Mean Reversion in Short-Horizon Expected Returns

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This article develops and estimates a simple model for monthly expected stock returns that relies on the rapidly decaying structure of shorter-horizon (weekly) expected returns. The most striking aspect of our findings is that the rapid mean reversion in short-horizon expected returns implies much greater variation through time in monthly expected returns than has been documented in earlier studies. For instance, during the 1962 to 1985 period, over 25 percent of the return variance of small firms can be explained by time variation in expected returns.

There is substantial evidence that expected returns vary through time. However, the conclusion of most studies is that the predictable component of short-horizon, usually monthly, returns explains only a small part (typically less than 3 percent) of return variances [see, e.g., Keim and Stambaugh (1986), Kaul (1987), and French, Schwert, and Stambaugh (1987)]. On the other hand, recent work by Fama and French (1988) and Poterba and Summers (1988) shows that time variation in expected stock returns accounts for substantial proportions (in excess of 30 percent) of return variance for holding periods greater than one year.

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In a recent paper, Conrad and Kaul (1988) show that weekly expected returns also exhibit substantial time variation. Using an autoregressive model of expected returns they find that the time variation in expected returns explains up to 26 percent of weekly return variances over the 1962 to 1985 period. More importantly, the variation in weekly expected returns appears to decay rapidly and is, therefore, quite different from (and perhaps unrelated to) the slow movements in long-horizon expected returns documented by Fama and French (1988) and Poterba and Summers (1988). Similar inferences about the mean reversion in weekly expected returns can be made from the variance ratios reported by Lo and MacKinlay (1988).

In this article, we develop a simple model for *monthly* expected returns that relies on the rapidly decaying structure of weekly expected returns. We propose that, because of the nature of the weekly returns process, the most recent weekly returns will have more information about the subsequent month's returns than the most recent monthly returns. We obtain monthly expected return estimates that are based on the observed rapid mean reversion in weekly expected returns and explore their statistical properties. In particular, we test (1) whether the expected return estimates are unbiased predictors of subsequent realized returns, (2) whether they explain a substantial percentage of return variances, and (3) whether the forecast errors are uncorrelated. Finally, we document the behavior of expected returns both over time and across portfolios containing securities of different market values.

The basic data are excess returns on five size-based portfolios over the 1962 to 1985 period. The evidence indicates that the mean reversion in shorter-horizon (weekly) expected returns implies substantial variation in monthly expected returns. For the overall period, over 25 percent of the small-firm return variance can be explained by variation through time in expected returns. There is also a dramatic change in the degree of expected return variation over time. The 1970s exhibited a much higher degree of time variation in expected returns compared to the 1960s and 1980s. For instance, for small firms, changes in estimated expected returns explain about 18 percent of return variance during the 1960s and about 11 percent in the 1980s, but this proportion was in excess of 40 percent during the 1970s. This phenomenon is common to all except the largest size portfolio. Finally, there is a monotonic relation between the size-rankings of the portfolios and the relative time variation in monthly expected returns: The smaller-sized firms exhibit the maximum variation. This systematic relation holds for each of the three subperiods, and corroborates the findings of Conrad and Kaul (1988) and Fama and French (1987).

Section 1 describes the model; the empirical evidence is presented in Section 2. Section 3 contains a brief summary and conclusions.

1. The Model

The choice of a particular model for the time-varying behavior of expected returns is, by its nature, somewhat arbitrary. Existing asset pricing models place few, if any, a priori restrictions on the behavior of expected returns over time. In a recent paper, Conrad and Kaul (1988) proposed a first-order autoregressive process for weekly expected returns:

$$R_t^w = E_{t-1}(R_t^w) + \epsilon_t^w \tag{1}$$

$$E_{t-1}(R_t^w) = \phi E_{t-2}(R_{t-1}^w) + u_{t-1}^w \tag{2}$$

where

R_t^w = the continuously compounded return on a security for week t

$E_{t-j}(R_{t-j+1}^w)$ = the expected return on the security for week $t-j+1$ as assessed at the end of week $t-j$

$\epsilon_t^w \sim \text{iid } N(0, \sigma_\epsilon^2)$

$u_t^w \sim \text{iid } N(0, \sigma_u^2)$

and $0 < \phi < 1$.

If expected returns are represented by the process in Equation (2), we can write realized returns as

$$R_t^w = \phi R_{t-1}^w + \epsilon_t^w - \phi\epsilon_{t-1}^w + u_{t-1}^w \tag{3}$$

which, in turn, implies that realized returns can be characterized by an ARMA (1,1) process of the form:

$$R_t^w = \phi R_{t-1}^w + a_t^w - \theta a_{t-1}^w \tag{4}$$

Conrad and Kaul (1988) use the Kalman filter technique to estimate the parameters of the model. Their results show that expected returns are well-characterized by a stationary autoregressive process: Estimates of ϕ vary inversely with the size rankings of the portfolios and are typically between 0.65 and 0.40. These estimates of ϕ indicate rapidly decaying variation through time in weekly expected returns.

