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The profitability of pairs trading strategies: distance, cointegration and copula methods

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We perform an extensive and robust study of the performance of three different pairs trading strategies—the distance, cointegration and copula methods—on the entire US equity market from 1962 to 2014 with time-varying trading costs. For the cointegration and copula methods, we design a computationally efficient two-step pairs trading strategy. In terms of economic outcomes, the distance, cointegration and copula methods show a mean monthly excess return of 91, 85 and 43 bps (38, 33 and 5 bps) before transaction costs (after transaction costs), respectively. In terms of continued profitability, from 2009, the frequency of trading opportunities via the distance and cointegration methods is reduced considerably, whereas this frequency remains stable for the copula method. Further, the copula method shows better performance for its unconverged trades compared to those of the other methods. While the liquidity factor is negatively correlated to all strategies' returns, we find no evidence of their correlation to market excess returns. All strategies show positive and significant alphas after accounting for various risk-factors. We also find that in addition to all strategies performing better during periods of significant volatility, the cointegration method is the superior strategy during turbulent market conditions.

1. Introduction

Gatev *et al*. (2006) show that a simple pairs trading strategy (PTS), namely the Distance Method (DM), generates profit over a long period. However, Do and Faff (2010) document that the profitability of the strategy is declining. They associate this decrease to a reduction in arbitrage opportunities during recent years, as measured by the increase in the proportion of pairs that diverge but never converge. Do and Faff (2012) show that the DM is largely unprofitable after 2002, once trading costs are taken into account. Jacobs and Weber (2015) find that the profitability of the DM is immensely time-varying. Nonetheless, there are other tools such as cointegration and copulas that can be used to implement statistical arbitrage trading strategies. Although such concepts are cursorily introduced in the pairs trading literature, their performance has not been robustly evaluated. Accordingly, our basic goal is to evaluate the performance of two sophisticated PTS, namely copula and cointegration methods, using a long-term and comprehensive data-set. We also assess if there is a decline in pairs trading profitability for these more sophisticated

methods and investigate the risk-factors that might influence their profitability.

Pairs trading strategies are comprised of two stages: first, the method applied to form pairs; and second, the criteria for opening and closing positions. In the DM, securities whose prices are closely correlated are grouped in pairs, and traded when their prices diverge by more than a pre-specified amount. This is the only strategy that has been tested thoroughly using extensive data-sets, a wide variety of securities and across different financial markets (Andrade *et al*. 2005, Gatev *et al*. 2006, Perlin 2009, Do and Faff 2010, Broussard and Vaihekoski 2012, Do and Faff 2012, Jacobs and Weber 2015). Cointegration can be employed in a pairs trading framework (Vidyamurthy 2004; Lin *et al*. 2006). Although Lin *et al*. (2006) implement a cointegration PTS, their empirical analysis only examines two Australian shares over a short sample period of one year. In the application of copulas in pairs trading, Xie and Wu (2013) propose a strategy, and Wu (2013) evaluates its performance using three pre-selected pairs. Xie *et al*. (2014) explore 89 US stocks in the utility industry over a sample period of less than a decade. Our study extends the literature by examining the performance of a cointegration-based and a

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copula-based PTS, using the CRSP data-set from 1962 to 2014. Using a comprehensive data-set spanning over five decades and containing all US stocks, our study is a robust examination of alternative PTS. By evaluating the performance of cointegration and copula-based trading strategies against the DM benchmark, we establish whether these complex methods yield better performance in the long-term.

Understanding the difference in performance outcomes between copula and cointegration PTS versus the benchmark, the DM, will provide valuable insight into the source of pairs trading profitability and the reasons behind the observed decline in the profitability of the DM. Has the market become more efficient and the availability of arbitrage opportunities diminished? Or are contemporary methodologies more sophisticated than the simple DM required to exploit market inefficiencies? For example, the simplicity of the DM might induce increased arbitrage activity, leading to fewer arbitrage opportunities to exploit thereby resulting in a drop in profitability. The answer to these questions sheds light on the direction of future research by academics and practitioners in order to build better performing strategies which will in turn further improve market conditions and efficiency.

We study how increasing the sophistication of methods by which pairs are selected and traded can affect the quality and precision of the captured relationship within the pair and, ultimately, the performance of PTS. In theory, the presence of a cointegration relation between two assets means that there is a long-term relationship between them. Exploiting this relationship should allow us to accurately model the co-movement of the pair and use that to implement a high-performance PTS. Equities are shown to exhibit asymmetric dependence (Longin and Solnik 1995, Patton 2004, Low *et al.* 2013). Using copulas for modelling the dependence structure between two assets, instead of restricting the framework towards the elliptical dependence structure of covariance matrix, would also possibly lead to a superior PTS by allowing for more flexibility in capturing asymmetries in the dependence structure within pairs. Nevertheless, more complex models can also result in inferior performance, especially out of sample, by introducing issues such as over-fitting. Moreover, the computational requirements necessary to process these mathematically complex algorithms may outweigh their relative performance improvements over simpler strategies.† This might result in weakening of motivation to adapt such strategies in practice.

The novel contributions of this paper to the relevant literature are fourfold. First, our study combines aspects of the DM and the cointegration or copula technique to produce a computationally efficient two-step approach to pairs trading that can be operationalized by practitioners. When considering

a trading strategy, speed and efficiency of computation is a vital consideration (Clark 2012, Angel 2014, Brogaard *et al.* 2014). Stock pairs are sorted and selected by sum of squared differences (SSD). In the copula (cointegration) strategy, for each selected stock pair, a range of copula and marginal models are fitted and selected based upon the AIC and BIC criterion (the cointegration coefficient is calculated). Wu (2013) only fits copulas and marginal models to one pair and Xie *et al.* (2014) use a data sample limited in both time span and number of stocks. We find that the Student-t copula is selected for 61% of the pairs. This highlights the fact that the dependence structure of the stock pairs exhibits fat tails, and therefore the classic linear correlation framework employed in simpler methods are inadequate in modelling their relation. Second, we perform a comprehensive evaluation of the performance of two alternative PTS (i.e. cointegration and copulas) against a data-set consisting of all the shares in the US market from 1962 to 2014. PTS more sophisticated than the DM, have not been empirically tested in a robust manner, and therefore their long-term performance remains unknown. Due to the broad and long data sample used, this longitudinal study presents the first extensive examination of the performance of two relatively new PTS, using cointegration and copula methods. Third, with various economic and risk-adjusted return metrics, we evaluate the performance of all three PTS and show if the increased complexity in the pairs selection and trading criteria improves performance. With respect to studies finding a recent decline in the performance of the DM, this comparison will lead to understanding if arbitrage opportunities are still available in the market, but perhaps due to increased arbitrage activity, more complex methods such as copulas are required to take advantage of them. Fourth, we examine the performance of the PTS in relation to findings in the asset pricing literature that show that momentum (Carhart 1997), liquidity (Pástor and Stambaugh 2003), and more recently profitability and investment patterns (Fama and French 2015) explain stock returns.

Our findings show that the cointegration method performs as well as the DM in economic and risk-adjusted performance measures. The two strategies also show very similar pair trade properties and risk profiles. Based upon lower partial moment and drawdown measures, the cointegration method performs better than the other strategies before transaction costs are taken into account, whereas after costs the DM is slightly superior. We find the copula method's economic and risk-adjusted performance to be weaker than the other two methods. The weaker performance of the copula method can be attributed to the high proportion of unconverged trades. A positive outcome of the copula method is that, unlike the other methods, the frequency of its trades have not fallen in recent years, thus its economic performance is more stable over time. We show that the liquidity factor is negatively correlated with the return of each strategy. No such correlation can be found with the market returns, which demonstrates the market neutrality of these strategies. The alphas of all PTS remain large and significant even after several asset pricing factors such as momentum, liquidity, profitability and investment (Fama and French 2015) are taken into account.

The remainder of this paper is structured as follows. In section 2, we review some of the relevant literature on pairs

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†In the last year of our study, 2014, there are an average of 2,377 stocks ($\bar{N}$) per day, resulting in a total of 2,823,876 ($\bar{N}[\bar{N} - 1]/2$) unique stock pairs to be analysed for selection into the strategy. When restricted to a single core processor, the average computation time for selecting the best copula model and fit for each stock pair is 0.44 seconds. Thus, analysing all unique stock pairs on a single day requires a total of 345 h for a single core processor. Performing such an analysis within 5 h requires a minimum of 70 core processors using parallel computing techniques. Our analysis is performed on Matlab 2014b with the Parallel Computing toolbox on a compute server with dual Intel Xeon Processors E5-2640 (24 hyper-threaded cores, 30 MB Cache, Max 3.00 GHz) and 128 GBs of RAM.

trading, copulas and cointegration. A description of our data-set is in section 3. Section 4 covers the research method. And finally, the results and conclusion are presented in sections 5 and 6, respectively.

