A General Equilibrium Model of Changing Risk Premia: Theory and Tests

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We derive and test a dynamic discrete-time model of asset returns. Both the risks of individual securities and equilibrium risk premia change predictably in the model, but these changes can be attributed to movements in the returns andprices of only two well-diversified portfolios. Any other components of returns should be unpredictable. Using the generalized method of moments, the model is estimated and tested on portfolios of equities. We find the data supportive of the model's restrictions, even when instruments designed to capture the January effect are employed.

In this article we derive and test a dynamic discrete-time asset pricing model in which security risks and risk premia change over time. The model relates these changes to movements in the prices and returns of two well-diversified portfolios. Thus, while it is genuinely dynamic, the model imposes restrictions that are similar in spirit to traditional, static asset pricing models.

Both the Sharpe/Lintner capital asset pricing model

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(CAPM) and Ross's arbitage pricing theory (ART) capture the intuition that large, well-diversified portfolios should serve as good proxies for the forces that cause security returns to move together. An individual asset's responsiveness to these comovements then becomes the "risk" that explains relative expected returns. Such restrictions are particularly fortunate in that they make good use of the quantity and quality of financial data available. Since the static models make predictions about relative returns, they can be tested using financial data alone. These features surely help to explain the remarkable durability of these models as important paradigms in financial economics.

In other respects, however, the static models are grossly inadequate. They fail to take the time-series characteristics of the data seriously and implicitly assume that the risks of securities, and the rewards for bearing those risks, do not change over time. There is now a large and growing literature which demonstrates that this is simply untrue.[1]

The continuous-time models of Merton (1973), Breeden (1979), and Cox, Ingersoll, and Ross (1985) impose restrictions similar in nature to those of the static models and allow risk premia to change. In their general forms, however, these models have proved difficult to deal with empirically. At any one point in time the conditional moments of assets' returns depend in very complicated, nonlinearways on agents' information sets. This makes it close to impossible to relate changes in risk premia to portfolio returns that are arguably "observable."

In attempts to construct dynamic, discrete-time models that can explain variability in risk premia many of the especially attractive features of the static models are typically lost. Following the lead of Hansen and Singleton (1982)) these models proceed by explicitly parameterizing the preferences of a "representative agent." The agent's first-order conditions then impose restrictions on the intertemporal evolution of asset returns. The parameters are estimated and overidentifying restrictions are tested using aggregate consumption measures as an argument for the preferences in these first-order conditions.

In the static or continuous-time models there is no need to deal directly with preference parameters or aggregate consumption, even though they may be implicit in the restrictions the models impose on relative returns. From an analytical point of view, the crucial feature of the single-period models is that the returns on well-diversified portfolios substitute for the marginal rate of substitution in the representative agent's first-order condition.[2] Thus, expected returns can be related to covariances with observable portfolios of financial assets, rather than to covariances with a marginal utility.

[1] That risk premia change in predictable ways has been shown by Hansen and Hodrick (1983, Gennotte and Marsh (1985). Gibbons and Ferson (1985). Person (1986). and Kelm and Stambaugh (1986) among others.

[2] See Chen and Ingersoll (1983) for an analysis of this phenomenon in connection with APT, or Bhattacharya (19131) for a thoughtful discussion of its role in continuous time.

dynamic structure, movements in the prices of two large portfolios "proxy" for changes in expectations about the future evolution of the marginal rate of substitution. Both the relative magnitudes of expected returns in cross section and their evolution in time series are driven by large, well-diversified portfolios. Thus, the model imposes rejectable restrictions on relative returns that, while dynamic in nature, can be tested without the use of aggregate consumption data or explicit parameterization of the preferences of a representative agent. This is achieved through strong assumptions concerning the distribution of endowments.

In our model, as in that of Lucas (1978), the primitive assumptions concern the payoffs, or "dividends," on each of several firms or technologies. The prices of claims on these payoffs are then determined endogenously, and the joint distribution for returns is studied. We assume the dividend process for each technology has a conditionally linear projection on the aggregate payoff with a serially uncorrelated residual, which is mean-independent of the aggregate payoff series. The intercept term in this linear relationship is assumed to evolve as a random walk, with innovations that are similarly unrelated to aggregates, while the slope coefficients are assumed constant through time. We show that in a representative consumer economy, equilibrium prices lead to a joint distribution of returns that inherits this conditional linear dependence on the market portfolio. A default-free consol bond serves a role analogous to the riskless asset in the static models. The "beta" associated with the returns for each asset varies stochastically, but this variation is determined very simply by the price of the asset relative to the current value of the market portfolio.

The arguments leading to our results owe an obvious debt to Connor's (1985) work developing an intertemporal version of the AFT. More recently, Connor and Korajczyk (1987) have developed results in a multiple factor context that, like ours, relax the parameterizations of preferences and unchanging investment opportunity set assumed in Connor's original paper. Our model, in addition, allows for stochastic variation in the intercept term of the dividend process. This proves important both to fitting the model empirically and in breaking the collinearity of prices in the earlier models, a "perfect fit" that is clearly rejected by the data.

The model implies that, in equilibrium, the conditional expected return on each asset is a fairly simple, albeit nonlinear, function of the prices and conditional expected returns of the market portfolio and a riskless consol. In addition, the model can be shown to impose rejectable restrictions without the need for data on consol bond returns. The estimation and tests of the model then exploit the fact that the deviations of assets' returns from their conditional expected return must be uncorrelated (unconditionally) with any lagged information. We test these restrictions using the generalized method of moments, developed by Hansen (1982) and Hansen and Singleton (1982).

Our empirical results suggest that the model does a reasonable job of capturing the sources of autocorrelation in returns and hence In explaining changes in risk premia through time in a parsimonious fashion. We test the model with portfolios formed both on the basis of size and on the basis of industry (SIC codes). The model is not rejected for these assets even when instrumental variables are included that lead to rejections in other tests of dynamic models, such as a dummy variable for January or the "junk bond premium." We attribute the apparent success of the model in part to its predictions about the relationship between an asset's current price and future returns. In our model conditional expected returns depend inversely on price, so that "big losers" will tend subsequently to perform relatively well. The January effect is known to be stronger for firms with poor past performance and similarly the types of "mean-reversion" observed in returns by Fama and French (1988) and Poterba and Summers (1988) are consistent with the model's restrictions.

The article is organized as follows. Section 1 describes the model and derives equilibrium prices based on the assumptions of a representative agent and a separating distribution for the "dividends." Section 2 then characterizes the joint distribution of returns and shows expected returns are linear in the assets' conditional "betas." Section 3 details our estimation strategy, and in Section 4 the data are described. Empirical results are discussed in Section 5. We interpret the results and conclude in Section 6.

1. The Model and Equilibrium Prices

We consider an economy with a single consumption good that is generated by the payoffs to n different technologies or firms. Investors are endowed with, and trade, "shares" in these technologies. For firm i, $i = 1, \dots, n$, let. q_i denote the number of shares outstanding. If d_{it+1} is the per-share payoff, or "dividend," on firm fin period $t + 1$, then the aggregate dividend is $\sum_{i=1}^{n} q_i d_{it+1}$. We will denote this as d_{wt+1}, the payoff on aggregate wealth. This is also the total 'amount of the good available for consumption in period $t + 1$. We assume it is related to the payoffs on individual firms in the following very simple way:

$$d_{it+1} = \alpha_{it} + \beta_i d_{wt+1} + \epsilon_{it+1} \tag{1}$$

The intercept in Equation (1), a, is assumed to evolve as a random walk:[3]

$$\alpha_{it+1} = \alpha_{it} + \nu_{it+1} \tag{2}$$

We take the innovations in the intercepts, as well as the "residuals" in the dividend processes, ϵ_{it+1}, to be conditionally mean-independent of current information and of current and future aggregate dividends. That is, if we let I_t be the information set at date t and let (d_w) denote the Information

[3] We thank both G. Sick and M. Brennan for suggesting this model for the intercept term, which did not appear in early versions of this article. It is similar In spirit to the model in Brennan (1973).

set generated by the sequence of aggregate dividends, then

$$E[\epsilon_{it+1} \mid I_t \vee \sigma(d_w)] \equiv E_t(\epsilon_{it+1}) = E(\epsilon_{it+1}) = 0 \tag{3}$$

and

$$E[\nu_{it+1} \mid I_t \vee \sigma(d_w)] \equiv E_t(\nu_{it+1}) = E(\nu_{it+1}) = 0 \tag{4}$$

As in the static models of portfolio separation [see Ross (1978) and Connor (1984)], the mean-independence condition here implies a linear dependence in the residuals and in the innovations In the α's, since

$$\sum_{i=1}^{n} q_i \epsilon_{it+1} = \sum_{i=1}^{n} q_i \nu_{it+1} = 0 \tag{5}$$

Alternatively one could consider a model, as Connor (1984) does, with a countably infinite number of firms in which the weight of any one firm in the market portfolio goes to zero on the order of $1/n$.