Given the process governing weekly expected returns, we can obtain forecasts of four-week returns based on the model for weekly returns in Equations (1) and (2). Define a one-month continuously compounded return, R_t^m, as

$$R_t^m = \sum_{j=0}^{3} R_{t-j}^w \tag{5}$$

where R_{t-j}^w = the continuously compounded return for week t - j. Then, using Equation (2), the conditional expected (monthly) return can be written as

$$E_{t-4}(R_t^m) = E_{t-4}\left(\sum_{j=0}^{3} R_{t-j}^w\right) = (1 + \phi + \phi^2 + \phi^3)E_{t-4}(R_{t-3}^w) \tag{6}$$

Hence, we can obtain estimates of monthly expected returns based on the expected return for the first week of a particular month, which, in the filter, uses information up to the last week of the previous month.

Using Equation (4), we can also express the conditional expected return for a particular month in terms of past weekly realized returns:

$$E_{t-4}(R_t^m) = \pi_1 R_{t-4}^w + \pi_2 R_{t-5}^w + \pi_3 R_{t-6}^w + \dots \tag{7}$$

where

$$\pi_i = \theta^{i-1}(\phi - \theta)(1 + \phi + \phi^2 + \phi^3) \qquad i = 1, 2, 3, \dots$$

Equation (7) demonstrates that as long as weekly expected returns exhibit mean reversion, better forecasts of monthly returns can be obtained by using the information structure in past weekly returns rather than past *monthly* returns. The basic intuition is that the quality of time-series forecasts are a function of the weights placed on past observations, and monthly time-series models put the same weights on all weeks within a particular month. However, if there is rapid mean reversion in weekly expected returns, the weights placed on past weeks should decay exponentially (and rapidly) because the most recent week(s) of a particular month contains a disproportionately large amount of information (compared to the earlier weeks comprising the month) about the subsequent month's return.

We use Equation (7) to obtain estimates of monthly expected returns. For convenience, in the presentation of the model we used continuously compounded returns to derive the monthly expected returns. However, in the tests reported in Sections 2 and 3 we use simple returns. The use-of simple returns in Equation (7) does not appear to bias the results because the replication of all the tests using continuously compounded returns reveals no significant differences.

2. The Evidence

2.1 Data description

We use the Center for Research in Security Prices (CRSP) daily return file, which lists data for both the AMEX and NYSE, to calculate weekly and monthly portfolio returns. In principle, we could estimate the model in Section 1 using daily data. However, we choose a one-week sampling interval as the basic (shortest-horizon) measurement period to estimate the model because daily data have potential biases associated with infrequent trading, the bid-ask effect, etc. We use portfolio returns, rather than individual security returns, because it is much more difficult to extract the (expected return) signal from noisy weekly returns of a single security.

At the end of each year, stocks are sorted into five portfolios based on market value (number of shares outstanding times price per share). For each week (Wednesday close to Wednesday close) of the following year, one-week simple returns are calculated for each security. Weekly holding-period security returns in each portfolio are value-weighted to form five series of portfolio returns. Each portfolio contains over 400 firms. We then calculate excess returns of each portfolio by subtracting the return on a one-week Treasury bill from the portfolio returns.

We use the same sample and portfolio formation procedure to calculate monthly excess returns of the five value-weighted portfolios. Specifically, we first calculate simple returns of each security for each calendar month using the CRSP daily return file. We then construct value-weighted monthly portfolio returns. This procedure minimizes the biases present in the daily rebalancing approach to calculating portfolio returns. [See Blume and Stambaugh (1983) and Roll (1983) for a discussion of the biases associated with alternative methods of computing returns.] In calculating monthly excess returns we use one-month Treasury-bill returns from the Fama files in the CRSP bond database. Finally, we also calculate daily portfolio returns to present some descriptive statistics for comparison with the characteristics of weekly and monthly data. (Hereafter, the word return implies "excess" return, unless otherwise specified.)

2.2 Autocorrelations

Table 1 shows the summary statistics for the daily, weekly; and monthly portfolio returns for the 1962 to 1985 period. For weekly returns, the first-order autocorrelations are large and significant and the higher-order autocorrelations (though significant) decay across longer lags. The weekly autocorrelation structure also displays a consistent pattern as we go from the smallest portfolio (RW1) to the largest (RW5): The magnitude and persistence of the autocorrelations decline monotonically. However, higher-order autocorrelations remain significant for all but the largest portfolio (which exhibits only first-order autocorrelation). Because we use value weights in forming portfolios, the autocorrelation structure of the largest portfolio is dominated by the return characteristics of the largest market value firms. For example, a portfolio of the largest 100 market value firms on the NYSE and AMEX exhibits virtually no autocorrelation.