2. Literature review

Research on PTS fall under the general finance banner of 'statistical arbitrage'. Statistical arbitrage refers to strategies that employ some statistical model or method to take advantage of what appears to be mispricing between assets while maintaining a level of market neutrality. Gatev *et al.* (2006) is the earliest comprehensive study on pairs trading. They test the most commonly used and simplest method of pairs trading, the DM, against the CRSP stocks from 1962 to 2002. Their strategy yields a monthly excess return† of 1.3%, before transaction costs, for the DM's top five unrestricted‡ pairs, and 1.4% for its top 20. In addition, after restricting the formation of pairs to same-industry stocks, Gatev *et al.* (2006) report monthly excess returns of 1.1, 0.6, 0.8 and 0.6% on top 20 pairs in utilities, transportation, financial and industrial sectors, respectively. This study is an unbiased indication of the strategy's performance as they interpret the simplest method that practitioners employ as PTS. To avoid any criticisms that a profitable trading rule is data-mined and applied in their study, the authors re-evaluate the original strategy after four years and show that it remains profitable.

Do and Faff (2010, 2012) further examine the DM strategy of Gatev *et al.* (2006) to investigate the source of its profits and the effects of trading costs on its profitability using CRSP data from 1962 to 2009. Do and Faff (2010) find that the performance of the DM peaks in 1970s and 1980s, and begins to decline in the 1990s. The two exceptions to the plummeting performance of the DM strategy both occur during bear markets of 2000–2002 and 2007–2009. During these two periods, the DM shows solid performance, which is a slight reversal in the declining trend in the strategy's profitability across the period from 1990 to 2009. Moreover, they show that increased performance during the first bear market, i.e. 2000–2002, is due to higher profits of pairs that complete more than one round-trip trade, rather than an increase in their number. By contrast, in the second bear market, i.e. 2007–2009, the increase in the number of trades that complete more than one round-trip trade is the driver of strategy's strong profitability. After taking into account time-varying transaction costs, Do and Faff (2012) conclude that DM on average is not profitable. However, for the duration of the sample period, the top 4 out of the 29 portfolios constructed show moderate monthly profits of average 28 bps or 3.37% per annum. In addition, for the period from 1989 to 2009, the DM is profitable, albeit the majority of its profits occur in the bear market from 2000–2002.

Several studies explore pairs trading using the DM in different international markets, sample periods and asset classes (Andrade *et al.* 2005, Perlin 2009, Broussard and Vaihekoski 2012). Jacobs and Weber (2015) comprehensively analyse the

DM in 34 countries and find that although the strategy generates positive return, this return varies considerably over time. They attribute the source of the strategy's profitability towards investors' under or over-reaction to news information.

PTS may be categorized under algorithmic trading strategies that presently dominate most markets' order books. Within this stream of literature, Bogomolov (2013) adapts technical analysis in pairs trading. Using two Japanese charting indicators (i.e. the Renko and Kagi indicators), the study has a non-parametric approach to pairs trading, and thus, does not rely on modelling the equilibrium price of a pair. These indicators model the variability of the spread process within a pair. Based on the premise that the pair expresses mean-reverting behaviour, this variability is used in calculating how much the spread should deviate, before a trade becomes potentially profitable. Thus, this strategy relies on the stability of the statistical properties of the spread volatility. The strategy yields a positive before-costs monthly return between 1.42 and 3.65%, when tested on the US and Australian markets. Yang *et al.* (2015) use limit orders to model the trading behaviour of different market participants in order to identify traders, and equivalently, algorithmic traders. They do so by solving an inverse Markov decision process using dynamic programming and reinforcement learning. They show that this method leads to accurate categorization of traders. All PTS apply the use of a threshold that when crossed by the spread, triggers a trade. Zeng and Lee (2014) aim to find the optimum value for this threshold, given the spread follows a Ornstein–Uhlenbeck process, by defining it as an optimization problem. They focus on maximizing the expected return per unit time. Several other studies also focus on deriving automated trading strategies from technical analysis or creating profitable algorithms based on different cross-disciplinary concepts (Dempster and Jones 2001, Huck 2009, 2010, Creamer and Freund 2010).

PTS can be implemented using cointegration. Vidyamurthy (2004) presents a theoretical framework for pairs trading using cointegration based upon the error correction model representation of cointegrated series by Engle and Granger (1987). Huck and Afawubo (2015) and Huck (2015) implement a PTS using the cointegration method, and using S&P 500 and the Nikkei 225 stocks, they show that it generates positive returns. Bogomolov (2011) studies the performance of three different PTS. DM, cointegration and stochastic spread strategies are implemented and tested on the Australian share market from 1996 to 2010. The study concludes that while all three trading strategies are profitable before transaction costs, much of the profits are diminished after costs and liquidity issues are taken into account. Caldeira and Moura (2013) also use a cointegration-based trading strategy on the Sao Paolo exchange. They find that the strategy generates a 16.38% excess return per annum with a Sharpe ratio of 1.34 from 2005 to 2012. Lin *et al.* (2006) also motivate the use of cointegration as a model that can capture the long-term equilibrium of the price spread, while addressing the deficiencies of simpler statistical techniques used in pairs trading such as correlation and regression analysis. Using cointegration coefficient weighting,§ Lin *et al.* (2006) implement a theoretical framework that ensures

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†Return on employed capital.
‡Unrestricted pairs are the pairs that have not been formed based on specific criteria such as belonging to the same industry.

§Cointegration coefficient weighting refers to the method in which position weights are calculated as a function of the cointegration coefficient.

some minimum nominal profit per trade (MNPPT). They proceed to introduce a five step set of trading rules to apply the framework in pairs trading. Finally, their empirical analysis uses a small data-set of a 20 month sample period for two Australian bank stocks, and concludes that the MNPPT does not put excessive constraints on trading if adapted along with commonly-used values for trading parameters such as open and close trade triggers. Galenko *et al.* (2012) implement a PTS based on cointegration and examines its performance with four exchange traded funds. Nevertheless, these studies share a common shortcoming, where the empirical evidence provided to support the cointegration-based PTS are either non-present (being theoretical constructs) or severely limited in their analysis. For example, the strategy proposed in Vidyamurthy (2004) is not analysed on real data and Caldeira and Moura (2013) use data from the Sao Paulo stock exchange for a period of less than seven years.

Due to the properties of copulas in allowing the freedom to select marginal distributions and to flexibly model joint distributions (particularly lower tail dependence) copulas have also been frequently used in risk management (Siburg *et al.* 2015, Wei and Scheffer 2015) and asset allocation (Patton 2004, Chu 2011, Low *et al.* 2013, Low, Faff *et al.* 2016).

Okimoto (2014) studies the asymmetric dependence structure in the international equity markets including the US and concludes that firstly, there has been an increasing trend in the dependence within the equity markets over the last 35 years. Secondly, with the aid of copulas, the study finds strong evidence of the asymmetry of upper and lower tail dependence structure and points out the inadequacy of the multivariate normal model in capturing the characteristics of equities. Given the recent developments in the equity dependence outlined by this study, employing traditional correlation in modelling the joint behaviour, as done in the majority of quantitative methods to date, would poorly represent the true relationship among assets, and therefore, is no longer appropriate. This motivates the use of copulas in our study, in order to overcome this deficiency and accurately model the joint behaviour of equity pairs.

The application of copulas in quantitative trading strategies such as PTS is limited. Xie and Wu (2013), Wu (2013), and Xie *et al.* (2014) attempt to address this limitation. Wu (2013) points out that the main drawbacks of distance and cointegration PTS lie in the linearity restriction and symmetry that correlation and cointegration enforce on the pairs' dependence structure. The application of copulas would be beneficial in relaxing these restrictions. Thus, they propose a PTS that uses copulas to measure the relative undervaluation or overvaluation of one stock against the other. Their study compares the performance of a copula method to those of the DM and a cointegration method, but using a limited analysis of 3 same-industry pairs with a sample period of 36 months. Furthermore, pre-specified stock pairs with the same SIC code whose prices are known to be related are used, and so the strategy performance when it has the freedom to form pairs cannot be evaluated. The results show that the copula approach yields higher returns than the other two approaches. The copula approach also presents more trading opportunities than the distance and cointegration methods. Xie *et al.* (2014) employ a similar methodology but use a broader data-set comprising of the utility stocks (a total of 89 stocks) from 2003 to 2012. Similarly, they show that the performance of the copula strategy is superior to that of the DM used in Gatev *et al.* (2006). They also observe fewer trades with negative returns for the copula strategy compared to the DM. Similar to the cointegration method, the main deficiency in these copula-based PTS studies, is the limited empirical evidence to robustly measure the performance of the strategies across a large number of stocks over an extensive sample period.

3. Data

Our data-set consists of daily data of the stocks in CRSP from 1 July 1962 to 31 December 2014. The data-set sample period is 13216 days (630 months) and includes a total of 23 616 stocks. In accordance with Do and Faff (2010, 2012), we restrict our sample to ordinary shares, which are identified by share codes 10 and 11 in the CRSP database. In order to avoid relatively high trading costs and complications, we have further restricted our sample to liquid stocks. This is done by removing the bottom decile stocks, in terms of market cap, in each formation period. For the same reason, stocks with prices less than \$1 in the formation period are also not considered. To increase the robustness of our results and to replicate practical trading environments as closely as possible, we use trading volume to filter out stocks that have at least one day without trading in any formation period in the respective trading period. In summary, our data-set is consistent with that of Do and Faff (2012).

