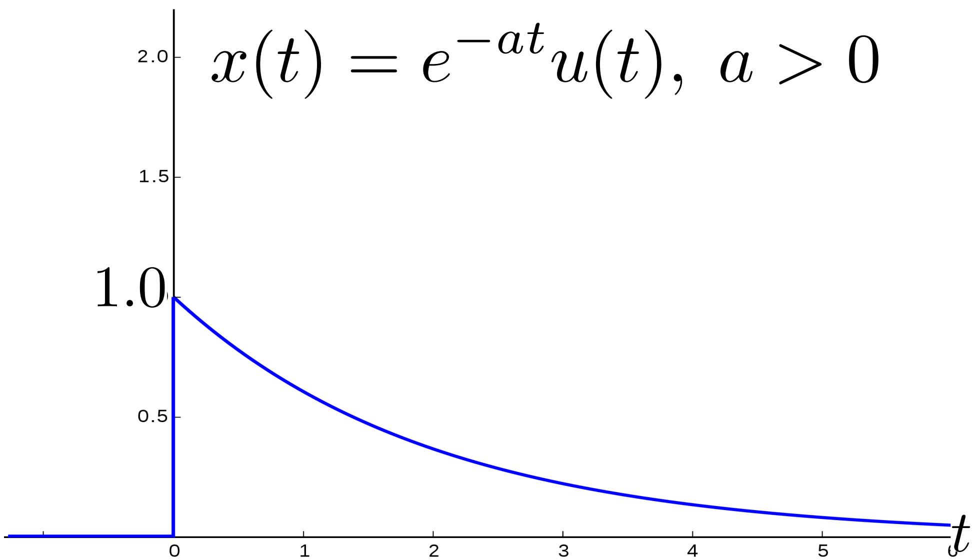
(2.1) Integrasi konvolusi

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

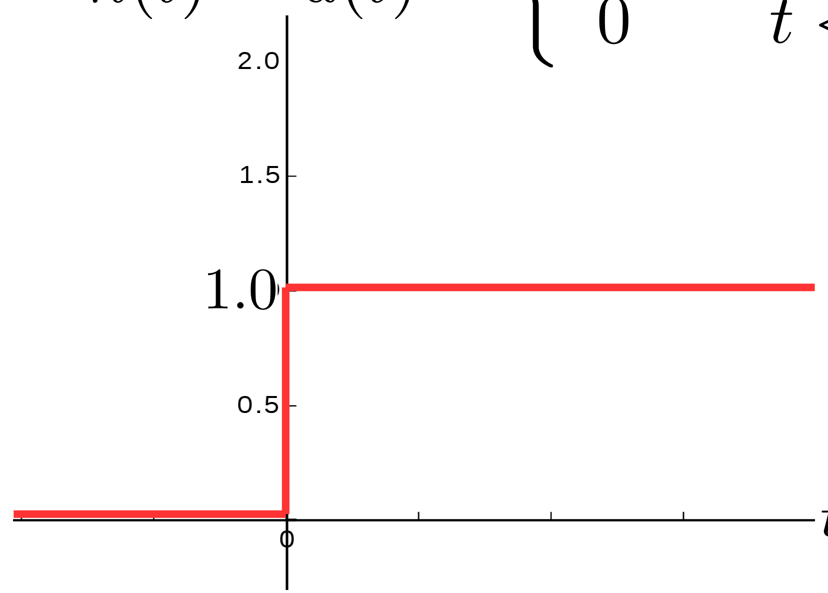
$$\boxed{\mathrm{y}\ (\mathrm{t}) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = x(t) * h(t)}$$

Contoh:

$$x(t) = e^{-at}u(t),\ a > 0$$



$$h(t) = u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 1 \end{cases}$$

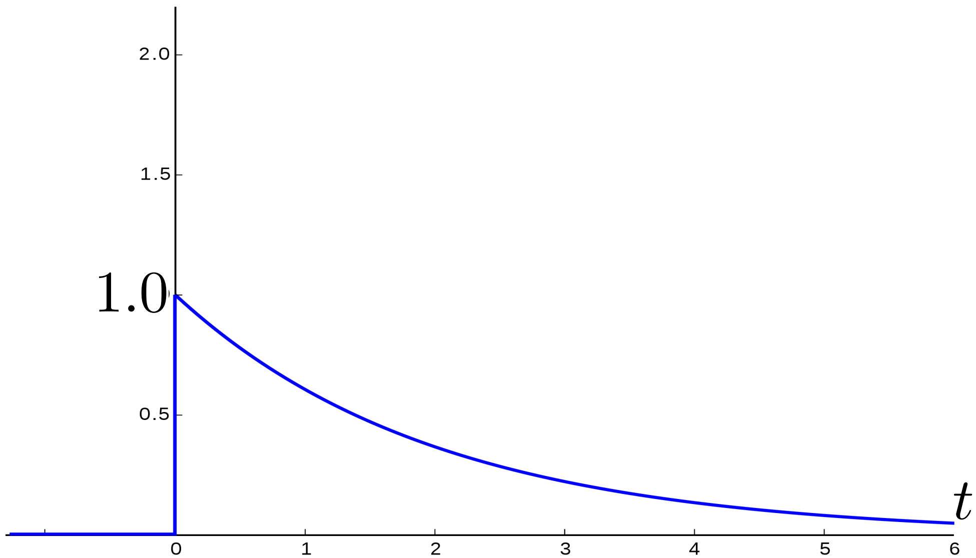


Tentukan $y(t) = x(t) * h(t)$!

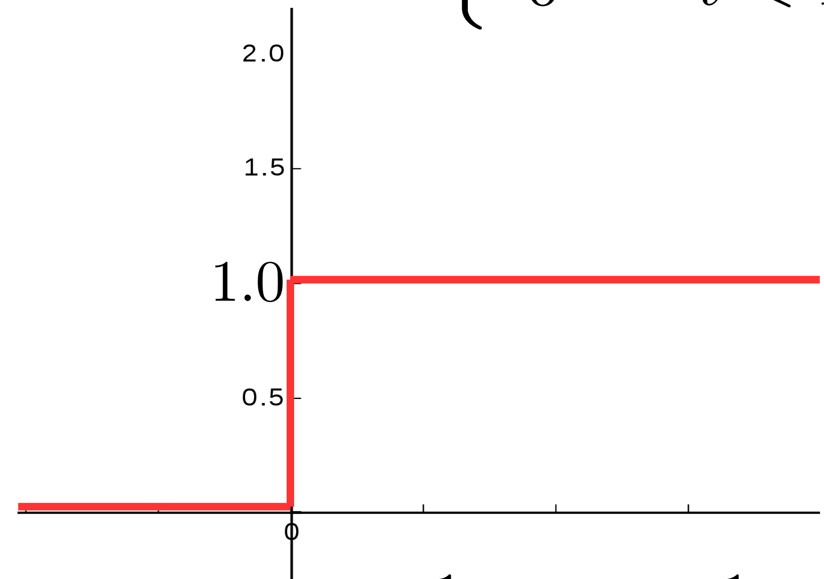
Sistem Waktu-invarian Linier

Solusi:

$$x(t) = e^{-at}u(t),\ a > 0$$



$$h(t) = u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 1 \end{cases}$$



$$y(t) = \int_{-\infty}^{\infty} \boxed{x(\tau)h(t-\tau)} d\tau$$

$$= \int_{0}^{t} e^{-a\tau} d\tau$$

$$= -\frac{1}{a} e^{-a\tau}\Big|_{0}^{t}$$

$$= \left(-\frac{1}{a}e^{-at} + \frac{1}{a}\right)$$

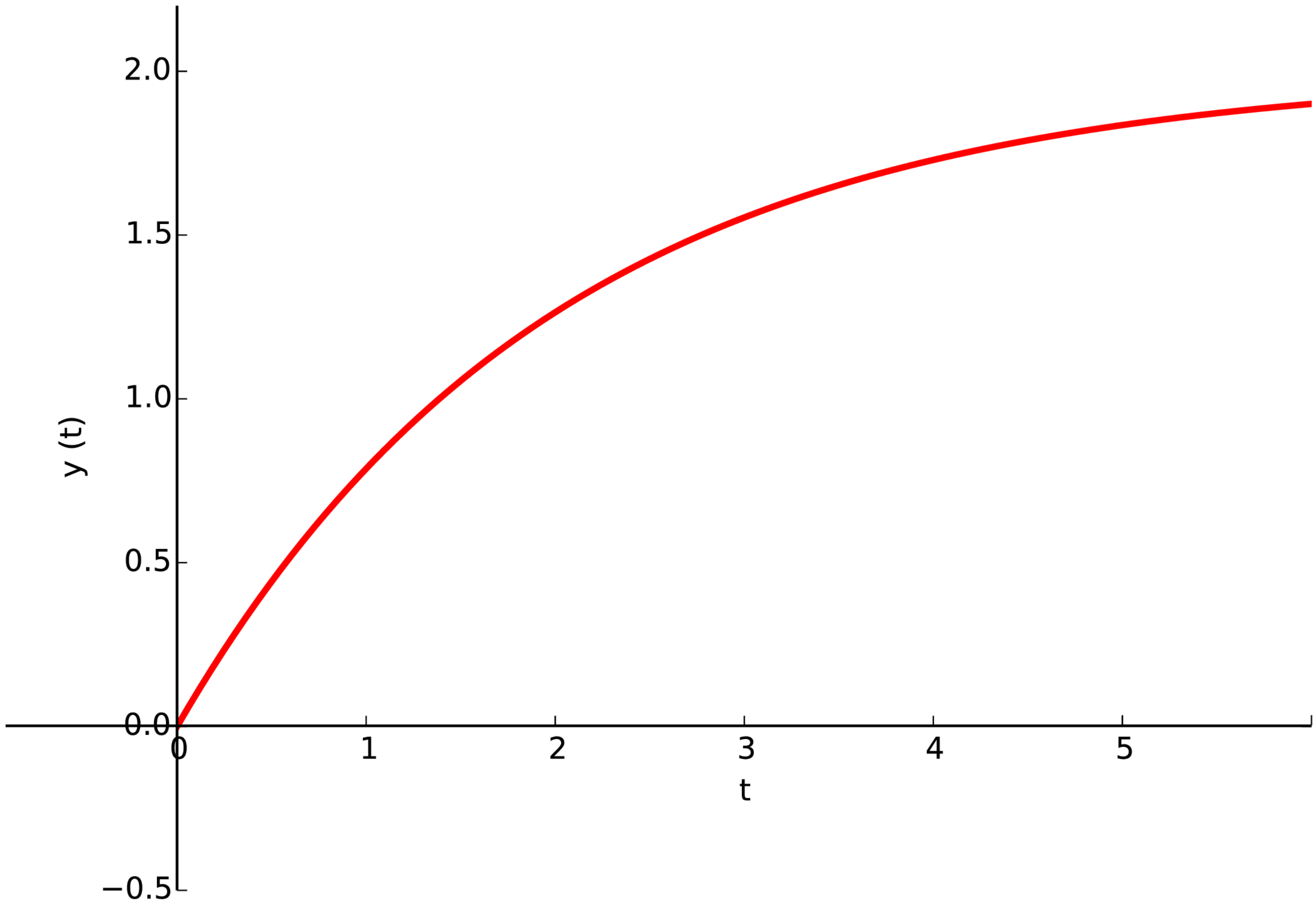
$$\Longrightarrow \quad y(t) = \frac{1}{a}(1 - e^{-at})$$

$$x(t)h(t-\tau) = \begin{cases} e^{-a\tau} & 0 < \tau < t \\ 0 & \text{lainnya} \end{cases}$$