The daily portfolio returns (RD1-RD5) exhibit characteristics similar to the weekly data, but both the magnitude and persistence (at longer lags) of the estimated autocorrelations are of a higher order. Note that even the largest market value portfolio exhibits high first-order autocorrelation. On the other hand, the monthly portfolio returns (RM1-RM5) exhibit significant autocorrelations only at the first lag. All higher-order serial correlations are close to zero. However, the magnitude of the first-order autocorrelations declines systematically with the size-rankings of the portfolios as in the daily and weekly data. The monthly returns of the largest portfolio behave like white noise.

Finally, to gauge the time-varying behavior of the size effect we also

[1] Lo and MacKinlay (1988) also find that value weighting affects only the properties of the portfolio containing the largest firms on the NYSE and AMEX. With equal weighting of security returns they reject the random walk hypothesis (for stock prices) for all quintiles, while with value weighting they reject the random walk hypothesis for all but the largest portfolio (fifth quintile).

Table 1
Summary statistics of daily, weekly, and monthly returns of five value-weighted portfolios of NYSE and AMEX common stocks, formed by quintile rankings of market value of equity outstanding at the end of the previous year, 1962–1985

Variable (x)	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	$\hat{\rho}_5$	$\hat{\rho}_6$	$\bar{x}$	$s(x)$
Daily returns ($n = 5903$)								
RD1	0.46	0.21	0.19	0.17	0.16	0.11	0.1088	0.7902
RD2	0.40	0.13	0.14	0.13	0.11	0.05	0.0791	0.7978
RD3	0.36	0.10	0.10	0.10	0.09	0.03	0.0649	0.8130
RD4	0.35	0.08	0.09	0.07	0.06	0.02	0.0561	0.7571
RD5	0.20	0.01	0.02	−0.00	−0.00	−0.02	0.0414	0.7792
DIF	0.27	0.18	0.14	0.10	0.10	0.10	0.0674	0.6538
Weekly returns ($n = 1226$)								
RW1	0.38	0.22	0.16	0.11	0.03	0.04	0.3207	2.5481
RW2	0.31	0.16	0.11	0.07	0.02	0.02	0.2415	2.3865
RW3	0.28	0.13	0.08	0.05	−0.00	0.01	0.1930	2.3140
RW4	0.23	0.10	0.07	0.04	0.00	−0.01	0.1604	2.1238
RW5	0.06	−0.00	0.03	0.00	−0.01	−0.03	0.0946	1.9222
DIF	0.29	0.20	0.14	0.07	0.05	0.01	2.2613	1.9765
Monthly returns ($n = 281$)								
RM1	0.18	−0.02	−0.01	0.02	0.03	0.02	1.1622	7.6938
RM2	0.14	−0.00	−0.01	0.02	0.06	−0.00	0.9140	6.6847
RM3	0.14	−0.03	−0.01	0.03	0.06	0.00	0.7368	6.1708
RM4	0.14	−0.03	−0.01	0.02	0.09	−0.02	0.6030	5.3645
RM5	0.03	−0.03	0.02	0.08	0.13	−0.07	0.3160	4.1849
DIF	0.10	0.02	−0.11	0.07	0.00	0.12	0.8463	5.9349

RD1 to RD5 are dally returns of five size-based portfolios in ascending order from smallest to largest. RW1 to RW5 and RM1 to RM5 are the weekly and monthly excess returns [the difference between the one-week (month) portfolio return and the one-week (month) Treasury bill rate]. DIP is the "size effect" variable defined as the difference between the returns of the smallest and largest-sized portfolios (R1-R5). $\bar{x}$ and $s(x)$ are the sample mean and standard deviation of the variable, and $\hat{\rho}_t$ is the sample autocorrelation at lag t. Under the hypothesis that the true autocorrelations arc zero, standard errors of the estimated auto. correlations are about 0.01 for the dally, 0.03 for the weekly. and 0.06 for the monthly returns, respectively. The returns are rates of return per period, in decimal fraction units $\times 10^2$.

report the autocorrelations of DIF, the difference between the returns of the smallest and largest firms on the NYSE and AMEX. Not surprisingly, DIF displays strong autocorrelation in both daily and weekly data. This evidence of high positive autocorrelations in daily and weekly returns is consistent with the results on variance ratios in Lo and MacKinlay (1988).

2.3 Infrequent trading and nonsynchronous prices

The positive autocorrelation in portfolio returns could conceivably be a reflection of infrequent trading, or the Fisher effect [see, e.g., Fisher (1966) and Scholes and Williams (1977)]. However, there are several reasons to believe that measurement errors due to nonsynchronous prices are not the basis for the significant positive autocorrelations shown in Table 1.

