Praktikum 1-c : Analysis of LTI Using Z-transform

Consider a discrete-time causal LTI (linear time-invariant) system whose input- output relationship is described by the following difference equation:

$$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n-1]$$

a) Find the system function G[z].

   Applying the linearity and time-shifting properties of the z-transform

$$G[z] = \frac{Y[z]}{X[z]} = \frac{z^{-1}}{1-(1/4)z^{-1}-(1/8)z^{-2}} = \frac{z}{(z-1/2)(z+1/4)}$$

b) Find the impulse response g[n].

   Noting that the system is causal and accordingly, the ROC of G[z] is z > 1/2 (the outside of the circle passing through the outermost pole), we obtain the inverse z-transform of G[z] as

$$g[n] = \mathcal{Z}^{-1}\left\{\frac{z}{(z-1/2)(z+1/4)}\right\} \overset{\text{partial fraction expansion}}{=}$$

$$\mathcal{Z}^{-1}\left\{\frac{4}{3}\left(\frac{z}{z-1/2} - \frac{z}{z+1/4}\right)\right\} \overset{\text{B.9(5)}}{=} \frac{4}{3}\left(\left(\frac{1}{2}\right)^n - \left(-\frac{1}{4}\right)^n\right)u_s[n]$$

   Alternatively, the impulse response can be obtained directly from the difference equation, which can be solved iteratively with the unit impulse input x[n] = δ[n] and zero initial conditions:

$$y[n] = \frac{1}{4}y[n-1] + \frac{1}{8}y[n-2] + x[n-1]$$

   where y[−1] = y[−2] = 0 and x[n − 1] = δ[n − 1] = 1 only for n = 1.

   n = 0: y[0] = (1/4)y[−1] + (1/8)y[−2] + x[−1] = 0 − 0 + 0 = 0

   n = 1: y[1] = (1/4)y[0] + (1/8)y[−1] + x[0] = 0 − 0 + 1 = 1

   n = 2: y[2] = (1/4)y[1] + (1/8)y[0] + x[1] = 1/4 − 0 + 0 = 1/4

   n = 3: y[3] = (1/4)y[2] + (1/8)y[1] + x[2] = 1/16 + 1/8 + 0 = 3/16