4. Research method

Pairs trading is a mean-reverting or contrarian investment strategy. It assumes a certain price relationship between two securities. Since pairs trading is a long-short strategy, modelling this relationship would allow us to take advantage of any short-term deviations by simultaneously buying the undervalued and selling short the overvalued security. Upon the restoration of the price relationship, we would close, or reverse, the two opened positions and realize the profit. We examine the performance of three different PTS using CRSP database consisting of US stocks from 1962 to 2014. For all strategies, we use a period of 6 months, the trading period, to execute the strategy using the parameters estimated in the previous 12 months, which we call the formation period. We run the strategies each month, without waiting six months for the current trading period to complete. As a result, we have six overlapping 'portfolios', with each portfolio associated with a trading period that has started in a different month.

In the DM (section 4.1), potential security pairs are sorted based on the SSD in their normalized prices during the formation period. After the pairs are formed, their spread is monitored throughout the trading period and any deviations beyond a certain threshold in that spread would trigger the opening of two simultaneous long and short positions. We use this strategy as our main benchmark to evaluate the cointegration and copula based PTS.

By definition, cointegrated time series maintain a long-term equilibrium and any deviation from this equilibrium is caused

by white noise and will be corrected as the series evolve through time (Vidyamurthy 2004). Using the statistical model of cointegration, we are able to incorporate the mean-reverting attribute of this statistical property in PTS. To allow for computational efficiency, our work employs a combination of the the cointegration framework outlined in Vidyamurthy (2004) and the DM to develop a cointegration-based trading strategy. After selecting nominated cointegrated pairs using the two-step Engle-Granger method (Engle and Granger 1987), we extract their stationary spread. Any deviation from this spread is by definition temporary and thus can be used to open long and short positions. We provide details of the cointegration model in PTS in section 4.2.

Our copula strategy is specifically designed to be computationally efficient to allow it to be easily operationalized in practice by traders. It combines aspects of the DM approach and the copula technique. Stocks pairs are sorted and selected by SSD, and we allow a range of marginal and copula distributions to be fitted to the resulting pair. Copulas are used (detailed in section 4.3) to model the relationship between stocks of a pair and to detect pairs deviations from their most probable relative pricing (Xie *et al.* 2014). We define two mispriced indices, which represent the relative under or overvaluation of stocks of a pair, and use them as criteria to open long-short positions when stocks move away from their relative fair prices.

4.1. The distance method

In DM, we calculate the spread between the normalized prices of all possible combinations of stock pairs during the formation period. The formation period is chosen to be 12 months.† The normalized price is defined as the cumulative return index, adjusted for dividends and other corporate actions, and scaled to \$1 at the beginning of the formation period. We then select 20 of those combinations that have the least sum of squared spreads, or SSD, to form the nominated pairs to trade in the following trading period, that is chosen to be 6 months. The standard deviation of the spread during the formation period is also recorded and used as the trading criterion. A specific stock can participate in forming more than one pair as long as the other stock of the pair varies.

Our implementation of the DM is in accordance with Gatev *et al.* (2006), Do and Faff (2010, 2012). At the beginning of the trading period, prices are once again rescaled to \$1 and the spread is recalculated and monitored. When the spread diverges by two or more historical standard deviation (calculated in the formation period), we simultaneously open a long and a short position in the pair depending on the direction of the divergence. The two positions are closed (reversed) once the spread converges to zero again. The pair is then monitored for another potential divergence and therefore can complete multiple round-trip trades during the trading period.

As the opening threshold is always set to two standard deviations, the divergence required to open positions are lower for less volatile spreads. Therefore, such positions can converge with a loss. Time-varying transaction costs (discussed in section 4.4) also contributes towards this effect. We analyse this issue further in section 5.5, sensitivity analysis.

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†Month refers to the calender month.

4.2. The cointegration method

4.2.1. Framework. A non-stationary time series X_t is called $I(1)$ if its first difference forms a stationary process, i.e. $I(0)$ (Lin *et al.* 2006). Consider $X_{1,t}$ and $X_{2,t}$ to be two $I(1)$ time series. If exists a linear combination of the two time series that is stationary, $X_{1,t}$ and $X_{2,t}$ are said to be cointegrated. Thus, $X_{1,t}$ and $X_{2,t}$ are cointegrated if there exists a non-zero real number β such that:

$$X_{2,t} - \beta X_{1,t} = u_t \tag{1}$$

where β is the cointegration coefficient and u_t is a stationary series known as the cointegration errors. Using the Granger's theorem (Engle and Granger 1987), the cointegration relationship can be equivalently shown in an Error Correction Model framework (ECM) (Vidyamurthy 2004). Based on ECM, the cointegrated series exhibits long-term equilibrium and, while short-term deviations from this equilibrium can occur, they will be corrected, through time, by the error term in the ECM. The ECM representation of the cointegration relationship between time series $X_{1,t}$ and $X_{2,t}$ is:

$$X_{2,t} - X_{2,t-1} = \alpha_{X_2}(X_{2,t-1} - \beta X_{1,t-1}) + \xi_{X_2,t} \tag{2}$$
$$X_{1,t} - X_{1,t-1} = \alpha_{X_1}(X_{2,t-1} - \beta X_{1,t-1}) + \xi_{X_1,t}$$

Equation (2) shows that the evolution of a time series, for example $X_{2,t}$, consists of a white noise, $\xi_{X_2,t}$, and an error correction term, $\alpha_{X_2}(X_{2,t-1} - \beta X_{1,t-1})$, which reverts the time series towards its long-term equilibrium as the series evolves through time. This mean-reverting property of cointegrated series can be used in implementing PTS.

Vidyamurthy (2004) provides a basic framework to apply cointegration to pairs trading. We describe this framework and extend his work to operationalize cointegration into an executable pairs trading strategy and provide details in section 4.2.2. The error correction term from equation (2) can be split into the rate of correction (α_{X_2}) and the cointegration relation (($X_{2,t-1} - \beta X_{1,t-1}$)). The cointegration relation shows the deviation of the process from its long-term equilibrium. The spread series is defined as the scaled difference in the price of two stocks:

$$\text{spread}_t = X_{2,t} - \beta X_{1,t} \tag{3}$$

Assume that we buy one share of stock 2 and sell short β share of stock 1 at time $t-1$. $X_{1,t}$ and $X_{2,t}$ represent the price series of stocks 1 and 2, respectively. The profit of this trade at time t, is given by:

$$(X_{2,t} - X_{2,t-1}) - \beta(X_{1,t} - X_{1,t-1}) \tag{4}$$

By rearranging the above equation we get:

$$(X_{2,t} - \beta X_{1,t}) - (X_{2,t-1} - \beta X_{1,t-1}) = \text{spread}_t - \text{spread}_{t-1} \tag{5}$$

Thus, the profit of buying one share of stock 2 and selling β share of stock 1 for the period Δt is given by the change in the spread for that period. From equation (1), the spread is stationary by definition and exhibits mean-reverting properties. We can use this to construct a quantitative trading strategy that uses deviations from the long-term equilibrium of a cointegrated pair to open long and short positions. Positions are unwound once the equilibrium is restored, as a consequence of being stationary.

4.2.2. Trading strategy.

We use two criteria to implement the pairs selection phase of the cointegration PTS. First, we sort all possible combinations of pairs based on their SSD in their normalized price during the formation period.† Second, we test each of the pairs with the least SSD for cointegration, using their cumulative return series in the formation period. Pairs that are not cointegrated are eliminated in the selection process. Pairs that are cointegrated will have their cointegration coefficient estimated. We continue until 20 cointegrated pairs with minimum SSD are selected to be traded in the following trading period.‡

We use the two-step Engle-Granger approach (Engle and Granger 1987) to test for the existence of cointegration between nominated pairs and to estimate cointegration coefficient β. In this procedure, the cointegration regression is estimated using OLS in the first step and the error correction model (ECM) is estimated in the second step. For each nominated pair, we then form the spread defined in equation (3) and calculate the spread's mean μ_e and standard deviation σ_e, with the data of the formation period. These parameters are used in the trading period as trades' open and close triggers. From equations (1) and (3) the spread is given by: $\text{spread}_t = e_t$ where $e_t \sim I(0)$, with mean μ_e and standard deviation σ_e. The normalized cointegration spread as:

$$\text{spread}_{\text{normalized}} = \frac{\text{spread} - \mu_e}{\sigma_e} \quad (6)$$

Similar to the DM, we simultaneously open and close long and short positions when the normalized spread diverges beyond 2. However, the values of long and short positions vary from those in the DM. By construction, if the spread drops below -2, we buy 1 dollar worth of stock 2 and sell short β dollar worth of stock 1. Equivalently, we sell short $1/\beta$ dollar worth of stock 2 and buy 1 dollar worth of stock 1, when the spread moves above the $+2$ threshold. We close both positions once the spread returns to zero, which translates into the pair returning to their long-term equilibrium. The pair is again monitored for other potential round-trip trades for the remainder of the trading period.