First, we use weekly, rather than daily, portfolio returns in our tests, and there is evidence that infrequent trading is likely to cause only minor measurement bias in weekly data. For example, Lo and MacKinlay (1988) model the nontrading phenomenon as a binomial process and show that even if (on average) 50 percent of stocks on the NYSE and AMEX do not trade each day, the theoretical first-order weekly autocorrelation of portfolio returns would be about 17 percent. This, in turn, implies that even with 50 percent nontrading the implied proportion of variation in weekly portfolio returns resulting from infrequent trading would be only about $(0.17)^2 = 2.89\%$. In contrast, our evidence shows that all of the value-weighted portfolios (except the largest) have first-order serial correlations and implied R^2's well in excess of such extreme theoretical values. For example, the R^2 of the regression of the smallest weekly portfolio returns on the extracted expected returns is in excess of 25 percent.

Second, Conrad and Kaul (1988) show that in bivariate regressions of size-based portfolio returns on portfolio expected returns and lagged market returns, the latter usually have insignificant marginal explanatory power. This implies that the high positive autocorrelation witnessed in the smaller portfolios, which potentially contain more infrequently traded firms, is a reflection of movements in expected returns rather than the Fisher effect.

Finally, and perhaps most importantly, to control for the potential dependence in returns caused by infrequent trading, we replicate all tests reported in this article using only trade prices from the CRSP daily master file. Specifically, we exclude all securities that do not actually trade on both the first and last days of a particular week. Hence, any remaining nonsynchronous trading bias would be restricted to differences in trading intervals of less than one day, and relative to weekly returns, such a bias should be small. The autocorrelation in portfolio returns and the implied R^2 resulting from infrequent trading should be even lower than the values of 17 percent and 2.89 percent, respectively, shown above. The results of all tests using the restricted set of firms are virtually identical to the evidence reported in this article.

2.4 Weekly estimates of the model

In this section we present and analyze estimates of a modified form of Equation (4) that take into account the January effect:

$$R_t^w = \delta_0 + \delta_i + \phi R_{t-1}^w + a_t^w - \theta a_{t-1}^w \tag{8}$$

where δ_i = coefficients on dummy variables for different weeks in January defined as $\delta_i = 1$ if ith week of January, $\delta_i = 0$ otherwise; and $i = 1, 2, 3, 4$. We use the Kalman filter and a Marquardt maximum-likelihood procedure proposed by Ansley (1980) to obtain weekly estimates of Equation (8). The results are shown in Table 2. We also estimate the model using the nonlinear procedure of Box and Jenkins (1970, chap. 7) and obtain very similar estimates.

A stationary autoregressive process for expected returns appears to be well-specified. The estimates of ϕ are significantly different from both zero and unity for all portfolios, and the residuals behave like white noise. Moreover, there is a monotonic relation between the estimates of ϕ and size: The magnitude of ϕ declines systematically from the smallest- to the largest-sized portfolios.

Table 2
Weekly estimates of the parameters of the model in which expected returns follow a stationary AR(1) process, August 1962 to December 1985, with standard errors in parentheses, $n = 1226$

Portfolio	$\hat{\delta}_0$	$\hat{\delta}_1$	$\hat{\delta}_2$	$\hat{\delta}_3$	$\hat{\delta}_4$	$\hat{\phi}$	$\hat{\theta}$	$s(\hat{a})$	Q
1	0.0014	0.0483	0.0290	0.0146	0.0082	0.6327	0.3332	0.02247	15.82
(smallest)	(0.0012)	(0.0047)	(0.0049)	(0.0049)	(0.0047)	(0.0587)	(0.0715)		(0.07)
2	0.0012	0.0283	0.0198	0.0106	0.0052	0.5459	0.2858	0.02227	11.51
	(0.0010)	(0.0047)	(0.0048)	(0.0048)	(0.0047)	(0.0782)	(0.0894)		(0.24)
3	0.0011	0.0172	0.0153	0.0097	0.0020	0.4668	0.2147	0.02202	7.48
	(0.0010)	(0.0046)	(0.0048)	(0.0048)	(0.0046)	(0.0908)	(0.1003)		(0.59)
4	0.0011	0.0096	0.0101	0.0077	0.0004	0.4647	0.2568	0.02059	5.85
	(0.0008)	(0.0043)	(0.0044)	(0.0044)	(0.0043)	(0.1080)	(0.1179)		(0.76)
5	0.0008	0.0009	0.0028	0.0033	−0.0010	0.0633	0.0000	0.01921	15.23
(largest)	(0.0006)	(0.0041)	(0.0041)	(0.0041)	(0.0040)	(0.0286)	(0.0000)		(0.12)
DIF	0.0004	0.0493	0.0280	0.0131	0.0112	0.6974	0.4954	0.01740	8.14
	(0.0008)	(0.0036)	(0.0037)	(0.0037)	(0.0036)	(0.0669)	(0.0811)		(0.52)

R_t^e = excess portfolio return for week t; ϕ = autoregressive parameter; θ = moving-average parameter; and δ_i = dummy variables for different weeks in January defined as $\delta_i = 1$ if ith week of January, $\delta_i = 0$ otherwise, $i = 1, 2, 3, 4$. $s(\hat{a})$ = residual standard error of the model. The last column reports the Q-statistic (with p values in parentheses) to test the hypothesis that all autocorrelations of the residuals up to lag 12 are

However, the important aspect (for this article) of the evidence in Table 2 is that the estimates of ϕ (which are all less than about 0.70) imply rapidly decaying time variation in weekly expected returns: The effects of an expected return shock are largely dissipated after a month. The rapid mean reversion in weekly expected returns, in turn, has implications for the relative importance of different weeks within a particular month in predicting the subsequent month's return.