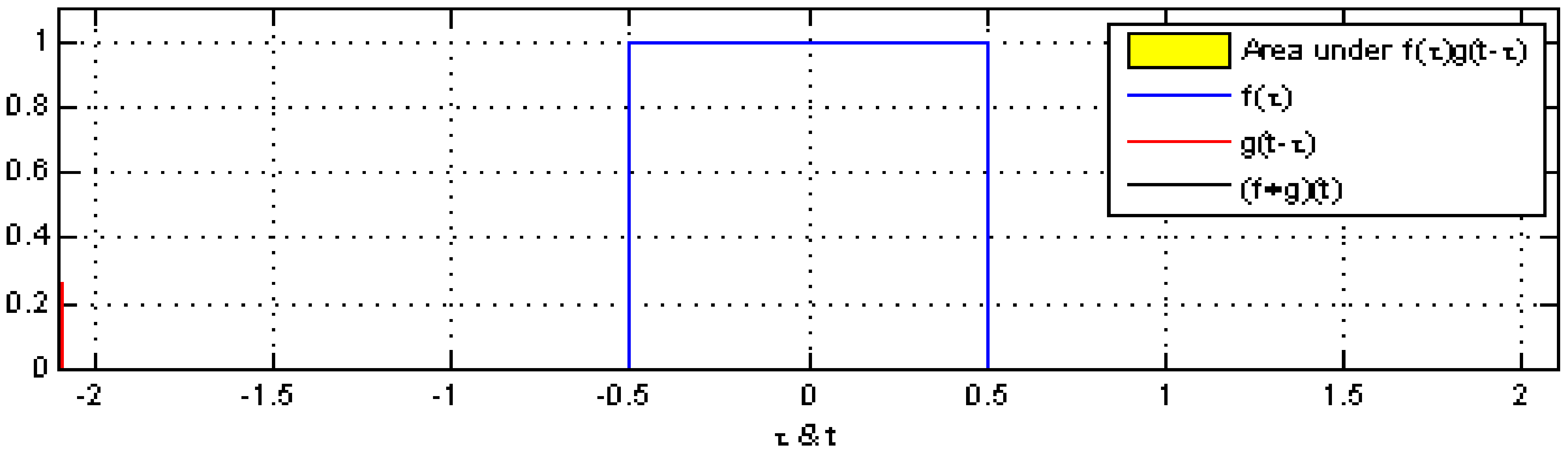
Sistem Waktu-invarian Linier

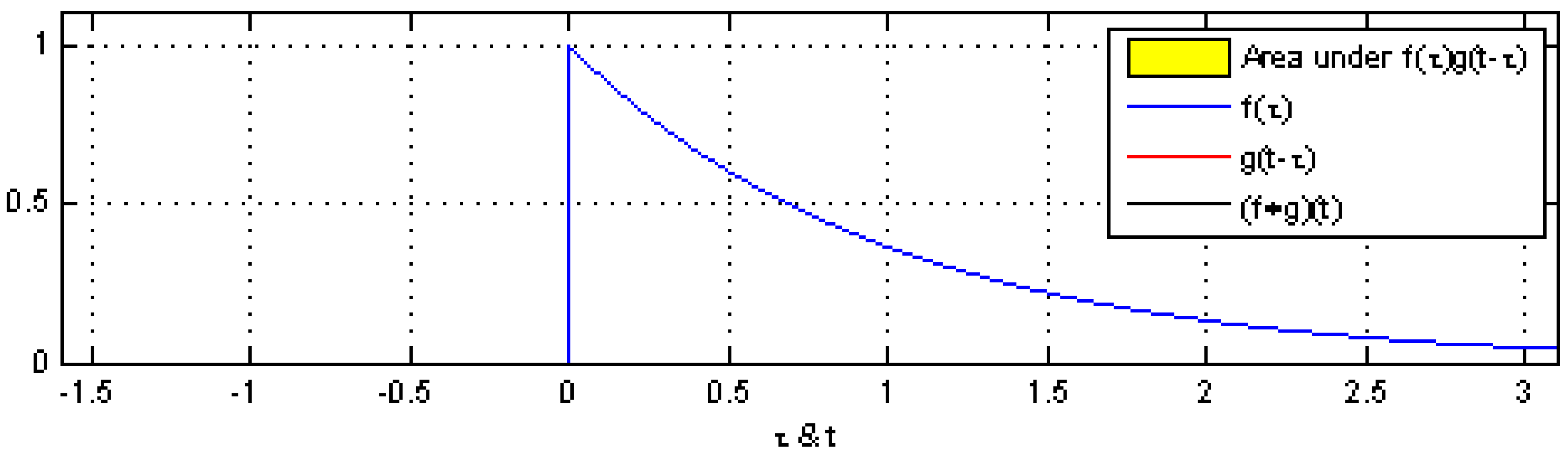
Hasil konvolusi:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \frac{1}{a}(1 - e^{-at})u(t)$$



Integral Konvolusi





Integral Konvolusi

1. $x(t) \rightarrow x(\tau)$
2. $h(t) \rightarrow h(t - \tau)$: dibalik dan digeser
3. Dicari irisannya

Referensi

(1) A. V. Oppenheim, A. S. Willsky, S. H. H. Nawab, *Sinyal dan Sistem jilid 1*, (Penerbit Erlangga, Jakarta, 2000)

(2) Plot grafik dibuat dengan bantuan program iPython dan Inkscape

Referensi pemrograman Python:

(1) Python Scientific Lecture Notes, http://scipy-lectures.github.io/index.html

(2) The Python Tutorial, https://docs.python.org/2/tutorial/index.html

(3) Matplotlib, http://matplotlib.org/index.html