4.3. Copula method

4.3.1. Framework.

A copula is a function that links marginal distribution functions to their joint distribution function. It captures the dependence structure between the marginal distributions. A copula function is defined as joint multivariate distribution function with uniform univariate marginal distributions:

$$C(u_1, u_2, \ldots, u_n) = P(U_1 \le u_1, U_2 \le u2, \cdots, U_n \le u_n), \quad (7)$$

where $u_i \in [0, 1]$, $i = 1, 2, \ldots, n$. Now, suppose $X_1, X_2, \ldots, X_n$ are n random variables with continuous distribution functions $F_1(x_1), F_2(x_2), \ldots, F_n(x_n)$. Since a random variable with arbitrary distribution can be transformed to a uniform random variable by feeding it into its distribution function, i.e. $U_i = F(X_i)$ where $U_i \sim Uniform(0, 1)$, we can define the copula function of random variables $X_1, X_2, \ldots, X_n$ as:

$$F(x_1, x_2, \ldots, x_n) = C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)) \quad (8)$$

If F_i and C are differentiable 1 and n times, respectively, we can write the joint probability density function (pdf) f as the product of marginal density functions $f_i(x_i)$ and the copula density function c:

$$f(x_1, x_2, \ldots, x_n) = f_1(x_1) \times f_2(x_2) \times \cdots \times f_n(x_n)$$
$$\times c(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)), \quad (9)$$

where the copula density function c is given by differentiating the copula function, C, n times with respect to each marginal:

$$c(u_1, u_2, \ldots, u_n) = \frac{\partial^n C(u_1, u_2, \ldots, u_n)}{\partial u_1 \partial u_2 \cdots \partial u_n} \quad (10)$$

Equation (9) allows us to decompose a multivariate distribution into two components, the individual marginal probability density functions, and the copula density function. Consequently, since all the characteristics of marginal distributions are captured in their pdfs and all the characteristics of the joint distribution are represented by the joint pdf, the copula density function should contain all the dependence characteristics of the marginal distributions.

Therefore, copulas allow for higher flexibility in modelling multivariate distributions. They allow the marginal distributions to be modelled independently from each other, and no assumption on the joint behaviour of the marginals is required. Moreover, the choice of copula is also not dependent on the marginal distributions. Thus, using copulas, the linearity restriction that applies to the dependence structure of multivariate random variables in a traditional dependence setting is relaxed. Thus, depending on the chosen copulas, different dependence structures can be modelled to allow for any asymmetries.

Now, let X_1 and X_2 be two random variables with probability functions $F_1(x_1)$ and $F_2(x_2)$ and joint bivariate distribution function $F(X_1, X_2)$. We have $U_1 = F_1(X_1)$ and $U_2 = F_1(X_2)$ where $U_1, U_2 \sim Uniform(0, 1)$ and their copula function $C(u_1, u_2) = P(U_1 \le u_1, U_2 \le u_2)$. By definition, the partial derivative of the copula function gives the conditional distribution function (Aas *et al.* 2009):

$$h_1(u_1|u_2) = P(U_1 \le u_1|U_2 = u_2) = \frac{\partial C(u_1, u_2)}{\partial u_2} \quad (11)$$

$$h_2(u_2|u_1) = P(U_2 \le u_2|U_1 = u_1) = \frac{\partial C(u_1, u_2)}{\partial u_1}$$

Using functions h_1 and h_2, we can estimate the probability of outcomes where one random variable is less than a certain value, given the other random variable has a specific value. The application of these functions in a PTS is that we can estimate the probability of one stock of the pair moving higher or lower than its current price given the price of the other stock.

4.3.2. Trading strategy.

Similar to the DM, we sort all possible pairs based on SSD in their normalized price during the formation period and nominate 20 pairs with the least SSD to trade in the trading period.§ We continue to fit nominated pairs to copulas using the Inference for Margins method

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†This is set to 12 months to allow for consistency with the DM.
‡This is set to 6 months to allow for consistency with the DM.

§The formation and trading periods are kept at 12 and 6 months, respectively, to be consistent with the distance and cointegration methods.

(Joe 1997) that is a two-step process. First, for each pair, we fit the daily returns of the formation period to marginal distributions and find the two distributions that best fit the each stock. Marginal distributions are selected from extreme value, generalized extreme value, logistic and normal distribution and independently fitted for each stock of the pair. In the second step, with the estimated marginal models' parameters from the previous step, we nominate the copula that best fits the uniform marginals and parameterize the copula. We allow for a range of copulas to be employed in this step, namely the Clayton, Rotated Clayton, Gumbel, Rotated Gumbel and Student-*t*. The best fitting copula is the copula that provides a parsimonious fit for the dependence structure between the stocks. In quantifiable terms, the best copula is chosen by maximizing the log likelihood of each copula density function and calculating the corresponding AIC and BIC. The copula associated with the highest AIC and BIC is then selected as having a parsimonious fit.

Table 1 reports the percentage frequency of copula and marginal models selected for the stock pairs during our empirical investigation. The Student-*t* is selected for 62% of all stock pairs and the Rotated Gumbel for 10.38%. The Logistic model is selected for 86% of the marginal models for each individual stock.

Each day during the trading period, using the daily realizations of random variables U_1 and U_2, that represent the daily returns of two stocks of a pair, we calculate the conditional probabilities, h_1 and h_2 functions defined in equation (11), for each nominated pair (conditional probability functions given in table 2). A value of 0.5 for h_1 is interpreted as 50% chance for the random variable U_1, which is the price of stock 1, to be below its current realization, which is its today's price, given the current price of stock 2. The same interpretation is valid for h_2, which demonstrates the same conditional probability for stock 2. Accordingly, conditional probability values above 0.5 show that chances for the stock price to fall below its current realization is higher than they are for it to rise, while values below 0.5 predict an increase in the stock price compared to its current value is more probable than a decrease. Similar to Xie *et al*. (2014), we define two mispriced indices:

$$m_{1,t} = h_1(u_1|u_2) - 0.5 = P(U_1 \leq u_1|U_2 = u_2) - 0.5 \quad (12)$$

$$m_{2,t} = h_2(u_2|u_1) - 0.5 = P(U_2 \leq u_2|U_1 = u_1) - 0.5$$

The cumulative mispriced indicies M_1 and M_2, which are set to zero at the beginning of the trading period are calculated each day:

$$M_{1,t} = M_{1,t-1} + m_{1,t} \quad (13)$$

$$M_{2,t} = M_{2,t-1} + m_{2,t}$$

Positive (negative) M_1 and negative (positive) M_2 are interpreted as stock 1 (stock 2) being overvalued relative to stock 2 (stock 1). We have arbitrarily set the strategy to open a long–short position once one of the cumulative mispriced indices is above 0.5 and the other one is below -0.5 at the same time. The positions are then unwound when both cumulative mispriced indices return to zero. The pair is then monitored for other possible trades throughout the remainder of the trading period.

4.4. Dynamic transaction costs

Transaction costs play a vital role in the profitability of PTS. Each execution of a complete pairs trade consists of two roundtrip trades. In addition to that, an implicit market impact and short selling costs are also applicable. Since, the sum of these costs can be large they can degrade the profitability of PTS when taken into account. We use a time-varying data-set of transaction costs in line with Do and Faff (2012). The motivation behind this is that commissions are the first element of transaction costs to be considered. As commissions have changed considerably over the last 50 years that we are using as our time span, a flat commission system distorts the accuracy of our study. As such, we use the institutional commissions that Do and Faff (2012) calculated that starts from 70 bps in 1962 and gradually declines to 9 bps for recent years. Similar to their study, we divided our time period into two sub-periods and use a different market impact estimate for each sub-period: 30 bps for 1962–1988 and 20 bps for 1989 onward.† As we screen out stocks that have low dollar value and low market capitalization, we assume the remaining stocks in our sample are relatively cheap to short sell and therefore do not take into account short selling costs explicitly. It is worth noting that we double these costs to cover each complete pairs trade which consists of two round-trip trades.

4.5. Performance calculation

The performance of the three PTS are recorded and compared based on various performance measures including returns. In accordance with Gatev *et al*. (2006) and Do and Faff (2010), two types of returns are calculated: return on employed capital (equation 14) and return on committed capital (equation 15). Return on employed capital for month m, R_m^{EC}, is calculated as the sum of marked-to-market returns on that month's traded pairs divided by the number of pairs that have traded during that month. Return on committed capital for month m, R_m^{CC}, is calculated as the sum of marked-to-market returns on traded pairs divided by the number of pairs that were nominated to trade in that month (20 in our case), regardless of whether they actually traded or not. In comparison to the return on employed capital, return on committed capital is a more conservative measure that mimics what a hedge fund might use to report returns, as it takes into account the opportunity cost of the capital that has been allocated for trading.

$$R_m^{EC} = \frac{\sum_{i=1}^n r_i}{n} \quad (14)$$

$$R_m^{CC} = \frac{\sum_{i=1}^n r_i}{NP}, NP = 20 \quad (15)$$

We execute the PTS each month and do not wait for a trading period to be complete, resulting in six overlapping 'portfolios' each month. The monthly excess return of a strategy is calculated as the equally weighted average return on these six portfolios. As the trades neither necessarily open at the beginning of the trading period nor close exactly at the end of the trading period, the full capital is not always locked in a

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†See Do and Faff (2012) section 3 for full details on commissions and market impact estimations.

Table 1. Proportion of selected copulas and marginal distributions.

Panel A: Copulas					
Copulas	Clayton	Rotated Clayton	Gumbel	Rotated Gumbel	Student-*t*
% of Pairs	6.59	11.54	9.85	10.38	61.64
Panel B: Marginal Distributions					
Marginal distributions:	Extreme value	Generalized extreme value	Normal	Logistic	
% of Stocks	0.15	2.61	11.10	86.14	

Notes: This table shows the proportion of copula models selected for each stock pair as a parsimonious fit (Panel A), along with the proportion of marginal models selected for the stocks as a parsimonious fit (Panel B).