Using Equation (7) and the estimates of ϕ and θ from Table 2, we can see that to predict monthly returns of a particular portfolio a disproportionately large weight should be given to the fourth (most recent) week of the previous month(s). For example, in the case of the smallest portfolio, the weights (in percentages) given to the fourth, third, second, and first weeks of the previous month should be 0.67, 0.22, 0.08, and 0.03, respectively.[2] A similar weighting pattern is applicable to all the portfolios. This pattern is very different from, for example, the weighting scheme implied by time-series models of one-month real returns on Treasury bills. In these models the decay in the time variation in expected returns is so gradual that giving equal weights to past observations results in forecasts that compete well with forecasts from more sophisticated models [see Fama and Gibbons (1984)].

Hence, the rapid decay in the variation through time of weekly expected stock returns implies that using monthly data to predict monthly stock returns understates the magnitude ofvariation in monthly expected returns.

[2] These weights are calculated using Equation (7) and noting that the sum of the weights given to all past weekly returns is $(\phi - \theta)/(1 - \theta)$.

The model in Section 1, on the other hand, takes into account the disproportionately large weight the market appears to place on the most recent intramonth information in obtaining forecasts of monthly stock returns.[3]

Table 2 also documents the January effect, which is significant for all but the largest portfolios. The findings corroborate Keim's (1983) results because the dummy variable for the first week of January has the largest positive coefficient. However, there is evidence of a significant positive effect even in the later weeks of January.[4] Finally, the mean excess return for all non-January weeks, $\hat{\delta}_0$, is positive for all portfolios but statistically indistinguishable from zero.

Before discussing the properties of the monthly expected return estimates, the procedure used to construct the forecasts should be clarified. If we used the same weekly return sample employed to estimate the time-series models, we would use information only up to the last Wednesday of the previous month, which coincides with the last day of the month merely 14 percent of the time. On average, therefore, we would lose three days' worth of information. However, given the rapid decay in the autocorrelations of daily returns shown in Table 1, the maximum weight should be given to the last day(s) of the previous month. We use two methods to correct for this loss of information:

Method 1. The first method uses a new set of (four) weekly returns for *each* month constructed so as to match weekly and monthly returns perfectly. We then use Equation (7) and estimates of ϕ, θ, and δ_i's from Table 2 to obtain forecasts of monthly conditional expected returns.

Method 2. The second method uses the original weekly return sample and avoids the loss of information by also including the return of the last day of each month in the forecasting function. Specifically, we estimate a regression of the form:

$$R_t^m = \alpha + \beta_1 ER_{t-1} + \beta_2 R_{t-1}^d + \epsilon_t \tag{9}$$

where

R_t^m = the excess return for month t

ER_{t-1} = the monthly expected return based on the original sample of weekly returns using Equation (7) (with no January dummies)

[3] To further investigate the existence of mean reversion in weekly expected returns we estimated Equation (8) for portfolios based on dividend yield. All of the (five) portfolio returns exhibit high autocorrelations, with the first-order autocorrelations ranging between 0.21 and 0.31. The estimated ϕ's range between 0.516 and 0.570, while $\hat{\theta}$'s vary between 0.294 and 0.329. These estimates Indicate that the rapid mean reversion in short-horizon expected returns is a "general" phenomenon in the stock market.

[4] We attempted to model the seasonality in portfolio returns by estimating time-series models with varying autoregressive, ϕ, and moving-average, θ, parameters in January. The results show that both parameters change in January, indicating that the autocorrelation structure of returns is different in January. However, the forecasts obtained by simply allowing a mean shift (i.e., in δ) statistically outperform forecasts based on models that allow both ϕ and θ to vary. One reason for these findings may be increased estimation error in more complicated time-series models.

R^d_{t-1} = the daily return on the last day of month $t-1$

ϵ_t = random disturbance term[5]

This approach, though not very elegant, is more informative compared with the first approach for two important reasons. First, it emphasizes the main implication of our model that the most recent intramonth (in this case daily) information should be the most important in forecasting subsequent monthly returns. Estimates of Equation (9), not reported, show that R^d_{t-1} always contains statistically significant information, over and above that contained in ER_{t-1}, about subsequent monthly returns. We use estimates of β_1, and β_2 as the weights on variables ER_{t-1} and R^d_{t-1} and thus obtain a second set of monthly portfolio return forecasts.