Note that we do not assume the joint distribution of dividends is constant through time, just that all the predictable intertemporal covariability is attributable to the aggregate dividend. Other changes in the investment opportunity set are "idiosyncratic" and unpredictable using past information. This description, and the results below, generalize in a straightforward way to allow for multiple "factors" or state variables along the lines of Connor and Korajczyk (1987).

We assume there is an infinitely lived representative consumer, who trades the shares of the n technologies to maximize time-additive von Neumann-Morgenstern utility of the form $E_t[\sum_{\tau=1}^{\infty} \rho^\tau u(C_{t+\tau})]$, $0 < \rho < 1$. Market clearing implies that, at an equilibrium, the agent's optimal consumption must equal the aggregate dividend, so $C_{t+\tau} = d_{wt+\tau}$. The agent's first-order conditions, then, yield the stochastic Euler equations

$$p_{it} = E_t\left[\rho \frac{u'(d_{wt+1})}{u'(d_{wt})} (d_{it+1} + p_{it+1})\right] \tag{6}$$

where, for each of the n firms, p_{it} is the price of one share at date t. The expectation in Equation (6) is conditional on information available at time t. Using recursive substitution and the law of iterated expectations, Equation (6) can be written alternatively as

$$p_{it} = E_t\left[\sum_{\tau=t+1}^{\infty} \rho^{\tau-t} \frac{u'(d_{w\tau})}{u'(d_{wt})} d_{i\tau}\right] \tag{7}$$

Finally, we assume that, as in Connor (1985) or Rubinstein (1981), investors have access to a riskless, real consol bond. Such a bond pays one unit of consumption at every date with certainty. It can be viewed as either an additional technology with $\alpha_{ct} = 1$, $\beta_c = 0$, and $\epsilon_{ct} = 0$ for all t, or as a contingent claim traded in zero net supply. Similarly, note that the portfolio that pays the aggregate dividend, which we will refer to as the "market," or "aggregate wealth," portfolio, satisfies Equation (1) with $\alpha_{wt} = 0$ and $\beta_w = 1$ for all t.

A random price sequence $\{p_{it}\}_{t=1}^{\infty}$ is a recursive equilibrium, as defined in Lucas (1978), if it solves Equation (6) for each of the i firms and at each time t. Our first result shows that a very simple recursive equilibrium exists. It represents a generalization of the results of Connor (1985) and Connor and Korajczyk (1987).

Theorem 1. *The following price sequence (assuming the expectations all exist) is a recursive equilibrium:*

$$p_{it} = \alpha_{it} p_{ct} + \beta_i p_{wt} \tag{8}$$

$$p_{ct} = E_t\left[\sum_{\tau=t+1}^{\infty} \rho^{\tau-t} \frac{u'(d_{w\tau})}{u'(d_{wt})}\right] \tag{9}$$

$$p_{wt} = E_t\left[\sum_{\tau=t+1}^{\infty} \rho^{\tau-t} \frac{u'(d_{w\tau})}{u'(d_{wt})} d_{w\tau}\right] \tag{10}$$

where p_{ct} is the price of a riskless consol bond and p_{wt} is the price of the market portfolio.

Proof: Equations (9) and (10) follow directly from Equation (7), given the definitions of the consol and the aggregate wealth or "market" portfolio. Rewrite (7), using (1) to substitute for $d_{i\tau}$ as

$$\begin{aligned} p_{it} = {} & \sum_{\tau=t+1}^{\infty} E_t\left[\rho^{\tau-t} \frac{u'(d_{w\tau})}{u'(d_{wt})} \alpha_{i\tau-1}\right] \\ & + \beta_i \sum_{\tau=t+1}^{\infty} E_t\left[\rho^{\tau-t} \frac{u'(d_{w\tau})}{u'(d_{wt})} d_{w\tau}\right] \\ & + \sum_{\tau=t+1}^{\infty} E_t\left[\rho^{\tau-t} \frac{u'(d_{w\tau})}{u'(d_{wt})} \epsilon_{i\tau}\right] \end{aligned} \tag{11}$$

The second summation in Equation (11) above is obviously $\beta_i p_{wt}$. We now show each term in the third summation is zero. Both $\rho^{\tau-t}$ and $u'(d_{wt})$ are in I_t. By iterated expectations and Equation (3),

$$E_t[u'(d_{w\tau})\epsilon_{i\tau}] = E_t\{u'(d_{w\tau})E[\epsilon_{i\tau} \mid I_t \vee \sigma(d_w)]\} = 0 \tag{12}$$

Now consider the terms in the first summation. Again, $\rho^{\tau-t}$ and $u'(d_{wt})$ are in I_t.

$$E_t[u'(d_{w\tau})\alpha_{i\tau-1}] = E_t[u'(d_{w\tau})\alpha_{it}] + E_t\left[u'(d_{w\tau}) \sum_{s=t+1}^{\tau-1} v_{is}\right]$$

$$= E_t[u'(d_{w\tau})\alpha_{tt}] + \sum_{s=t+1}^{\tau-1} E_t[u'(d_{w\tau})\nu_{ts}] \tag{13}$$

Each term in the final summation of Equation (13) is zero because

$$E_t[u'(d_{w\tau})\nu_{ts}] = E_t\{u'(d_{w\tau})E[\nu_{ts} \mid I_t \vee \sigma(d_w)]\} = 0 \tag{14}$$

Therefore, the first summation in Equation (11) can be rewritten as

$$\sum_{\tau=t+1}^{\infty} E_t\left[\rho^{\tau-t}\frac{u'(d_{w\tau})}{u'(d_{wt})}\alpha_{t\tau-1}\right] = \sum_{\tau=t+1}^{\infty} E_t\left[\rho^{\tau-t}\frac{u'(d_{w\tau})}{u'(d_{wt})}\alpha_{tt}\right]$$

$$= \alpha_{tt}\sum_{\tau=t+1}^{\infty} E_t\left[\rho^{\tau-t}\frac{u'(d_{w\tau})}{u'(d_{wt})}\right] \tag{15}$$

and the infinite sum of future marginal rates of substitution is just the price of a riskless consol. ■

Theorem 1 states that, at any given time, pricing is conditionally linear in the parameter that measures the sensitivity of the dividend process to the aggregate dividend. The other component of the value is due to the intercept in the dividend process, which changes through time. The market values of both this intercept and the portion of future dividends attributable to the aggregate dividend also vary through time. They do so, however, in a manner that is easy to relate to changes in the values of only two financial instruments, namely the market portfolio and the consol. Since changes in the intercept are assumed to be unpredictable, it follows that all the predictable variation in the value of the firm's future dividends must be attributable to changes in the value of the market portfolio and in the value of the consol. The variability in the intercept is important, however, in avoiding the perfect collinearity of prices which characterizes similar models such as that of Connor and Korajczyk (1987).[4]

The value of the intercept term in the dividend process is just the time t expectation of the discounted marginal utilities. Because the innovations in the intercept are mean-independent of the aggregate dividend,

$$E_t[u'(d_{w\tau})\alpha_{t\tau}] = E_t[u'(d_{w\tau})]E_t(\alpha_{t\tau}) \tag{16}$$

and $E_t(\alpha_{t\tau}) = \alpha_{tt}$, which clearly does not depend on τ, the index of summation in Equation (11). Thus, the expectation of the future intercept terms factors out of the summation, and one is left with just the infinite sum of expected future marginal rates of substitution-that is, with the value of a riskless consol. Note that to price dividend processes with more complicated sources of persistence than a random walk would require more information about the term structure of interest rates than just the

[4]Earlier circulated versions of this article also have this limitation. This type of degeneracy, which is characteristic of intertemporal models where changes in the investment opportunity set are lower dimensional, w-as first analyzed in Rosenberg and Ohlson (1976).

current value of a consol. For example, if $\alpha_{i\tau}$ was a first-order autoregressive process with a coefficient. less than 1, then $E_t(\alpha_{i\tau})$ would depend geometrically on t. To price it we would require a riskless security displaying geometric decay at that rate. Alternatively, with a full set of riskless, pure-discount bonds, we could value any process where the innovations $\alpha_{i\tau} - E_t(\alpha_{i\tau})$ were "idiosyncratic" in the sense assumed above.[5]

Similarly, the value of the technology's responses to fluctuations in the aggregate dividend depends on the discounted expected product between the marginal utility and its argument. This quantity can be written

$$E_t[u'(d_{w\tau})d_{w\tau}] = E_t[u'(d_{w\tau})]E_t(d_{w\tau}) + \mathrm{cov}_t[u'(d_{w\tau}), d_{w\tau}] \qquad (17)$$

The covariance term in the above must be negative under risk aversion, since the marginal utility is decreasing in its argument. Greater degrees of risk aversion will obviously tend to make the covariance more negative, as would higher levels of volatility in the aggregate dividend (or consumption). Thus, as one would expect, higher risk aversion or greater volatility in future aggregate cash flows decrease the relative value of technologies that are highly sensitive to these fluctuations.

