

CMPE 300 - Analysis of Algorithms

Fall 2022

Assignment 1

Due 01.11.2022 11:59pm

This assignment is designed to help you understand some of the important topics of the course. Please give your answers to the following questions clearly and thoroughly.

Do not just give the final answer or only some of the steps in deriving the answer. You must show all the steps clearly, including those that may seem simple, during the derivation. Otherwise, no points will be given for that question.

Please work on the questions on your own. Upload your answer to **Moodle** as a **PDF file** until **01.11.2022 Tuesday 11:59pm**. Note that the deadline is strict and late submissions will not be considered. For any questions, please contact the assistant.

Question 1 (50 points)

Consider the given function $f(n)$ and determine whether the following cases are true or false. Justify your answers formally. (*Use Stirling's Approximation*)

$$f(n) = n^4 \log(n^8 n!) + 4n^3$$

- A. $f(n) \in \Theta(n^5 \log(n))$
- B. $f(n) \in \Omega(n^5 \sqrt{n})$
- C. $f(n) \in \omega(n^4)$
- D. $f(n) \in O(n^4 \log(n))$

Question 2 (50 points)

Consider the algorithm below. Do a fine (exact) analysis and calculate the number of operations for the worst case and average case, and express them in the form of Θ notation. The basic operations are assignments. In addition to the assignments in the function *anonymous*, assume

that the *find_all_subsets* function (which finds all the subsets of the array) executes 1 assignment statement for each subset. Note that each invocation of *find_all_subsets* finds all the subsets of the array. You can ignore the array initialization, condition checks, and the assignments in the loop headers.

You may assume that:

- k is a uniformly distributed random integer between 1 and n
- n is a positive odd integer

```
function anonymous(k, n)
  x = 1
  arr = initialize_random_array(n)
  if k = n/2 (round to ceil) then
    for i=0 to (n-1) do
      find_all_subsets(arr)
    end for
  else if k < n/2 then
    for j=0 to (n-1) do
      for k=n-1 down to 1 by k=k/2 do
        x=x*2
      end for
    end for
  else
    for i=0 to (n-1) do
      x = x + 1
    end for
  end function
```