Second, unlike the earlier approach, this method does not incorporate any January mean effects. Hence, by relying solely on information contained in past portfolio returns and without any special treatment for January weeks, this method underscores the importance of the rapid mean reversion in expected returns in all months of the year. At the same time, the comparative forecasting performance of the expected return estimates obtained from the two methods also gives an idea of the extent to which our model can (or cannot) explain the January effect.

2.5 Forecasts of monthly expected returns

To analyze the properties of the monthly return forecasts, we estimate a regression of monthly portfolio returns, R^m_t, on the forecasts obtained from each of the two methods, ER^m_{t-1}:

$$R^m_t = \alpha + \beta_1 ER^m_{t-1} + \eta_t \tag{10}$$

Table 3 shows the estimated regression parameters for all market value portfolios and the size effect variable, DIF. Panels A and B report regressions using expected return forecasts based on Methods 1 and 2, respectively. The entire discussion of the statistical properties of the forecasts will be confined to an *in-sample* evaluation.

For all portfolios the slope coefficients are close to one and the intercepts are close to zero, so that the extracted expected returns obtained from both methods are conditionally unbiased. The heteroskedasticity test of White (1980) produces chi-square statistics that are only occasionally above conventional significance levels. Also, both the ordinary least squares (OLS) standard errors and the (usually) more conservative standard errors based on White's (1980) heteroskedasticity-consistent method provide similar inferences. Hence, we only report the OLS standard errors.

The regressions also appear to be well-specified because the autocorrelation in monthly portfolio returns (see Table 1) is absorbed by the estimated expected returns. The Q -statistics are not significant at conven-

[5] We use only the last day's return in Equation (9) to avoid data dredging.

Table 3
Estimates of regressions of monthly excess returns on extracted expected returns, with ordinary least squares standard errors in parentheses, August 1962 to December 1985

Portfolio	$\hat{\alpha}$	$\hat{\beta}_1$	R^2	Q
Panel A: Monthly expected returns based on Method 1				
1 (smallest)	0.0011 (0.0041)	1.098 (0.112)	0.257	11.50 (0.49)
2	0.0025 (0.0038)	1.100 (0.156)	0.151	9.28 (0.68)
3	0.0030 (0.0036)	1.117 (0.203)	0.098	11.87 (0.46)
4	0.0033 (0.0032)	1.126 (0.259)	0.063	12.31 (0.42)
5 (largest)	0.0030 (0.0026)	0.349 (1.129)	0.000	12.27 (0.43)
DIF	−0.0019 (0.0031)	1.058 (0.096)	0.303	31.57 (0.00)
Panel B: Monthly expected returns based on Method 2				
1 (smallest)	−0.0026 (0.0045)	1.000 (0.127)	0.183	11.23 (0.51)
2	0.0004 (0.0041)	1.001 (0.171)	0.109	8.94 (0.71)
3	0.0024 (0.0037)	1.000 (0.215)	0.072	9.99 (0.62)
4	0.0019 (0.0033)	1.001 (0.238)	0.059	9.96 (0.62)
5 (largest)	0.0014 (0.0028)	1.002 (0.668)	0.008	12.85 (0.38)
DIF	−0.0005 (0.0035)	0.999 (0.137)	0.161	59.90 (0.00)

R_t^e = excess portfolio return for month t; ER_{t-1}^e = expected portfolio return for month t as of month t - 1; and η_t = random disturbance term. R^2 = coefficient of determination. The last column reports the Q- statistic (with p values In parentheses) to test the hypothesis that all autocorrelations of the residuals up to lag 12 are jointly zero. $R_t^e = \alpha + \beta_1 ER_{t-1}^e + \eta_t$.

tional significance levels for all five portfolios. In the DIF regression, however, forecasts based on our model do not absorb all the autocorrelation in monthly returns for the overall period (the Q- statistic is statistically significant). One explanation for this result may be the invalidity of our implicit assumption that the parameters of the time-series model, ϕ and θ, are constant over the entire 1962 to 1985 period. There is some empirical support for this conjecture in that the subperiod Q- statistics for the DIF regression are all statistically insignificant (see Table 5).

The most striking aspect of the regressions in Table 3 is the large proportion of variance of monthly stock returns explained by time variation in expected returns. The R^2 values of over 25 percent in Panel A are much larger than the maximum values of 3 percent typically found in earlier studies [see, e.g., Keim and Stambaugh (1986), Kaul (1987),and Fama and French (1987)].[6] The results in Panel B show that using the most recent daily information, over and above the weekly returns, helps explain over

[6] Campbell (1987) is the only study with evidence that suggests that expected return movements may explain more than 3 to 5 percent of monthly return variance.