2. Equilibrium Rates of Return

In this section we describe the joint distribution of returns through time. The dividend process we have assumed for each firm has a conditionally linear regression on the payoff of the aggregate endowment. Here we show the returns, on claims to the dividend processes have a conditionally linear dependence on the returns to two "funds." One of these funds is the "market portfolio," which represents the return to aggregate wealth implied by the endogenous price process. The other is the riskless consol bond. Thus, the return-generating process can be viewed as an intertemporal version of a two-factor model. Both the risk premia on the funds and the responsiveness of individual securities to their movements change through time. All these changes are attributable to two sources: changes in the prices of the two funds, which were derived in the previous section. This time variation proves to be empirically, as well as theoretically, tractable.

Let $1 + r_{it+1}$ denote the gross real return on the shares of the ith firm or technology between dates t and $t + 1$. Substituting for the prices from Equation (8) yields

$$\begin{aligned} 1 + r_{it+1} &= \frac{\alpha_{it+1} p_{ct+1} + \beta_i p_{wt+1} + \alpha_{it} + \beta_i d_{wt+1} + \epsilon_{it+1}}{p_{it}} \\ &= \frac{1}{p_{it}}[\alpha_{it}(1 + p_{ct+1}) + \beta_i(d_{wt+1} + p_{wt+1}) + \epsilon_{it+1} + \nu_{it+1} p_{ct+1}] \end{aligned}$$

[3]These Issues are pursued in Jagannathan and Viswanathan (1988).

$$= \alpha_{it}\left(\frac{p_{ct}}{p_{it}}\right)(1 + r_{ct+1}) + \beta_t\left(\frac{p_{wt}}{p_{it}}\right)(1 + r_{wt+1})$$
$$+ \frac{1}{p_{it}}\epsilon_{it+1} + \nu_{it+1}\frac{p_{ct+1}}{p_{it}} \tag{18}$$

The next theorem describes the implications of the above in detail.

Theorem 2. *At any time t, the return-generating process is described by*

$$r_{it+1} = r^*_{it+1} + \epsilon^*_{it+1} \tag{19}$$

where

$$r^*_{it+1} = (1 - \beta^*_{it})r_{ct+1} + \beta^*_{it}r_{wt+1} \tag{20}$$

$$\epsilon^*_{it+1} = \frac{1}{p_{it}}(\epsilon_{it+1} + \nu_{it+1}p_{ct+1}) \tag{21}$$

$$\beta^*_{it} = \beta_t\frac{p_{wt}}{p_{it}} \tag{22}$$

$$1 - \beta^*_{it} = \alpha_{it}\frac{p_{ct}}{p_{it}} \tag{23}$$

Conditional expected returns are given by

$$E_t(r_{it+1}) = E_t(r_{ct+1}) + \beta^*_{it}[E_t(r_{wt+1}) - E_t(r_{ct+1})] \tag{24}$$

Proof. Equations (19) to (23) follow from Equation (18) on noting that by Equation (8)

$$1 = \alpha_{it}\frac{P_{ct}}{P_{it}} + \beta_t\frac{p_{wt}}{p_{it}} \tag{25}$$

Equation (24) is just the conditional expectation of Equation (19), provided we can show that

$$E_t(\epsilon^*_{it+1}) = \frac{1}{p_{it}}[E_t(\epsilon_{it+1}) + E_t(\nu_{it+1}p_{ct+1})] = 0 \tag{26}$$

The first expectation in parentheses is zero by assumption. Consider, then the second expectation:

$$E_t(\nu_{it+1}p_{ct+1}) = E_t\left\{\nu_{it+1}E_{t+1}\left[\sum_{\tau=t+2}^{\infty}\rho^{\tau-t-1}\frac{u'(d_{w\tau})}{u'(d_{wt+1})}\right]\right\}$$
$$= E_t\left\{E_{t+1}\left[\sum_{\tau=t+2}^{\infty}\rho^{\tau-t-1}\frac{u'(d_{w\tau})}{u'(d_{wt+1})}\nu_{it+1}\right]\right\}$$
$$= E_t\left[\sum_{\tau=t+2}^{\infty}\rho^{\tau-t-1}\frac{u'(d_{w\tau})}{u'(d_{wt+1})}\nu_{it+1}\right]$$

$$= E_t\left\{\sum_{\tau=t+2}^{\infty} \rho^{\tau-t-1} \frac{u'(d_{w\tau})}{u'(d_{wt+1})} E[\nu_{it+1} \mid I_t \vee \sigma(d_w)]\right\}$$

$$= 0 \quad \blacksquare \tag{27}$$

Note that from Equation (25) the dynamics for the intercept term, a, can be inferred from changes in the price series. In fact, as we will show in the next section, the model developed above is empirically testable even without the ability to observe a real consol bond. First, though, we develop as a corollary a special case that imposes additional restrictions on the data and relates the intertemporal restrictions more directly to those that are familiar from the static asset pricing models.

Corollary. If α_{it} is constant (and $\nu_{it+1} = 0$) then the returns on the consol bond and the market portfolio are conditionally mean-variance efficient each period.

Proof: In this case $\epsilon^*_{it+1} = \epsilon_{it+1}/p_{it}$. We must show that this error term is not only a forecast error, but also that it is the projection error from the conditional linear projection of the returns of the assets on to the returns on the cons01 and the market. Since this projection error has both zero price and zero mean for every asset the mean-variance efficiency of the market and consol follows from results in Hansen and Richard (1987). To show that ϵ^*_{it+1} is the projection error, we must simply show that the following two expressions are equal to zero:

$$\begin{aligned} E_t(\epsilon^*_{it+1} r_{ct+1}) &= E_t[\epsilon^*_{it+1}(1 + r_{ct+1})] \\ &= \frac{1}{p_{it}p_{ct}}[E_t(\epsilon_{it+1} 1) + E_t(\epsilon_{it+1} p_{ct+1})] \end{aligned} \tag{28}$$

and

$$\begin{aligned} E_t(\epsilon^*_{it+1} r_{wt+1}) &= E_t[\epsilon^*_{it+1}(1 + r_{wt+1})] \\ &= \frac{1}{p_{it}p_{wt}}[E_t(\epsilon_{it+1} d_{wt+1}) + E_t(\epsilon_{it+1} p_{wt+1})] \end{aligned} \tag{29}$$

The error ϵ_{it+1} is clearly conditionally uncorrelated with any constant since it is in time *t* information. Similarly, it is assumed to be mean-independent of, and hence uncorrelated with, d_{wt+1}. Thus, all we need to show is that it is conditionally uncorrelated with the end-of-period prices of the consol and the market. Using Equation (10),

$$\begin{aligned} E_t(\epsilon_{it+1} p_{wt+1}) &= E_t\left\{\epsilon_{it+1} E_{t+1}\left[\sum_{\tau=t+2}^{\infty} \rho^{\tau-t-1} \frac{u'(d_{w\tau})}{u'(d_{wt+1})} d_{w\tau}\right]\right\} \\ &= E_t\left\{E_{t+1}\left[\sum_{\tau=t+2}^{\infty} \rho^{\tau-t-1} \frac{u'(d_{w\tau})}{u'(d_{wt+1})} d_{w\tau}\epsilon_{it+1}\right]\right\} \end{aligned}$$

$$= E_t\left[\sum_{\tau=t+2}^{\infty} \rho^{\tau-t-1} \frac{u'(d_{w\tau})}{u'(d_{wt+1})} d_{w\tau}\epsilon_{it+1}\right]$$

$$= E_t\left\{\sum_{\tau=t+2}^{\infty} \rho^{\tau-t-1} \frac{u'(d_{w\tau})}{u'(d_{wt+1})} d_{w\tau}E[\epsilon_{it+1} \mid I_t \vee \sigma(d_w)]\right\}$$

$$= 0 \tag{30}$$

Duplicating the steps in Equation (27) with ϵ_{it+1} in place of v_{it+1} leads to the conclusion that $E_t(p_{ct+1}\epsilon_{it+1}) = 0$. ■

While this version of the model does impose interesting additional restrictions on returns, in another sense it restricts the data too much. With a,, constant through time, Equation (8) implies prices are unconditionally as well as conditionally collinear. This implies a "perfect fit" for prices that is easily rejected by casual observation.

3. Estimating the Model

The model summarized by Equations (19) to (24) in Theorem 2 implies that the following orthogonality conditions hold:

$$E_t(z_t\epsilon^*_{it+1}) = 0 \tag{31}$$

where z_i is any variable in the time t information set. If, for each i, α_{it} is a constant, α_i, then the corollary implies further that

$$E_t(\epsilon^*_{it+1}r_{wt+1}) = E_t(\epsilon^*_{it+1}r_{ct+1}) = 0 \tag{32}$$

because, under the conditions of the corollary, both the market and the consol are conditionally mean-variance efficient. For both versions of the model, iterated expectation implies that the unconditional expectations of the above quantities will also be zero. If the model is "true," then the sample quantities that correspond to these unconditional expectations should be close to zero. The generalized method of moments (GMM), developed by Hansen (1982), chooses parameters to make them as close to zero as possible by minimizing a quadratic form in these sample covariances. As long as there are more orthogonality conditions than parameters, the model will be overidentified and sample' statistics can be computed that allow one to make inferences under certain regularity conditions. In particular, the distance of the sample covariances from zero, as measured by a quadratic form, is asymptotically distributed as a chi-square. If we could observe real prices and returns for a number of securities, a consol, and the market, then we could compute the residuals ϵ^*_{it+1} and attempt to impose orthogonality requirements such as these to estimate the parameters associated with each security, β_i, $i = 1, \ldots, n$. Before imposing the above restrictions on the data, the following problems need to be addressed.