Table 2. Copula conditional probability functions.

Copula	$P(U_1 \le u_1 \mid U_2 = u_2)$	
Student-*t*	$h(u_1, u_2; \rho, \nu) = t_{\nu+1}\left(\dfrac{x_1 - \rho x_2}{\sqrt{\frac{(\nu + x_2^2)(1 - \rho^2)}{\nu + 1}}} \right)$	$\rho \in (-1, 1)$
	$x_i = t_\nu^{-1}(u_i), u_i \in (0, 1) i = 1, 2$	$\nu > 0$
	t_ν: Student's *t*-distribution cumulative distribution function with ν degrees of freedom.	
Clayton	$h(u_1, u_2; \theta) = u_2^{-(\theta+1)}\left(u_1^{-\theta} + u_2^{-\theta} - 1 \right)^{-\frac{1}{\theta} - 1}$	$\theta > 0$
Rotated Clayton	$h(u_1, u_2; \theta) = 1 - \left((1 - u_2)^{-(\theta+1)}\left((1 - u_1)^{-\theta} + (1 - u_2)^{-\theta} - 1 \right)^{-\frac{1}{\theta} - 1} \right)$	$\theta > 0$
Gumbel	$h(u_1, u_2; \theta) = C_\theta(u_1, u_2) * [(-\ln u_1)^\theta + (-\ln u_2)^\theta]^{\frac{1-\theta}{\theta}} * (-\ln u_2)^{\theta - 1} * \frac{1}{u_2}$ $C_\theta(u_1, u_2) = exp\left(-[(-\ln u_1)^\theta + (-\ln u_2)^\theta]^{\frac{1}{\theta}} \right)$	$\theta > 0$
Rotated Gumbel	$h(u_1, u_2; \theta) = 1 - C_\theta(1 - u_1, 1 - u_2) * [(-\ln(1 - u_1))^\theta + (-\ln(1 - u_2))^\theta]^{\frac{1-\theta}{\theta}}$ $* (-\ln(1 - u_2))^{\theta - 1} * \frac{1}{1 - u_2}$ $C_\theta(u_1, u_2) = \exp\left(-[(-\ln u_1)^\theta + (-\ln u_2)^\theta]^{\frac{1}{\theta}} \right)$	$\theta > 0$

Note: This table shows the conditional probability functions of copulas used in the copula method.

trade. In addition, there are months where no trading occurs. As interest is not accrued to the capital when it is not involved in a trade, the performance outcomes are underestimated.

Positions that are opened in the distance and copula methods are $1 long–short positions. $1 long–short positions are defined as opening a long positions worth $1 and a short positions worth $1 simultaneously. Since the money raised from shorting a stock can be used to buy the other stock, these positions are self-financing and do not require any capital to trade. However, for the sake of calculating returns, we adapt the widely used concept of using $1 as the total value of each long–short position. In the cointegration method, by definition, long and short positions are not valued equally. However, since we have designed the method to ensure a $1 long position for every trade, we assume an average $1 value for each long–short position.

5. Results

5.1. Descriptive statistics

Table 3 reports the monthly excess return distribution for each of the three strategies from 1962 to 2014 both before and after transaction costs in two sections. Section 1 of the table shows the return on employed capital, while section 2 reports return on

committed capital (see section 4.5 for details on calculations). As both return measures achieve similar results and rankings for the strategies, we use return on employed capital, hereafter simply referred to as return, to report results for the remainder of this paper, unless stated otherwise. The average monthly excess return of the DM before transaction costs is 0.91%.† Results presented in section 1 of table 3 show that, while the DM and cointegration method both show statistically and economically significant and very similar average monthly excess returns (before and after transaction costs), the copula method's after-cost excess return is relatively small at 5 bps. However, before transaction costs, the copula method is producing a significant 43 bps average excess return. Moreover, all three strategies show small standard deviations, with the lowest belonging to the copula method with 0.0067 after costs and the highest to DM the highest with 0.0110 before costs. The DM and the cointegration method exhibit similar Sharpe ratios. While neither of the strategies show normally distributed returns, the cointegration method is the only strategy whose returns are positively skewed after transaction costs. The return on committed capital measure, presented in section 2 of table 3, produces very similar results.

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†Do and Faff (2010) report a similar monthly excess return of 0.90% before transaction costs.

Table 3. Pairs trading strategies' monthly excess return.

Strategy	Mean	*t*-stat	Std. Dev.	Sharpe ratio	Skewness	Kurtosis	VaR (95%)	CVaR (95%)	JB test *p*.value
Section 1: Return on employed capital									
Panel A: After transaction costs									
Distance	0.0038	5.3937***	0.0110	0.3498	−0.3491	13.2586	−0.0106	−0.0190	0
Cointegration	0.0033	5.1444***	0.0099	0.3497	0.3565	8.5718	−0.0121	−0.0173	0
Copula	0.0005	1.5797	0.0067	0.0749	−0.5127	6.9598	−0.0107	−0.0150	0
Panel B: Before transaction costs									
Distance	0.0091	7.4260***	0.0122	0.7517	0.2231	9.0419	−0.0075	−0.0150	0
Cointegration	0.0085	7.3268***	0.0111	0.7703	0.8130	7.3561	−0.0068	−0.0126	0
Copula	0.0043	7.1598***	0.0071	0.6032	−0.2073	5.7007	−0.0066	−0.0111	0
Section 2: Return on committed capital									
Panel A: After transaction costs									
Distance	0.0032	5.4941***	0.0089	0.3566	−0.3723	24.1712	−0.0074	−0.0135	0
Cointegration	0.0030	5.1963***	0.0085	0.3483	0.6462	12.3600	−0.0091	−0.0144	0
Copula	0.0005	2.1477**	0.0053	0.1008	−0.4790	6.8850	−0.0082	−0.0115	0
Panel B: Before transaction costs									
Distance	0.0068	7.0284***	0.0102	0.6729	0.8152	14.6456	−0.0050	−0.0104	0
Cointegration	0.0068	7.0365***	0.0097	0.6969	1.3934	10.8547	−0.0058	−0.0105	0
Copula	0.0030	7.1436***	0.0055	0.5410	−0.3170	6.3584	−0.0056	−0.0092	0

Notes: This table reports key distribution statistics for the monthly excess return time series of three different PTS - i.e. the distance, cointegration, and copula methods - for July 1962 to December 2014. The formation and trading period for all strategies are set to 12 and 6 months, respectively. The column labeled 'JB Test' tests the null hypothesis of normality of the series using the Jarque-Bera Test.
***significant at 1% level.
**significant at 5% level.
*significant at 10% level.

To further demonstrate the relative performance of the strategies, figure 1 compares the cumulative excess return for each of the three strategies from 1963 to 2014. It can be seen that the DM and cointegration methods' performance are almost identical to each other with the cointegration method slightly underperforming compared to the DM. In contrast, the copula method performs poorly in terms of cumulative excess return. However, the gap between the copula method and the other two strategies is narrower on a risk-adjusted basis, as shown in figure 2.

The five-year rolling sample Sharpe ratio (figure 2) also confirms the nearly identical performance of the DM and the cointegration method. In addition, it shows the risk-adjusted performance of all three strategies fluctuate greatly, but generally maintain an upward trend until around 1985, where they experience their peak and the trend reverses. More importantly, the reversal in this trend occurs in the copula, cointegration and the DM, thus none of the strategies avoid this decline. However, after the downward trend begins, the gap between the risk-adjusted performance of the copula method and the other two strategies becomes smaller than before. It appears that as we move closer to recent years, the three strategies show a very close risk-adjusted performance and there is no clear winner among them.

5.2. Risk-adjusted performance

As the return series of neither of the strategies are normally distributed, the Sharpe ratio, as the classic risk-adjusted measure, has the potential to underestimate risk thereby, overestimating the risk-adjusted performance (Eling 2008). Thus, we further analyse the risk profile of the three PTS, using downside performance measures. Table 4 reports various risk-adjusted metrics for each of the strategies, before and after transaction costs. The measures are divided into two main groups: lower partial moment measures and drawdown measures. Lower partial moment measures take into account only the negative deviations of returns from a specified minimum threshold value.† They appropriately account for downside risk compared to the Sharpe ratio that considers both positive and negative deviations equally. Omega is defined as the ratio of returns above a threshold, to returns below that threshold. The Sortino ratio is the ratio of average excess return, to the absolute value of second lower partial moment (or negative standard deviation). Kappa 3, is the ratio of average excess return to the third lower partial moment‡ (or negative skewness). Drawdown metrics measure the magnitude of losses of a portfolio over a period of time. Maximum drawdown is defined as the maximum possible loss that could have occurred during a time period. The Calmar ratio is defined as the ratio of average excess return to the maximum drawdown. Sterling ratio is the ratio of average excess return to the average of *n* most significant continuous drawdowns, thus reducing the sensitivity of the measure to outliers. The continuous drawdown is defined as the maximum incurred loss that is not interrupted by positive returns. Finally, the Burke ratio is the ratio of average excess return to the square root of sum of the squared *n* most significant drawdowns.§