18 percent of the monthly return variance of the smallest portfolio for the overall period. This proportion jumps to over 40 percent during the 1970s (see Section 2.6). These results are especially interesting because the expected return estimates used in Panel B do not explicitly incorporate the January effect and hence emphasize the ability of our model to demonstrate time variation in expected returns throughout the year.

The results also reveal a monotonic (inverse) relation between the relative degree of variation in expected returns and firm size. The smallest firms exhibit the maximum variation in expected returns. The R^2 values decline systematically from a high of over 25 percent to a low of 0 percent for the largest firms. The evidence that the largest market value firms on the NYSE and AMEX exhibit almost no variation in expected returns is simply a reflection of the low autocorrelations for the returns of the largest portfolio reported in Table 1. Hence, based on our model, the assumption of constant monthly expected returns is strongly rejected for all but the largest market value firms. Finally, the size effect also shows a high degree of variation through time: The regression with DIF as a dependent variable has an R^2 of over 30 percent.

In Table 4, we show regressions that test whether inclusion of other ex ante information significantly improves our ability to predict stock returns. There is no basic principle that suggests that rational agents use only an asset's own past returns (as presumed in our model) to form expectations about its future returns. We use two variables-a dividend yield variable, DIV_{t-1}, and a term premium variable, $TPREM_{t-1}$ -that have demonstrated an ability to track expected returns.[7] The estimated regressions show that DIV_{t-1} and $TPREM_{t-1}$ have an ability, over and above ER^m_{t-1} (based on Method 1), to track expected returns, especially of the larger firms. However, both DIV_{t-1} and $TPREM_{t-1}$ together make only a marginal contribution to the explanatory power of the regressions. Hence, viewed by themselves, such ex ante variables would understate the degree of variation through time in short-horizon expected returns.

2.6 Subperiod issues

Table 5 shows evidence of the relative degree of time variation in monthly expected returns in three subperiods: August 1962 to May 1970, June 1970 to March 1979, and April 1979 to December 1985. We report regressions of monthly portfolio returns on the forecasts, ER^m_{t-1} , obtained using Method 1. We reestimate the time-series model in Equation (8) for each of the three subperiods and obtain the forecasts based on the subperiod parameter estimates. This procedure is adopted to take account of parameter instability over time, especially since the evidence does reveal changes in

[7] DIV_{t-1} is the ratio of (annualized) dividends paid for the three months ending at month t - 1 to the price at the end of month t - 1 for the value-weighted portfolio of NYSE stocks. $TPREM_{t-1}$ is the difference between the Aaa (annualized) yield and the (annualized) yield on a one-month Treasury bill. These measures are similar to those used by Fama and French (1987) and Kandel and Stambaugh (1988).

Table 4
Estimates of regressions of monthly excess returns on extracted expected returns and other ex ante information, with OLS standard errors in parentheses, August 1962 to December 1985

Portfolio	$\hat{\alpha}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	R^2	Q
1	−0.0308	1.081	0.340	0.659	0.269	13.26
(smallest)	(0.0170)	(0.112)	(0.266)	(0.410)		(0.35)
2	−0.0352	1.066	0.392	0.786	0.173	10.76
	(0.0157)	(0.155)	(0.246)	(0.378)		(0.55)
3	−0.0358	1.065	0.432	0.793	0.126	13.45
	(0.0149)	(0.202)	(0.233)	(0.359)		(0.38)
4	−0.0319	1.049	0.457	0.690	0.097	13.49
	(0.0131)	(0.257)	(0.206)	(0.317)		(0.33)
5	−0.0220	0.075	0.542	0.393	0.049	13.46
(largest)	(0.0105)	(1.108)	(0.165)	(0.254)		(0.34)
DIF	−0.0079	1.057	−0.078	0.186	0.304	30.48
	(0.0128)	(0.097)	(0.199)	(0.309)		(0.00)

R_t^m = excess portfolio return for month t; ER_{t-1}^m = expected portfolio return for month t as of month $t-1$; TPREM$_{t-1}$ = difference between end-of-month Aaa (annualized) yield and the (annualized) yield on a one-month Treasury bill; DIV$_{t-1}$ = the ratio of (annualized) dividends paid for the three months ending at month $t-1$ to the price at the end of month $t-1$ for the value-weighted portfolio of NYSE stocks; and η_t = random disturbance term. R^2 = coefficient of determination. The last column reports the Q-statistic (with p values in parentheses) to test the hypothesis that all autocorrelations of the residuals up to lag 12 are jointly zero. $R_t^m = \alpha + \beta_1 ER_{t-1}^m + \beta_2 \text{TPREM}_{t-1} + \beta_3 \text{DIV}_{t-1} + \eta_t$.

the estimates of the parameters. For comparative purposes, we also report R^2 values (in parentheses) based on regressions that use forecasts from Method 2 as the independent variables.