3.1 Absence of data for consol bonds

The model assumes agents have access to a real cons01 bond for which no obvious empirical proxy exists. The lower-dimensional structure of the model, however, implies that other securities or portfolios can substitute for the consol, while still leaving the model overidentified. Consider some other security *j*. By Theorem 2,

$$r_{jt+1} = \alpha_{jt}\left(\frac{p_{ct}}{p_{jt}}\right)(r_{ct+1}) + \beta_j\left(\frac{p_{wt}}{p_{jt}}\right)(r_{wt+1}) + \epsilon^*_{jt+1} \tag{33}$$

Algebraic rearrangement yields an expression for the quantities we cannot observe:

$$p_{ct}r_{ct+1} = \frac{p_{jt}}{\alpha_{jt}}\, r_{jt+1} - \beta_j\left(\frac{p_{wt}}{\alpha_{jt}}\right)(r_{wt+1}) - \frac{p_{jt}}{\alpha_{jt}}\,\epsilon^*_{jt+1} \tag{34}$$

We can then substitute this into the return for any security *i* to obtain

$$r_{it+1} = \left(\frac{\alpha_{it}}{\alpha_{jt}}\right)\left(\frac{p_{jt}}{p_{it}}\right)(r_{jt+1}) + \left(\beta_i - \beta_j\frac{\alpha_{it}}{\alpha_{jt}}\right)\left(\frac{p_{wt}}{p_{it}}\right)(r_{wt+1}) + \hat{\epsilon}_{it+1} \tag{35}$$

where

$$\hat{\epsilon}_{it+1} = \epsilon^*_{it+1} - \frac{\alpha_{it}p_{jt}}{\alpha_{jt}p_{it}}\,\epsilon^*_{jt+1} \tag{36}$$

Clearly, $\hat{\epsilon}_{it+1}$ is still orthogonal to time tinformation. Also, under the conditions of the Corollary to Theorem 2, when $\alpha_{it} = \alpha_i$ for all t, $\hat{\epsilon}_{it+1}$ will be orthogonal to contemporaneous values of r_{wt+1}. It will obviously not, however, be orthogonal to r_{jt+1}. Thus, if we are willing to assume the α's are constant, we can directly estimate them along with the β's in Equation (35), and the overidentifying restrictions could be regarded as a test of the conditional mean-variance efficiency of the "market portfolio." Alternatively, we can allow the a's to change through time, but infer their dynamics through some additional structure imposed by the model, namely the conditional collinearity of prices.

3.2 Inferring the dynamics for the intercept terms

Theorem 1 states that the prices for financial assets are conditionally, though not unconditionally, two dimensional:

$$p_{jt} = \alpha_{jt}p_{ct} + \beta_j p_{wt} \tag{37}$$

which implies

$$p_{ct} = \frac{1}{\alpha_{jt}}(p_{jt} - \beta_j p_{wt}) \tag{38}$$

Substituting this into Equation (8) gives (after rearrangement)

$$\frac{\alpha_{it}}{\alpha_{jt}} = \frac{p_{it} - \beta_i p_{wt}}{p_{jt} - \beta_j p_{wt}} \tag{39}$$

This expression in turn can be used to substitute for the α's in the expression for r_{it+1}, Equation (35):

$$r_{it+1} = \left(\frac{p_{it} - \beta_i p_{wt}}{p_{jt} - \beta_j p_{wt}}\right)\left[\frac{p_{jt}}{p_{it}} r_{jt+1} - \beta_j\left(\frac{p_{wt}}{p_{it}}\right)(r_{wt+1})\right] + \beta_i\left(\frac{p_{wt}}{p_{it}}\right)(r_{wt+1}) + \hat{\epsilon}_{it+1} \tag{40}$$

Equation (40) gives the form in which we actually estimate the model using ex-post real returns and requiring that the error terms $\hat{\epsilon}_{it+1}$ be unconditionally uncorrelated with instrumental variables in the information set at time *t*. Note that there is one parameter to be estimated per security, β_i, and one more which must be the same across securities, β_j. By using large portfolios sorted by Industry and firm size for the dependent variables, we can keep the number of parameters and orthogonality conditions at a computationally feasible level.

In this form the model is rather more nonlinear, and hence less intuitive, than if we were to assume real returns on a consol are observable and directly employ the expressions in Theorem 2 in our empirical tests. Nevertheless, since Equation (40) follows from these expressions, under the maintained assumptions of the model it inherits their properties; and one might expect these properties to help with the data. Expected returns in the model are inversely related to an asset's own price. The reason for this is quite simple and intuitive. When the price of an asset rises relative to the value of the market portfolio and the consol, this is due to a series of fortunate draws for the innovations in the intercept of the dividend process. Since these innovations persist, they add to expected future cash flow, but this also decreases the percentage of the firm's future cash flows that can be attributed to movements in the aggregate dividend. It is these movements that are "risky" in the sense of being correlated with the marginal utility of the representative agent. Thus, at the level of returns, the security has less "systematic" risk. A smaller portion of its cash flows are responsive to market movements, and its conditional expected return is accordingly lower.

The inverse dependence of the conditional expected return on the asset's previous price is fortuitous, given behaviors documented by DeBondt and Thayler (1985), Fama and French (1988), and Poterba and Summers (1988), and also the correlation between January returns and poor performance in previous months. The model will tend to predict that big losers have, ceteris paribus, high subsequent expected returns and vice versa.

Other aspects of the model as written in Equation (40) can reasonably be expected to capture some important regularities in asset returns. First, if a security *j* is chosen that has price changes that move with interest rates, Equation (40) ends up including the types of instruments that have typitally been employed in other empirical studies of changing risk premia. Indeed, the first term in Equation (40). which involves the return on security j plays an important and intuitive role in fitting the model to data, despite its nonlinearity. If there are persistent, idiosyncratic components to the firm's cash flows, which affect conditional expected returns beyond just scaling up their sensitivity to the market, these will be picked up through this term.

To better understand this aspect of the model consider some polar cases. First, suppose that asset i has no systematic risk and that asset j has none either, so $\beta_i = \beta_j = 0$. Then the expected return on the asset would be proportional to the expected return on asset j. Any predictable variation in asset i would be due to predictable variation in the reference asset j. Such a model would be easy to reject with, for example, industry-sorted portfolios. Alternatively, we could develop a version of the model where all the intercept terms in the dividend processes were constant and zero. This would give us Equation (40), without the nonlinear intercept, as a return-generating process:

$$r_{it+1} = \beta_i\left(\frac{p_{wt}}{p_{it}}\right)(r_{wt+1}) + \hat{\epsilon}_{it+1} \tag{41}$$

This is just a market model with a "beta" that varies according to the relative prices of the market and the asset. Since it holds when the a's are constant, the Corollary to Theorem 2 implies that under these circumstances the error should be orthogonal to the contemporaneous market return, as well as to lagged instruments. Our empirical tests show this model is rejected by the data even without using r_{wt+1} as an instrument.[6] Now consider Equation (40) with $\beta_i \neq 0$ but $\beta_j = 0$. Note that this amounts to simply treating the reference asset as if it were indeed a cons01 bond. It yields a return-generating process of the form:

$$r_{it+1} = r_{jt+1} + \beta_i\left(\frac{p_{wt}}{p_{it}}\right)(r_{wt+1} - r_{jt+1}) + \hat{\epsilon}_{it+1} \tag{42}$$

This model looks very much like an intertemporal CAPM with time-varying "betas" and asset j playing the role of the riskless asset. It allows for time variation in expected returns to come from more than one source but denies us use of any information that might come from changes in the price of asset i relative to the reference asset j. Additionally, and in contrast to Equation (40), no parameters that need to be estimated in Equation (42) are common across assets. We report tests of this model, which is also rejected, below.

Note that Equation (42) will also hold under the conditions of the

[6] This version also implies an extreme degeneracy in pricing. Theorem 1 would Imply the relative prices of the asset and the market do not change in this case.

Corollary, which assumes the α's are constant through time and shows the market portfolio is then conditionally mean-variance efficient. Consider Equation (35) with $\alpha_{it} = \alpha_i$ and $\alpha_{ft} = \alpha_f$. Clearly these two parameters are not separately identified, since they only appear as ratios. Similarly, β_i can always be adjusted, given different value for β_f, to obtain a desired value for the coefficient multiplying $(p_{wt}/p_{it}) r_{wt+1}$. Thus without loss of generality, the conditions that ensure the conditional efficiency of the market portfolio allow us to set $\beta_f = 0$, which delivers Equation (42). Additional testable restrictions are implied, however, as in this case the error must be uncorrelated with contemporaneous realizations of the market return, r_{wt+1}, as well as with lagged instruments.