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†The threshold value is also known as the minimum acceptable return where we use 0%.
‡For detailed explanation and calculation of lower partial moment measures, refer to Eling and Schuhmacher (2007).
§For detailed explanation and calculation of drawdown measures, refer to Schuhmacher and Eling (2011).
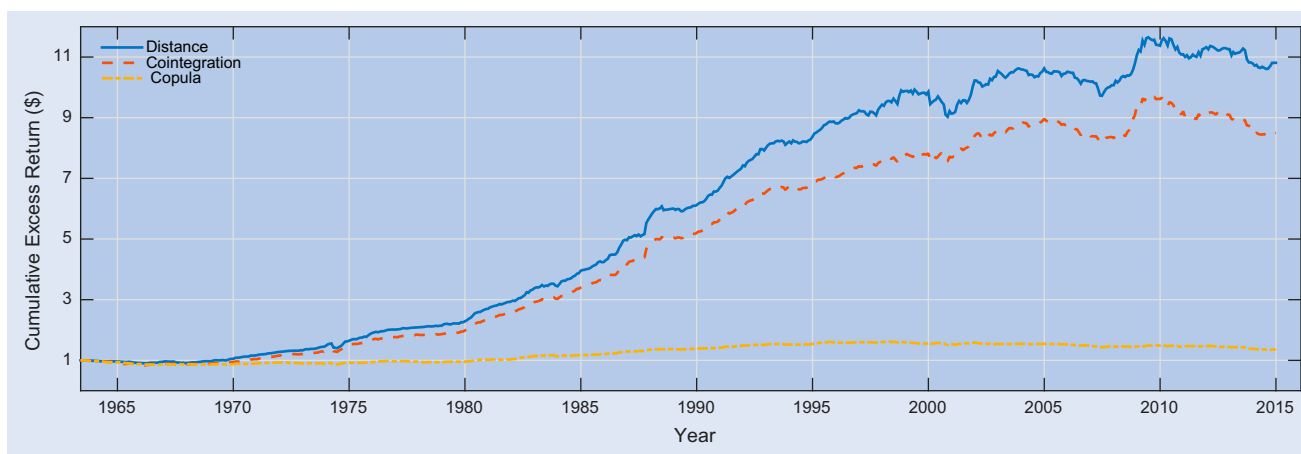
Figure 1.   Cumulative excess return.
Notes: This figure shows the evolution of wealth based upon an investment of $1 in each strategy. The return on employed capital after transaction costs is applied for the calculation of cumulative excess returns.
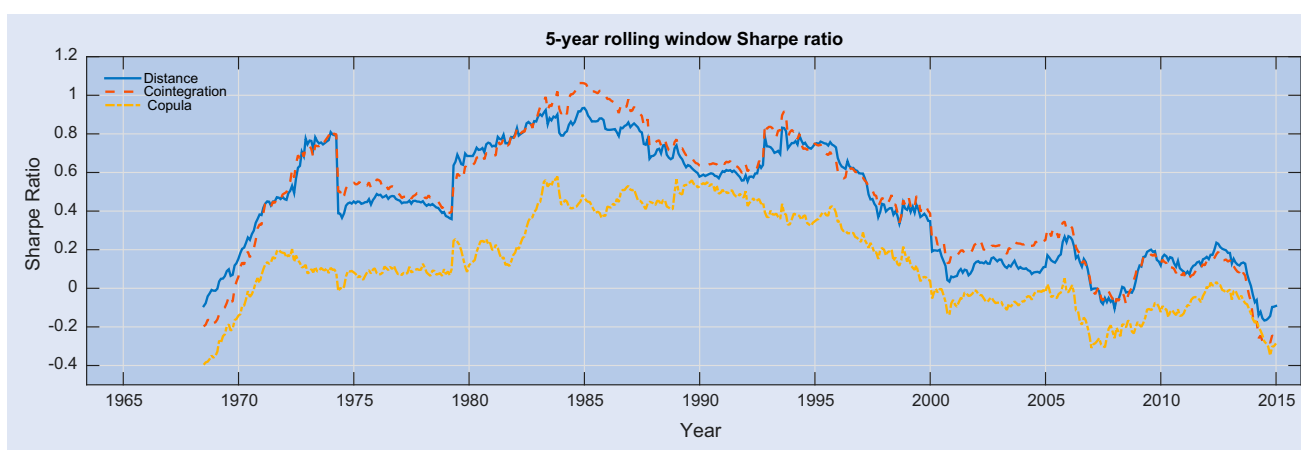

Figure 2.   Five-year rolling sample Sharpe ratio.
Note: This figure shows the 5-year (60-month) rolling sample Sharpe ratio for the three pairs trading strategies.

Table 4.   Pairs trading strategies' risk-adjusted performance.

	Lower partial moments measures			Drawdown measures			
	Omega	Sortino ratio	Kappa 3	Max drawdown	Calmar ratio	Sterling ratio	Burke ratio
Panel A: After transaction costs							
Distance	2.7733	0.6419	0.3350	0.1052	0.0366	0.3434	0.0195
Cointegration	2.6825	0.6774	0.4093	0.1718	0.0201	0.3499	0.0180
Copula	1.2257	0.1083	0.0705	0.1845	0.0027	0.0530	0.0028
Panel B: Before transaction costs							
Distance	9.2018	1.9965	0.9520	0.0737	0.1241	1.0415	0.0687
Cointegration	9.4562	2.4345	1.3344	0.0459	0.1858	1.0335	0.0826
Copula	5.0044	1.3443	0.7606	0.0418	0.1023	0.6229	0.0430

Notes: This table reports key risk-adjusted performance measures for the monthly excess return time series of three different PTS—i.e. the distance, cointegration, and copula methods—for July 1962 to December 2014. The formation and trading period for all strategies are set to 12 and 6 months, respectively.

The cointegration method exhibits the best before-cost risk-adjusted performance with the best figures for all but the maximum drawdown and Sterling ratio measures. However, its after-cost performance is similar to that of the DM. The copula method is the poorest PTS among the three, except for showing the least before-cost maximum drawdown. The major contributor to the low performance of the copula method is the insignificance of its mean returns. This can be verified by the fact that the strategy transforms from having the best maximum drawdown to the worst when transaction costs are taken into account. At a lower magnitude, the other two strategies also suffer considerably when transaction costs are taken into account. We can see the after-costs Omega ratio decreases by 70% or more for all strategies. The same decrease is observed in the Sortino ratio of DM and the cointegration method, while for the copula method this figure rises above 90%.

5.3. Properties of pair trades

Studying the properties of converged and unconverged pairs allows us to further investigate the sources of each PTS' profitability.

Figure 3 illustrates the trade distributions after transaction costs for each strategy. All the strategies have fatter left tails than the right which makes extreme negative returns more likely than extreme positives. This can be attributed to the fact that for all strategies, we use some criteria to close a trade once it has converged, however unconverged trades remain open for the duration of the trading period. Albeit infrequently, higher profits do occur when a pair's spread suddenly diverges overnight by much more than just the triggering amount. A similar scenario can occur upon the convergence of a pair. Contrary to profit-making trades, trades that do not converge can accumulate big losses before being forced to close by the strategy at the end of their trading period, which results in fat left tails.

Specifically, for the DM and cointegration methods, where the criteria for opening and closing positions are directly related to the prices of stocks, the magnitude of profit for each trade is bound to the scale of divergence that is used in the opening trade criteria. Therefore, the profits per each trade is generally limited thus, we do not observe fat right tails. In the copula method, the opening and closing criteria are based on the probability of relative mispricing within pairs, rather than being directly related to the spread. Therefore, the observed fatter right tail in its distribution is expected. In practice, an additional risk-limiting criteria for closing a trade, i.e. a stop-loss measure, can potentially limit the extreme losses. In that case, strategies such as the copula method have the potential to perform well as their profits are not bound to some specific amount.

Table 5 reports further statistics on the converged and unconverged trades. As expected, all strategies show positive average return for their converged and negative average return for their unconverged trades. For converged trades, the cointegration method has the highest average return (4.37%), but the second-highest Sharpe ratio (1.62) after the DM's (1.79). The DM has the highest percentage of converged trades at 62.53%, while the cointegration method is second with 61.35%. It is in fact this higher percentage of converged trades that puts the DM first among the three strategies in terms of overall monthly return, as shown in section 5.1.

Interestingly, for converged trades, the copula method also shows a significant average return (Sharpe ratio) of 3.95% (1.05), that are both comparable to the other two methods. The copula method exhibits the highest mean return and Sharpe ratio for unconverged trades among the three strategies, which demonstrates its lower risk profile. However, the considerably high proportion of unconverged trades for the copula method degrades its performance to only 0.3% average return for all trades compared to 1.2 and 1% for the DM and cointegration methods, respectively. In fact, the copula method is the only strategy that has less converged trades than unconverged. Therefore, the performance of the copula method can be vastly improved if the methodology can be enhanced to reduce the frequency of unconverged trades.

The DM has the lowest average number of days that takes for its converging trades to converge (21 days), followed by the cointegration (23 days) and the copula method (26 days). Intuitively, the DM is less exposed to risks as the trades are open for shorter periods. More than 98% of the converged trades in the DM and cointegration methods generate positive return that is slightly higher than the copula method at 94%.