The variation in monthly expected returns changes rather dramatically over time. In the first subperiod, ER_{t-1}^m explains between 18.3 percent and 2.7 percent of monthly return variances for portfolios 1 through 4. In the second subperiod, however, the proportion of return variance explained by ER_{t-1}^m jumps to between 41.3 percent and 14.8 percent. The third subperiod shows a drop in the proportions to between 10.8 percent and 0.5 percent. The size effect variable, DIF, shows a similar pattern across the three subperiods, with the R^2 values jumping from 31.8 percent to 44.9 percent and then dropping to 17.4 percent. Finally, forecasts based on Method 2 also show a similar dramatic pattern in explaining movements in monthly portfolio returns.

Hence, the evidence shows distinctly higher variation in expected returns during the 1970s. This suggests that agents' expectations varied to a much larger extent during the volatile 1970s, when the variance of realized returns was, on average, 1.5 times the variance in the 1960s and 1980s.

3. Summary and Conclusions

In this article, we have developed a simple model for monthly expected returns that relies on the rapidly decaying structure of weekly expected returns. Excess returns of five size-based portfolios over the 1962 to 1985

Table 5
Subperiod estimates of regressions of monthly excess returns on extracted expected returns, with OLS standard errors in parentheses

Portfolio	$\hat{\alpha}$	$\hat{\beta}_1$	R^2	Q
August 1962 to May 1970				
1 (smallest)	0.0045 (0.0067)	1.019 (0.224)	0.183 (0.094)	16.68 (0.16)
2	0.0042 (0.0061)	0.930 (0.301)	0.094 (0.063)	14.63 (0.26)
3	0.0038 (0.0058)	0.744 (0.370)	0.042 (0.052)	15.90 (0.20)
4	0.0029 (0.0050)	0.684 (0.428)	0.027 (0.038)	16.83 (0.16)
5 (largest)	0.0022 (0.0037)	0.028 (0.812)	0.000 (0.003)	15.24 (0.23)
DIF	−0.0000 (0.0048)	1.161 (0.177)	0.318 (0.120)	19.54 (0.08)
June 1970 to March 1979				
1 (smallest)	−0.0080 (0.0078)	1.185 (0.147)	0.413 (0.404)	5.27 (0.95)
2	−0.0049 (0.0073)	1.221 (0.203)	0.282 (0.266)	5.71 (0.93)
3	−0.0033 (0.0068)	1.270 (0.251)	0.217 (0.204)	6.26 (0.90)
4	−0.0009 (0.0061)	1.264 (0.316)	0.148 (0.182)	6.50 (0.89)
5 (largest)	−0.0002 (0.0051)	0.954 (0.910)	0.012 (0.041)	9.06 (0.70)
DIF	−0.0079 (0.0061)	1.098 (0.127)	0.449 (0.267)	19.80 (0.70)
April 1979 to December 1985				
1 (smallest)	0.0076 (0.0066)	0.736 (0.222)	0.108 (0.060)	4.63 (0.97)
2	0.0091 (0.0061)	0.609 (0.341)	0.034 (0.025)	4.69 (0.97)
3	0.0089 (0.0060)	0.599 (0.504)	0.015 (0.012)	6.03 (0.91)
4	0.0083 (0.0053)	0.565 (0.837)	0.005 (0.002)	7.43 (0.83)
5 (largest)	0.0052 (0.0045)	−1.639 (2.034)	0.007 (0.007)	10.26 (0.59)
DIF	0.0010 (0.0045)	0.833 (0.190)	0.174 (0.091)	7.83 (0.80)

R_t^m = excess portfolio return for month t; ER_{t-1}^m = expected portfolio return for month t as of month $t-1$; and η_t = random disturbance term. R^2 = coefficient of determination. The last column reports the Q-statistic (with p values in parentheses) to test the hypothesis that all autocorrelations of the residuals up to lag 12 are jointly zero. $R_t^m = \alpha + \beta ER_{t-1}^m + \eta_t$.

period were used to estimate the model and obtain the monthly return forecasts. The results show that the rapid mean reversion in shorter-horizon (weekly) expected returns implies substantially higher variation through time in monthly expected returns than has been documented in earlier studies. For the overall period, over 25 percent of small-firm return variance can be explained by time variation in monthly expected returns.

There is a dramatic change in the degree of time variation in expected returns over the sample period. For instance, the proportion of return variation explained by movements in expected returns of small firms jumps from around 18 percent during the 1960s to over 40 percent during the 1970s and drops to about 11 percent in the 1980s. This pattern is common to all except the largest portfolio, which shows virtually no variation in expected returns. Finally, there is a monotonic relation between. the size rankings of the portfolios and the degree of relative time variation in monthly expected returns: The smaller firms exhibit the maximum variation. This systematic relation is found both in the overall period and during each of the three subperiods.

Future research is needed to develop models that capture the substantial degree of time variation in both short-horizon and long-horizon expected returns within a single framework. [See Kandel and Stambaugh (1988) for an example of such a unifying framework.] Such models may help us understand the determinants of short- and long-horizon expected returns.

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