4. The Data

The tests described above require time-series observations on the returns and ex-dividend prices of a set of individual assets. We need similar series for the "market portfolio," and for a benchmark portfolio to serve as an alternative to the consol bond. Finally, we require a price-level series to recover real returns and prices.

As an empirical proxy for the aggregate wealth portfolio we use the value-weighted portfolio of NYSE stocks from the CRSP database, from January 1926 to December 1984. Choosing an appropriate proxy for the ex-dividend price of this portfolio raises some issues regarding the interpretation of the model. Two possibilities present themselves: (1) using the series of ex-dividend portfolio values provided by CRSP, and (2) computing wealth relatives using the value-weighted market returns without dividends provided by CRSP. The former would seem to provide a sensible measure of "aggregate wealth." It increases as new shares are issued in response to new Investment. On the other hand, a portfolio with these prices could not actually be held without periodic infusions of cash, as well as periodic rebalancing of existing asset holdings.

With the second alternative, we obtain a measure of the value of the representative agent's wealth, assuming the dividends are "consumed" when paid. The wealth relatives on the returns without dividends would represent the actual values of a portfolio that, after being purchased at an initial date, was held to some horizon date with no intermediate investment or withdrawals aside from the dividend payments accruing to the position.

The model itself is developed in a pure-exchange setting and thus does not explicitly address the reinvestment decisions of either firms or individuals. Any reinvestment is assumed to be accounted for in the modeling of the dividend process, which allows for negative as well as positive real cash flows. What the model actually values through time is the price of one "share" of this process, and thus growth in the number of shares should not be reflected in its price, after adjustments for stock dividends and splits. Accordingly, we have chosen to employ the wealth relatives calculated from the returns without dividends as, on balance, more con-

Table 1
Descriptive statistics for size- and industry-based portfolios

	Size				
	1	2	3	4	5
Mean	.0158	.0126	.0113	.0092	.0070
Standard deviation	.0699	.0662	.0641	.0670	.0595
Correlations:					
1		.950	.926	.878	.822
2			.960	.931	.882
3				.964	.928
4					.965
Autocorrelations (lags):					
1	.232	.281	.256	.226	.189
2	.012	−.011	−.037	−.049	−.045
3	−.131	−.164	−.176	−.215	−.182
6	−.077	−.064	−.059	−.050	−.039
9	.242	.292	.274	.204	.149
18	.118	.164	.147	.129	.086

Statistics were computed from real monthly returns Feburary 1926 to December 1984 ($n = 707$). Size-based portfolios are ordered from smallest quintile firms (first portfolio) to largest quintile firms (fifth portfolio). Industry-based portfolios are constructed as follows. Portfolio 1: Mining and Construction (SIC

sistent with the spirit of the model. In summary, for the quantity r_{wt+1} in the model we employ the monthly series VWRET from the CRSP in our tests. We compute p_{wt+1} as the accumulated value at month $t + 1$ of a \$1 investment in a portfolio for which the monthly returns are given by VWRETX from CRSP, the value-weighted returns without dividends. Our construction of other return and price series also reflects this choice. Nominal returns and prices were converted to real quantities using the price-level series provided in Ibbotson (1985).

The individual assets employed in our tests are sets of five portfolios. The first set is constructed on the basis of firm size. Beginning in January 1926, all firms with continuous return records over the subsequent 10-year period were sorted on the basis of market value of equity and divided into five quintiles. For the next 10 years, monthly returns on two equally weighted portfolios of the firms in each quintile were then calculated, one from the monthly total return series and the other from the returns without dividends. Both are provided in the CRSP monthly returns files. We use the returns on the portfolio without dividends, then, to compute wealth relatives that serve as the price of the portfolio. Subsequently, in January 1936 firms were resorted and returns for the next 10 years calculated. This process was repeated through 1984. The second set of assets employed consists of five industry portfolios constructed on the basis of the SIC code. Again portfolio grouping was done at 10-year intervals starting in January 1926. Table 1 shows the five industry classifications and contains descriptive statistics for both the size- and industry-based portfolios. Portfolios are employed to keep the number of orthogonality conditions and parameters

Table 1
Extended

Industry				
1	2	3	4	5
.0119	.0111	.0103	.0122	.0110
.0770	.0638	.0564	.0605	.0783
	.920	.826	.829	.818
		.896	.907	.873
			.852	.820
				.793
.207	.245	.255	.215	.250
−.026	−.029	.006	−.105	−.030
−.114	−.192	−.089	−.199	−.148
−.078	−.058	−.105	−.037	−.049
.212	.235	.291	.238	.192
.054	.140	.156	.217	.108

codes 10-19); Portfolio 2: Manufacturing, Transportation, Communications, and Utilities (SIC codes 20-49); Portfolio 3: Trade (SIC codes 50-59); Portfolio 4: Finance, Insurance, and Real Estate (SIC codes 60-69); Portfolio 5: Services (SIC codes 70-89).

to a manageable number and also, hopefully, to increase the power of the tests by reducing residual variance.[7]

The conditional collinearity of prices implied by the model means any asset with $\alpha_{jt} \neq 0$ can substitute for the consol return in the pricing equation. We choose the long-term government bond portfolio constructed in Ibbotson (1985). The same source reports a capital appreciation index for the portfolio, which we use for its "ex-dividend" price p_{jt}. Keep in mind that the bonds in the portfolio are turning over. As bonds mature new bonds are purchased with the proceeds, and the computed returns and capital appreciation series account for this reinvestment. Thus, while it is obvious that individual bonds do not have real payments that could be reasonably approximated by the dividend process assumed by the model, it may not be unreasonable to assume that the portfolio does make such payments.

In fact, while we do not view this as an appropriate "test" of the model, informal analysis of the bond portfolio's real coupon income and the real dividend on the value-weighted index are reassuring on these dimensions. We compute the real dividend or coupon as follows:

$$d_{jt+1} = \left[\prod_{s=1}^{t} (1 + r^*_{js})\right](1 + r_{jt+1}) - \prod_{s=1}^{t+1} (1 + r^*_{js}) \qquad (43)$$

[7]The results of Lehmann and modest (1985) suggest that, in tests of the static models, the loss in power due to idiosyncratic risk more than offsets the gains due to less within-group variability when the number of portfolios is increased beyond five.

where r^{c}_{jt} and r_{jt} are the ex-post real rates of capital appreciation and total return, respectively. We constructed a similar series for the market index, denoted d_{wt+1}, and then temporally aggregated both monthly series to quarterly dividends and coupon income to eliminate the obvious quarterly seasonal in dividend payouts on the market index. An ordinary least squares regression of d_{jt+1} on d_{wt+1} yields residuals that display autocorrelations that are positive and decay very slowly, suggesting nonstationarity. The Durbin-Watson statistic is 0.539. This is consistent with the assumed dividend process if α_{jt} is not constant. After differencing the variables once, however, the regression should yield an MA(1) error that should not show obvious evidence of serial correlation over more than one lag. As a quick check on this, we simply omitted first even- then odd-numbered quarters from the series of first differences and ran OLS. The Durbin-Watson statistics for these regressions were 1.888 and 1.541, and any obvious evidence of serial correlation appeared to have been purged from the autocorrelation function.[8]

Equations (40) to (42) all involve variables that one might reasonably expect to be nonstationary. The estimation procedure we employ, GMM, requires that the forecast error in the model be both stationary and first-moment continuous in the parameters. The latter restriction would fail, for example, in a model involving co-integrated variables in which the error is stationary only at the parameter values associated with the null hypothesis, while its variance explodes with slight deviations from those values. Some algebraic manipulations yield the following form for Equation (40), which provides some reassurance concerning these questions.

$$r_{it+1} = A_t\left[r_{jt+1} - \beta_j\left(\frac{p_{wt}}{p_{jt}}\right)(r_{wt+1}) - \epsilon^{*}_{jt+1}\right] + \beta_i\left(\frac{p_{wt}}{p_{it}}\right)(r_{wt+1}) + \epsilon^{*}_{it+1} \quad (44)$$

where

$$A_t = \frac{(p_{it} - \beta_i p_{wt})/p_{it}}{(p_{jt} - \beta_j p_{wt})/p_{jt}} \quad (45)$$

A_t is of course not observable, but there appears to be no obvious reason for it to be nonstationary. It is clearly stationary at $\beta_i = \beta_j = 0$, and with both parameters nonzero but of the same sign nonstationary movements in p_{wt} will affect both the numerator and denominator of A_t. We have computed the sample autocorrelations of the terms in Equation (44) that involve price ratios multiplied by r_{wt+1} for all 10 portfolios in our data set and for the long-term government bond portfolio. These all exhibit significant autocorrelations, but none display the extreme persistence typically associated with a unit root process.