Although the two-step selection method that we have used leads to similarities in the distribution of the returns of the three strategies, there are some notable differences in their behaviour. In table 5, we report the difference in the number of distinct pairs for each strategy to highlight this point. On a monthly basis, the cointegration method leads to the highest number of distinct pairs at 4421, while the DM and the copula method have a lower number of distinct pairs (approximately 4100 pairs). The different sum of distinct pairs per month for each strategy indicates that even if the same pairs are selected in different strategies, the opening and closing characteristic of each strategy leads to those pairs exhibiting different outcomes for each strategy in terms of the month in which the trades are initiated. Analysing the entire sample, the cointegration method has the largest number of distinct pairs totaling 17307 pairs, followed by the DM and the copula method at 15026 and 13450 pairs, respectively. In other words, based on this measure, the copula (distance) method involves more than 20% (10%) fewer distinct pairs compared to the cointegration strategy.

When considering the converged and unconverged trades, as shown in figure 4, the DM strategy generates the most profitable trades (71% of all trades), followed by the cointegration method (69%). However, since the average returns of the cointegration method are smaller in magnitude, the effect of this high percentage is reduced, thus resulting in a slightly lower monthly return compared to the DM as demonstrated in section 5.1.

5.4. Risk profile

We further investigate the source of pairs trading profitability to determine if performance is compensation for risk. Thus, we regress the strategies' monthly excess return series against several risk factors. First, we examine the strategies against the Fama and French (1993) three factors plus the momentum and a liquidity factors. We added the liquidity factor since it is argued that liquidity shock is a source of pairs trading profitability (Engelberg *et al*. 2009). Similar to Do and Faff (2012), we use the 'Innovations in Aggregate Liquidity' series (Pástor and Stambaugh 2003).

Results from Panels A and B of table 6 show that, the profits of PTS are not fully explained by the risk factors. In fact, large and significant alphas are observed for all the strategies. It is worth mentioning that alphas are very close to their corresponding strategy's mean of monthly excess return, implying that after being adjusted for these risk-factors, the PTS' profits remain unaffected. Moreover, the momentum factor is negatively correlated with strategies' profits, with significant *t*-statistics both before and after transaction costs for all but the copula methods. Similarly, liquidity has also a considerable
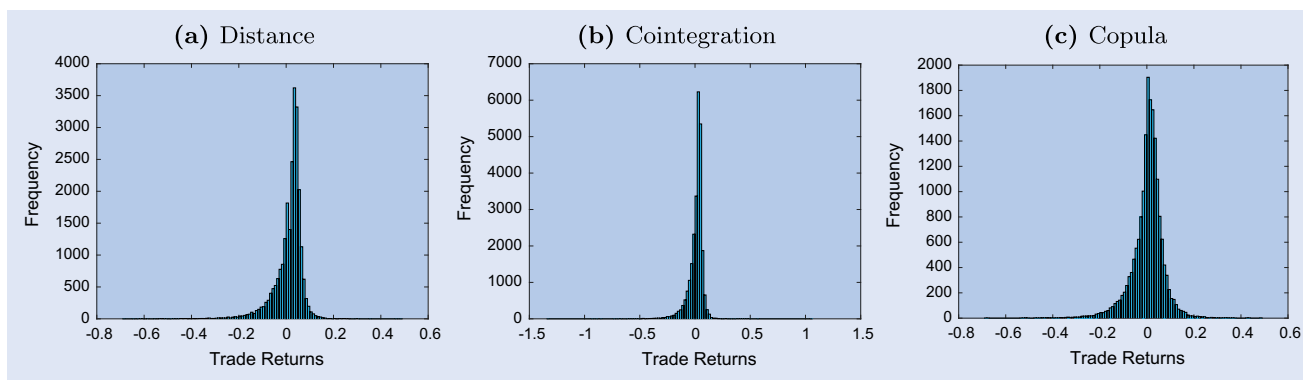
Figure 3.   Distribution of trade returns.
Notes: This figure shows the distribution of trade returns after transaction costs for each of the three strategies.

Table 5.   Converged and unconverged trades return series.

Strategy	# Distinct pairs	Sum of D.P.P.M.	Trade type	% of trades	Mean	St.D.	S.R.	Skewness	Days open Mean	Median	Positive trades (%)
Distance	4061	15087	C	62.53	0.0426	0.0238	1.7889	2.1702	21.15	14	98.42
			U	37.47	−0.0399	0.0702	−0.5691	−2.3428	–	–	–
Cointegration	4421	17348	C	61.35	0.0437	0.0270	1.6173	5.8236	22.65	15	98.64
			U	38.65	−0.0436	0.0754	−0.5782	−2.7620	–	–	–
Copula	4100	13463	C	39.98	0.0395	0.0377	1.0485	1.7030	26.30	17	94.41
			U	60.02	−0.0215	0.0752	−0.2855	−0.9488	–	–	–

Notes: This table reports key distribution statistics for converged and unconverged trade return series after transaction costs. The 'Trade Type' column classifies trades into converged (C) and unconverged (U). '# Distinct Pairs' and 'Sum of D.P.P.M' columns report the number of distinct pairs and sum of distinct pairs per month for each trading strategy throughout the study period, respectively. The 'St.D' and 'S.R' columns report standard deviation and Sharpe ratio, respectively. Two columns under the 'Days Open' title show the mean and median number of days that a converged trade remains open. 'Positive Trades (%)' shows the percentage of converged trades with positive returns. Note that all calculations are based on after-transaction cost returns.
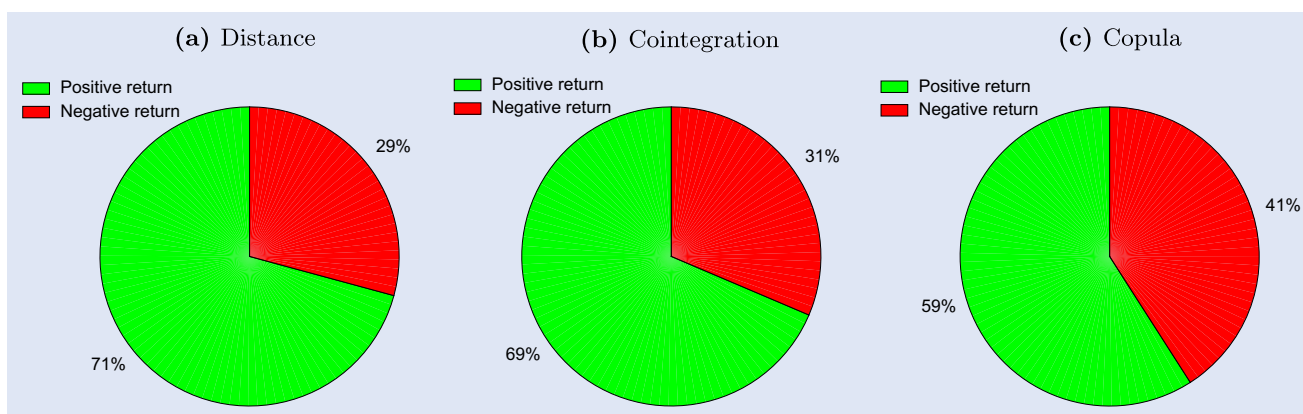

Figure 4.   Proportion of positive and negative trades.
Note: This figure shows the proportion of trades with positive and negative returns after transaction costs for each of the three strategies.

negative effect on profits, but only for the DM and the cointegration method. Interestingly, liquidity is unable to explain the profits of the copula method neither before nor after costs, as opposed to postulations of Engelberg *et al.* (2009) who suggest otherwise. This further demonstrates the unique nature of the copula method in comparison to the other PTS. Surprisingly, no other factor, including the market excess return in the copula method, is correlated to the strategies' returns, which is further evidence for the market-neutrality of these strategies. From an investor's perspective, this market neutrality can have diversifying effects on investment portfolios, by reducing risk,

particularly those that have strong correlations with market returns.