[8] At various stages in this research we have employed other reference assets, including ATT common and the large-firm portfolio. Generally speaking this left the qualitative results unchanged, although at times we encountered numerical problems due to the high degree of collinearity between these alternatives and the market index

5. Empirical Results

The pricing model developed above imposes, first, the restriction that the residuals in Equation (40), $\hat{\epsilon}_{it+1}$, are one-step-ahead forecast errors. They should accordingly be unconditionally uncorrelated with any variables known at date *t*. If, in addition, the conditions of the corollary hold and the a's are constant through time, then returns on the consol bond and the market are conditionally mean-variance efficient at each point In time. In this case, the residuals should also be unconditionally uncorrelated with realizations of the return on the market portfolio. Finally, the first term in Equation (40) is highly nonlinear in the parameters, and in an attempt to develop some intuition about its role in the evolution of risk premia in the model we considered some special cases. Equation (41) treats this term as constant and zero. Equation (42) is implied by the model if we simply treat the reference asset as if it were indeed a consol. The tests reported below explore all these restrictions.

First, however, it seems reasonable to ask about the ability of this data to generate tests with GMM powerful enough to reject the static CAPM. Accordingly, we first estimated the following equation:

$$r_{it+1} - r_{ft+1} = \hat{\beta}_i(r_{mt+1} - r_{ft+1}) + e_{it+1} \tag{46}$$

where for the one-period riskless rate, r_{ft+1}, we use the Treasury-bill returns from Ibbotson (1985). The tests are done with ex-post real returns, and the instruments employed are a constant, the market excess return at time $t + 1$, and a dummy variable for January. Using GMM in this setting is equivalent to running a market-model regression for excess returns forcing a zero intercept and allowing for heteroskedastic errors. For each set of five portfolios we estimate the model in Equation (46) as a system. In each system there are five parameters (the market betas) and 15 orthogonality conditions, since the error for each portfolio is required to be uncorrelated with three instruments. The chi-squared statistic, which measures the distance of these 15 sample covariances from zero, has 10 degrees of freedom. For the industry-based portfolios the chi-squared statistic is 25.8733. Under the null hypothesis the probability of a chi-squared statistic as large or greater than this is .004. For the size-based portfolios the chi-squared statistic and probability values are 21.343 and .019, respectively. We also tested Equation (46) using instruments more directly comparable to those we actually employ in the tests of our model, namely, the lagged return on the market index, the first and third lag of the portfolio's own return, a January dummy, and a constant. Given the relatively low autocorrelations in financial asset returns one might suspect that all the power to reject the static CAPM is due to the use of the contemporaneous market return. This does not appear to be the case. Using only these instruments, which are known at the beginning of each period, the chi-squared statistic for the industry portfolios still rejects at the 1 percent level. The chi-squared statistic for the size-based portfolios, however, is only significant at the 10 percent level. In summary, the data and the instruments provide substantial evidence against the static CAPM, though the industry-based grouping appears to lead to more powerful tests than does the size grouping.[9] These results are not advanced to provide a systematic study of the static CAPM, which would involve more detailed analysis of subperiods, but rather as a rough benchmark with which to compare the dynamic model under study here and demonstrate that our data generates results similar to previous findings. For example, Gibbons, Ross, and Shanken (1989) also report more evidence of a January effect for portfolios grouped by industry codes.

Table 2 shows the central empirical results of the article, the estimation of the dynamic model with time-varying intercepts, Equation (40), for both the size- and industry-based portfolios. In panel A four instruments are employed for each portfolio. These include the constant vector (since the unconditional means of the residuals should be zero), the lagged return on the market portfolio; and the return on the portfolio itself, at lags one and three. The third lag is included because of the evidence in Table 1 that this autocorrelation tends to be fairly large. All of these variables are in the information set at date t and should therefore be unconditionally orthogonal to the residuals simply because they are forecast errors, Thus, for each set of portfolios there are six parameters to be estimated, the β's for each of the five portfolios and for the reference asset. There are four orthogonality conditions per portfolio, leaving us with an asymptotic chi-square which has 14 degrees of freedom. In panel B a January dummy is added to the instrument list.

Table 2 shows that the model performs quite well. The parameter estimates are all significant at the 1 percent level. For the size-based portfolios the β's tend to be close to 1, while for the industry-based portfolios they show more variation. Thus, for these instruments the model is not rejected by the data despite its apparent ability to estimate the parameters tightly. This presents a sharp contrast to a number of other studies of dynamic asset pricing models, in particular those which employ consumption data. The results do not change appreciably when the January dummy is included as an instrument. Since our model predicts expected returns are inversely related to price, and since the January effect is largely driven by the behavior of big losers in the previous year, one might hope that the model could perform relatively well with regard to this feature of the data. We also estimated the model using data from 1926-1978 employing the "junk bond spread," Ibbotson's (1980) junk bond series less the long-term government bond series, as an additional sixth instrument. The model was not rejected at standard significance levels for either portfolio group. The failure to reject the model does not appear to be due to solely a lack of power in

[9] The same tests without the January dummy as an instrument do not lead to rejection, consistent with the results of Gibbons, Ross, and Shanken (1989). Tests of the CAPM, without January as an instrument but using raw returns and restricting the intercept for each portfolio to be proportional to unity minus its beta times the riskless rate lead to rejection at the 1 percent level for the she-based portfolios and at the 10 percent level for the industry-based portfolios.

Table 2
Estimates of basic model with and without the January dummy as an instrument

	Parameter							Prob
	β_j	β_1	β_2	β_3	β_4	β_5		$(\chi^2 < X)$
	Panel A: January dummy excluded						χ^2 (14)	
Size-based portfolios	1.0035 (.0029)	1.0011 (.0934)	0.7953 (.0670)	0.7839 (.0658)	1.0916 (.0639)	1.1058 (.0296)	7.9086	0.8940
Industry-based portfolios	1.0000 (.0008)	0.6468 (.0114)	0.8487 (.0128)	0.7663 (.0079)	1.1423 (.0184)	0.7012 (.0234)	16.8315	0.2653
	Panel B: January dummy included						χ^2 (19)	
Size-based portfolios	0.9260 (.0070)	1.2354 (.2055)	0.8447 (.0966)	0.8338 (.0977)	1.1278 (.0723)	1.0278 (.0426)	21.7940	0.9246
Industry-based portfolios	1.2765 (.0025)	1.1909 (.1678)	1.5093 (.1931)	1.6355 (.1912)	2.4620 (.3162)	0.5987 (.0915)	14.9501	0.7258

The equation estimated is

$$r_{it+1} = \left[\frac{p_{it} - \beta_i p_{mt}}{p_{it} - \beta_j p_{mt}} \frac{p_{pt}}{p_{it}} (r_{pt+1}) - \beta_j \left(\frac{p_{mt}}{p_{it}}\right)(r_{mt+1})\right] + \beta_i \left(\frac{p_{mt}}{p_{it}}\right)(r_{mt+1}) + \hat{\epsilon}_{it+1} \tag{40}$$

Estimation uses real monthly returns from April 1926 to December 1984 (n = 704). Chi-squared statistics test the hypothesis that the errors are orthogonal to the first and third lag of the portfolios' own returns, the lagged return on the CRSP value-weighted Index, and a constant. In panel B a January dummy is added to the instrument list. Standard errors of parameters are in parentheses. p_{mt} and r_{mt+1} denote the price and return on the market portfolio (due-weighted CRSP Index). p_{pt} and r_{pt+1} denote the price and return on the reference asset [long-term government bond portfolio from Ibbotson (1985)]. Size-based portfolios are ordered, from smallest quintile firms (first portfolio) to largest quintile firms (fifth portfolio). Industry. based portfolios are as follows. Portfolio 1: Mining and Construction; Portfolio 2: Manufacturing, Transportation, Communiations, and Utilities; Portfolio 3: Trade Portfolio 4: Finance, Insurance, and Real Estate; Portfolio 5: Services. All parameter estimates are significant at the 1 percent level. Initial values employed are row 1: 0.9945. 1.0477, 0.9739, 0.9210, 1.1115, 1.0503; row 2: 0.9999, 0.6394, 0.8278, 0.7520, 1.1093, 0.7066; row 3: 0.9347, 1.3143, 0.8937, 0.9011, 1.1913, 1.0912; row 4: 1.3000, 0.6360, 0.7937, 0.8969, 1.1312, 0.0500.

our instruments. The tests reported below do reject special cases of the model using the same data and instruments.[10]

One feature of Table 2 deserves additional comment. The standard errors for β_j in both tables appear to be almost absurdly small. The t-statistics for β_j vary across 'the four rows of the table from 132 to 1250. This seems surprising, given the way β_j enters Equation (40). Though it is identified theoretically, it appears only in terms that involve ratios or differences with β_i, so that one might expect to have difficulty identifying β_j in a particular sample. Figures 1 and 2 help to resolve this conundrum. Using the five

[10] Tests reported in Bossaerts (1988) could be viewed as additional evidence in favor of our model. Let $\alpha_{it}^{*} = \Sigma_{\tau=1} \nu_{i\tau}$ and rewrite Equatlon (8) as follows:

$$p_{it} = \alpha_{i0} p_{ct} + \beta_i p_{mt} + \alpha_{it}^{*} p_{ct} = \alpha_{i0} p_{ct} + \beta_i p_{mt} + \xi_{it}$$

ξ_{it} is deady nonstationary. Using the same data set as in this article. Boasaerts (1988) tests the nonstationarity of E. explicitly assuming that p, and p, are unit root processes, and cannot reject it. It is difficult, however, to assess the significance of this finding for the validity of the present model, since the test in Bossaerts (1988) is constructed under the null hypothesis that ξ_{it} is a unit root process with weakly dependent Increments. That assumption may he violated in the present context, which would suggest ξ_{it} is the product of two unit root program. Further research should clarify the relationship between the present article and Bossaerts (1988).

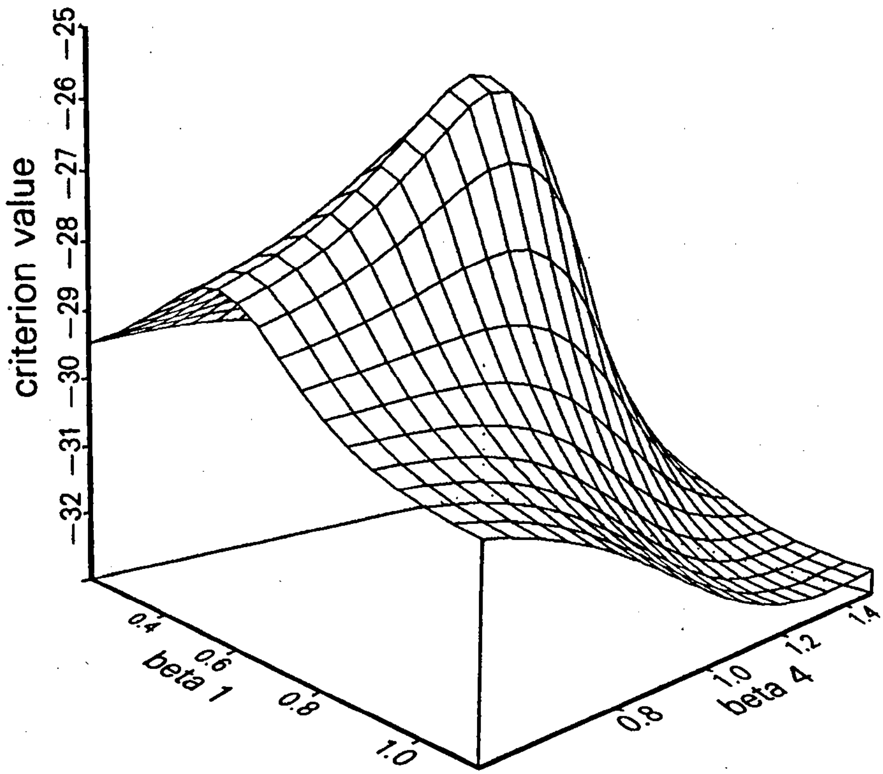

Figure 1
Plot of the negative of the chi-squared statistic against the betas for portfolios 1 and 4
The plot Illustrated that the criterion function is well behaved in these parameters around a local minimum. The estimation procedures employed In the article will choose the values at which the surface attains its maximum.

instruments from panel B of Table 2, we computed chi-squared statistics for different values of the parameters. The weighting matrix was constructed from the errors in Equation (40) at the assumed parameter values.[11] Figure 1 plots these against different values of β_1 and β_4 for the industry-based sample, holding the other parameters constant at their values at a particular local optimum. We chose to plot the negative of the chi-squared statistic to better illustrate the behavior in the region of an optimum, so

[11] The results from such a “grid search” arc not directly comparable to those from the usual two-step procedure, in which the optimum from a first minimization, which uses the identity matrix as weighting matrix, forms the basis for calculating the weighting matrix In a second minimization. Since both procedures use consistent estimates of the weighing matrix, they will provide consistent estimates of the parameters, as well as a quadratic form that is asymptotically chi-squared distributed. The “grid search” procedure is somewhat like maximum-likelihood analysis. In order to obtain correct maximum-likelihood estimates, however, other Instruments should have been used. Maximum-likelihood estimation is very complicated here since some of the explanatory variables In Equation (40) arc nonstationary. Moreover, the parametera of the stochastic process of these variables depend on the parameters In Equation (40), so that it Is inappropriate to just use the conditional-likelihood function.

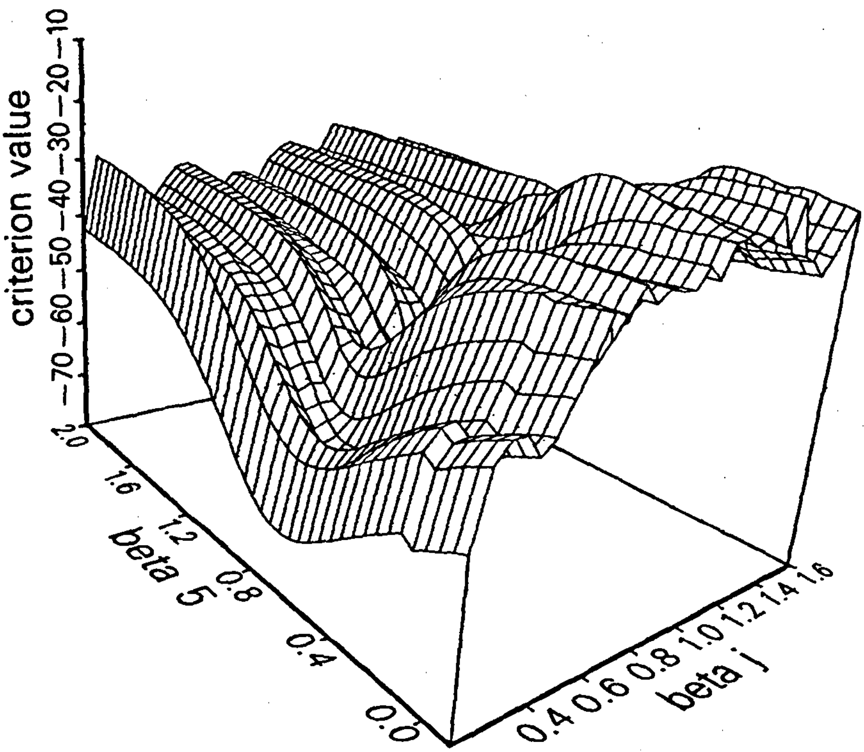

Figure 2
Plot of the negative of the chi-squared statistic against the beta for a portfolio and the beta of the reference asset
The plot illustrates that the criterion function is ill behaved in the beta for the reference asset. It has many local optima, and the slope of the surface in the neighborhoods of these optima is extremely steep. The beta for the reference asset is therefore difficult to estimate precisely, despite its low reported standard errors.

minimal values of the chi-squared statistic correspond to maxima in the plots. The criterion function appears very well behaved in Figure 1. Evidently, it is a simple matter numerically to estimate the relative values of these two parameters with precision. In contrast, the shape of the function in Figure 2, which plots the criterion function in the same way against β_5 and β_j, is highly irregular. It exhibits many local optima. In this space the numerical optimization will clearly be sensitive to initial conditions. Nevertheless, the surface has a steep slope near the various local optima in the directions parallel to the β_j axis. This leads to a very low standard error for β_j, despite the apparent numerical difficulties in pinning down its value. The local steepness of the criterion function in β_j is presumably due to the fact that this is the only parameter that is common across equations. Holding the other parameters constant, a slight change in the β associated with one of the portfolios increases the chi-squared statistic, but only through its effect on the sample covariances between the Instruments and

Table 3
Estimates of model with intercept term constrained

	Parameter						Prob
	β_1	β_2	β_3	β_4	β_5	χ^2 (20)	($\chi^2 < X$)
Panel A: $r_{it+1} = \beta_i \left(\frac{p_{mt}}{p_{it}}\right)(r_{mt+1}) + \hat{\epsilon}_{it+1}$ ($\alpha_i = 0$)							
Size-based portfolios	6.0964 (1.8495)	3.3112 (.8176)	3.2752 (.7031)	2.1000 (.2027)	1.3309 (.0537)	24.7899	0.2096
Industry-based portfolios	3.6590 (.8426)	3.6791 (.7377)	3.5574 (.7449)	5.4445 (1.1998)	2.3368 (.3522)	44.2337	0.0014
Panel B: $r_{it+1} = r_{ft+1} + \beta_i \left(\frac{p_{mt}}{p_{it}}\right)(r_{mt+1} - r_{ft+1}) + \hat{\epsilon}_{it+1}$ ($\beta_f = 0$)							
Size-based portfolios	2.5355 (.6117)	4.0425 (.6496)	5.5959 (.9566)	3.0095 (.2967)	1.3758 (.0632)	76.6937	<0.0001
Industry-based portfolios	2.6363 (.8286)	2.9767 (1.1161)	3.0331 (1.3131)	4.3834 (1.9579)	2.2450 (.5668)	37.5215	0.0101

Estimation uses real monthly returns from April 1926 to December 1984 (n - 704). Chi-squared statistics test the hypothesis that the errors are orthogonal to the first and third lag of the portfolios' own returns, the lagged return on the CRSP value-weighted index, a January dummy, and a constant. Standard errors of parameters are In parentheses. p_{mt} and r_{mt+1} denote the price and return on the market portfolio (value-weighted CRSP index). p_{ft} and r_{ft+1} denote the price and return on the reference asset [tong-term government bond portfolio from Ibbotson (1985)]. Size-based portfolios are ordered, from smallest quintile firms (first portfolio) to largest quintile firms (fifth portfolio). Industry-based portfolio are as follows. Portfolio 1: Mining and Construction; Portfolio 2: Manufacturing, Transportation. Communications, and Utilities; Portfolio 3: Trade; Portfolio 4: Finance, Insurance, and Real Estate; Portfolio 5: Services. All parameter estimates are significant at the 1 percent level. Initial values employed are rows 1 and 3: 1.3143, 0.8937, 0.9011, 1.1913, 1.0912; rows 2 and 4: 0.9251, 1.1304, 1.0342, 1.5781, 0.9044.

the errors for that particular portfolio. A slight change in β_f, however, affects all the orthogonality conditions simultaneously. Hence it has a much more dramatic effect on the chi-squared statistic, and this is reflected in its t-statistics. Our experiments with different initial conditions were consistent with this interpretation, and suggest that β_f is in fact difficult to Identify in this sample. Changes in initial conditions lead to minima at which the relative values of β_1–β_5 change very little. Across these local minima, however, their absolute values, as a group, and the absolute value of β_f tend to be very sensitive to initial conditions. Indeed, this is evident from examining the two panels of Table 2, where the ordinal magnitudes of the portfolio β's across the two panels are similar, though the absolute values are generally higher in panel B.

Table 3 shows the results of estimating some restricted versions of the model, using the same instruments as were employed in Table 2 including the January dummy. This allows us to evaluate whether our instruments have any power and also provides some insight into how particular features of the model add to its ability to fit the data. For example, the persistence in the intercept of the assumed model for the securities' cash flows allows for returns to have predictable components not attributable directly to the market's return and price. To assess the importance of this we estimate, in panel A of Table 3, Equation (41), which results from assuming all the intercept terms are zero in the cash flow streams. This increases the chi-squared statistics for both sets of portfolios. For the industry-based portfolios the model is rejected at the 1 percent level. Further, the parameter estimates seem unreasonably high, given that the model predicts the average β is unity. The same could be said of the estimates in panel B. The equation used here results from setting $\beta_j = 0$, so that the information associated with the relative price of the market portfolio and the reference asset is lost. This model is rejected at conventional significance levels for both sets of portfolios. Thus, simply introducing the reference asset's return does not seem to reduce the predictability of the error term sufficiently. As noted above this version of the model is Implied by the conditions of the corollary to Theorem 2, which implies the market portfolio is mean-variance efficient. We could accordingly add the contemporaneous market return as an instrument, but given the clear rejections obtained without it, we do not report these results in detail.[12]

6. Conclusions

In this article we have developed and tested a dynamic, discrete-time general equilibrium model in which market risk premia and the riskiness of individual securities change through time in predictable ways. The sources of these changes, however, can be attributed to variations in the prices and expected returns of two "funds," the market or aggregate wealth portfolio and a riskless cons01 bond. Any components of returns not attributable to movements in these variables should be unpredictable. This leads to very natural tests of the model using the generalized method of moments.

We interpret the results reported above as, in general, supportive of the model's ability to summarize the time variation in risk premia for equities in a parsimonious fashion. Our tests of more restrictive versions of the model and the static CAPM with the same techniques and instruments suggest that our failure to reject is not simply a matter of the power of the tests. Indeed, our instruments included series with documented predictive power in similar settings, such as the January dummy and the junk bond premium. These tests also illustrate some of the sources of the model's explanatory power, in particular the importance of allowing for persistence in the idiosyncratic shocks to firms within the model.

References

Bhattacharya, S., 1981, "Notes on Multiperiod Valuation and the Pricing of *Options," Journal of Finance,* 36, 163-180.

Bossaerts, P., 1988, "Common Nonstationary Components of Asset Prices," *Journal of Economic Dynamics and Control* 12, 347-364.

[12] We have estimated the model directly under these conditions by assuming the α's in Equation (35) are constant and estimating the five which are Identified using the contemporaneous market return as an tnstrument. This approach also leads to rejection of the model at high levels of significance.

Breeden, D. T., 1979, "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," *Journal of Financial Economics* 7, 265-296.

Brennan, M., 1973, 'An Approach to the Valuation of Uncertain Income Streams," *Journal of Finance,* 28, 661-674.

Chen, N. F, and J. E. Ingetsoll, 1983, "Exact Pricing in Linear Factor Models with Finitely Many Assets: A *Note," Journal of Finance,* 38, 985-988.

Connor, G., 1984, "A Unified Beta Pricing Theory," *Journal of Economic Theory,* 34, 13-31.

Connor, G., 1985, "An Intertemporal Equilibrium Beta Pricing Model," working paper, Kellogg Graduate School of Management, Northwestern University.

Connor, G., and R. A. Korajcayk, 1987, "An Intertemporal Equilibrium Beta Pricing Model," working paper, Kellogg Graduate School of Management, Northwestern University, August.

Cox. J. C., J. E. Ingersoll, and S. A. Ross, 1985, "An Intertemporal Genenl Equilibrium Model of Asset Prices," *Econometrica* 53, 363-384.

DeBondt. W. P. M., and R. Thayler, 1985. "Does the Stock Market Overreact?" *Journal of Finance,* 40, 793-805.

Pama, E. F., and K. R. French, 1988, "Permanent and Temporary Components of Stock prices." *Journal of Political Economy,* 96, 246-273.

Person, W. E., 1986, "Changes in Expected Risk Premiums and Security Risk Measures," working paper, Graduate School of Business, University of Chicago.

Gennotte, M. R., and T. A. Marsh, 1985, "'Variations in Ex-ante Risk Premiums on Capital Assets," working paper, M.I.T. Sloan School of Management.

Gibbons, M. R., and W. Person, 1985, "Testing Asset Pricing Models with Changing Expectations and an Unobservable Market Portfolio," *Journal of Financial Economics,* 14, 217-236.

Gibbons, M. R.. S. A. Ross, and J. Shanken, 1989, 'A Test of the Efficiency of a Givcn Potrfolio," *Econometrica,* 57, 1121-1153.

Hansen, L P., 1982, 'Large Sample Properties of Generalized Method of Moments Estimaton." *Econometrica,* 50, 1029-1054.

Hansen, L P., and R. J. Hodrick, 1983, "Risk Averse Speculaion In the Forward Foreign Exchange Market: An Econometric Analysis of Linear Models," in J. A. Frenkel (ed.), *Exchange Rates and International Economics,* University of Chiago Press, Chiago, Ill.

Hansen, L. P., and S. F. Richard, 1987, "The Role of Conditioning Information In Deducing Testable Restrictions Implied by Dynamic Asset Pricing Models," *Econometrica* 55, 587-613.

Hansen, L P., and K. J. Singleton, 1982, "Generalized Instrumental Variables Estimation of Non-linear Rational Expectations Models," *Econometrica,* 50,1269-1286.

Ibbotson, R. G., 1980, *Corporate Bond Study 1926-1978,* R. G. Ibbotson and Co., Chicago, Ill.

Ibbotson Associates. 1985, *Stocks, Bonds, Bills and Inflation: 1985 Yearbook,* Chicago, Ill.

Jagannathan, R., and S. Viswanathan, 1988, "Linear Factor Pricing, Term Structure of Interest Rates and the Small Firm Anomaly," Working Paper 57, Department of Finance, Kellogg Graduate School of Management, Northwestern University, September.

Keim, D. B., and R. F. Stambaugh, 1986, "Predicting Returns in the Stock and Bond Markets," *Journal of Financial Economics,* 17, 357-391.

Lehmann, B., and D. Modest, 1985, "The Empirical Foundation of the Arbitrage Pricing Theory I: The Empirical Tests," Stanford University Research Paper 821, forthcoming in *Journal of Financial Economics*

Lucas, R. E., 1978, "Asset Prices in an Exchange Economy," *Econometrica,* 46, 1429-1445.

Merton, R. C., 1973, "'An Intertemporal Capital Asset Pricing Model," *Econometrica,* 41, 867-887.

Poterba, J.M., and L.H. Summers, 1988, "Mean Reversion in Stock Prices: Evidence and Implications," *Journal of Financial Economics,* 22, 27-60.

Rosenberg, B., and J.A. Ohlson, 1976, "The Stationary Distribution of Returns and Portfolio Separation In Capital Markets: A Fundemental Contradiction," *Journal of Financial and Quantitative Analysis,* 11, 393-401,

Ross, S., 1978, "Mutual Fund Separation in Financial Theory - The Separating Distributions," *Journal of Economic Theory,* 17, 254-286.

Rubinstein, M., 1981. “A Discrete-Time Synthesis of Financial Theory,” in H. Levy (ed.), *Research In Finance* (vol. 3), JAI Press Inc., pp. 53-102.