Next, we regress the monthly excess returns of the three strategies against the recent Fama and French (2015) 5 factor model. This model is an attempt by the authors to improve their well-known three factor model by introducing two additional factors: profitability and investment factors. Robust Minus Weak (RMW), a measure of profitability, is defined as the difference between returns on robust and weak profitability portfolios, while Conservative minus Aggressive (CMA), which represents the investment factor, is defined as the differ-
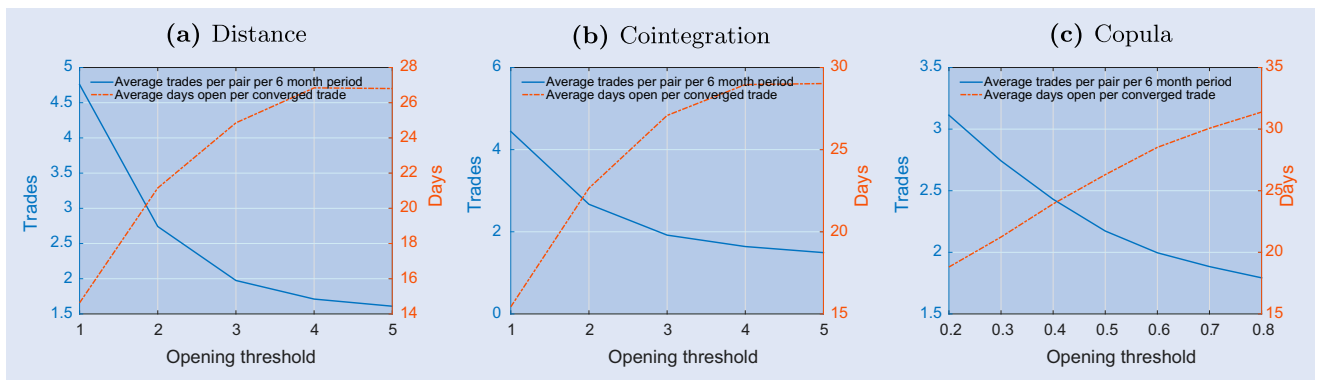
Figure 5. Threshold sensitivity analysis.
Note: This figure shows the average number of trades per pair per six month period and the average convergence time for each strategy. The solid lines represent 'Trades' and the staggered lines represent 'Days'.
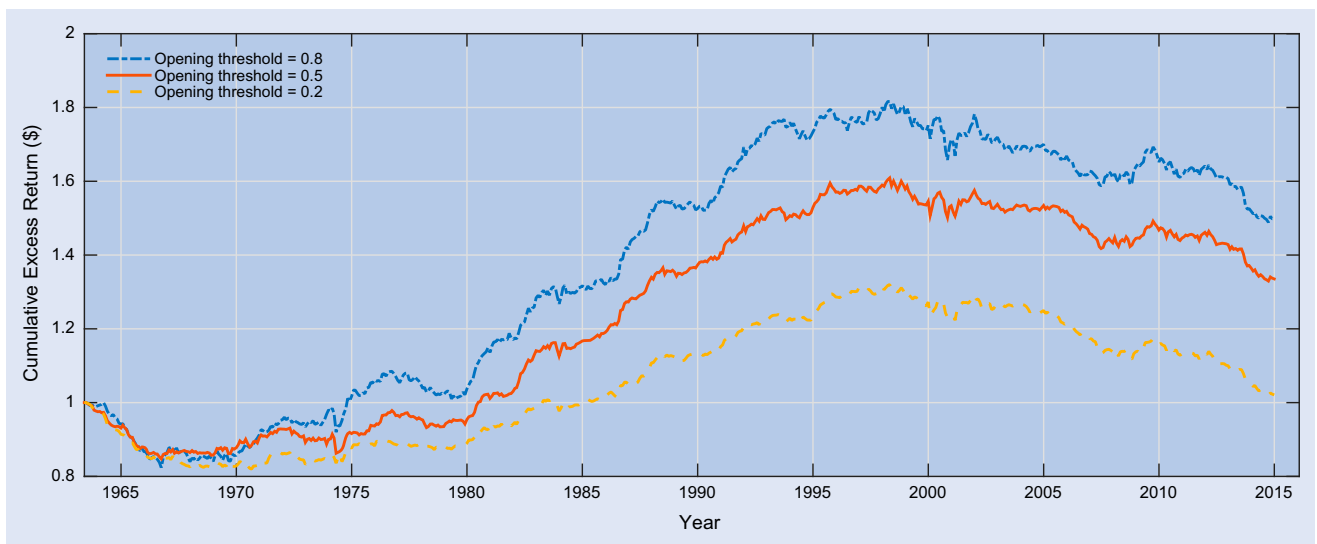

Figure 6. Copula method's cumulative excess return.
Notes: This figure shows the evolution of wealth based upon an investment of $1 for the copula method using three different opening thresholds. Return on employed capital after transaction costs is applied to calculate the cumulative excess return.
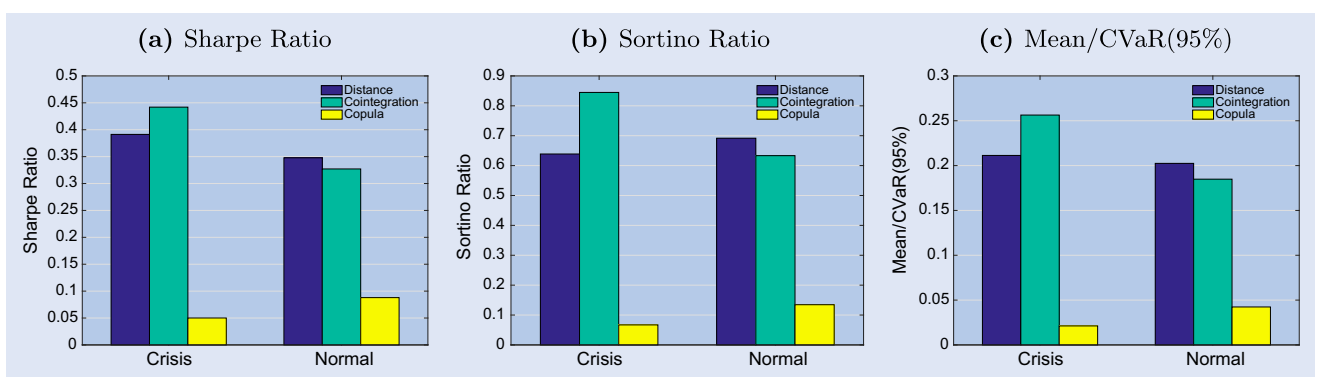

Figure 7. Average monthly performance in crisis and normal periods.
Notes: This figure shows the performance of each strategy during 'Crisis' and 'Normal' periods. 'Crisis' is defined as the lowest quintile of the US S&P 500 stock market index returns on our entire sample (11 worst-performing years out of 53) and 'Normal' consists of the remaining annual sub-periods.

ence between the returns of conservative and aggressive portfolios. Results, reported in Panels C and D of table 6, show that there is a negative correlation between before-cost pairs profit and RMW for all three strategies. This relationship is statistically significant at 1, 5 and 10% for the DM, cointegration and

copula methods, respectively, albeit the size of the regression coefficients pointing out the smallness of these correlations' economic significance. Nonetheless, RMW only affects the after-cost profitability of the DM, which is again economically insignificant. Notably, the size factor is related to some PTS'

Table 6.　Monthly return risk profile.

Strategy	Alpha	MKT	SMB	HML	MO	LIQ	RMW	CMA
Fama French 3 Factors + Momentum + Liquidity								
Panel A: After transaction costs								
Distance	0.0042	0.0113	−0.0174	−0.0095	−0.0341	−0.0395		
	(6.9770)***	(0.3229)	(−1.1739)	(−0.5373)	(−2.9074)**	(−3.2232)***		
Cointegration	0.0038	0.0042	−0.0124	−0.0093	−0.0311	−0.0391		
	(6.5738)***	(0.2326)	(−0.8623)	(−0.6125)	(−2.5911)***	(−3.7265)***		
Copula	0.0005	0.0183	−0.0031	0.0026	−0.0133	−0.0053		
	(1.5958)	(2.2306)**	(−0.3530)	(0.2449)	(−1.8706)*	(−0.9097)		
Panel B: Before transaction costs								
Distance	0.0095	0.0091	−0.0106	−0.0041	−0.0344	−0.0480		
	(12.8246)***	(0.5870)	(−0.6342)	(−0.2036)	(−2.3164)**	(−3.6379)***		
Cointegration	0.0089	0.0013	−0.0042	−0.0033	−0.0316	−0.0471		
	(12.3023)***	(0.0977)	(−0.2520)	(−0.1874)	(−2.6092)***	(−4.1281)***		
Copula	0.0043	0.0156	0.0027	0.0054	−0.0105	−0.0076		
	(10.9267)***	(1.9104)*	(0.2855)	(0.5180)	(−1.4562)	(−1.1992)		
Fama French 5 Factors								
Panel C: After transaction costs								
Distance	0.0042	−0.0000	−0.0004	0.0001			−0.0006	−0.0002
	(6.4904)***	(−0.1574)	(−2.2536)**	(0.4173)			(−2.1017)**	(−0.3922)
Cointegration	0.0037	−0.0001	−0.0003	−0.0000			−0.0003	0.0001
	(5.8411)***	(−0.3598)	(−1.6281)	(−0.2170)			(−1.2827)	(0.3341)
Copula	0.0005	0.0002	−0.0001	0.0000			−0.0003	0.0002
	(1.3806)	(2.3186)**	(−1.1523)	(0.0244)			(−1.6446)	(0.6542)
Panel D: Before transaction costs								
Distance	0.0097	−0.0001	−0.0004	0.0002			−0.0008	−0.0002
	(12.0386)***	(−0.5037)	(−1.9671)**	(0.5878)			(−2.4880)**	(−0.4487)
Cointegration	0.0089	−0.0001	−0.0002	−0.0000			−0.0005	0.0001
	(11.3104)***	(−0.7528)	(−1.2680)	(0.1123)			(−1.7344)*	(0.1707)
Copula	0.0044	0.0001	−0.0001	0.0001			−0.0004	0.0001
	(10.7402)***	(1.6047)	(−0.7304)	(0.4058)			(−2.0318)**	(0.2532)
Fama French 5 Factors + Momentum + Liquidity								
Panel E: After transaction costs								
Distance	0.0043	0.0085	−0.0298	−0.0086	−0.0320	−0.0379	−0.0005	0.0000
	(7.0850)***	(0.5365)	(−1.7779)*	(−0.3225)	(−2.3645)**	(−3.1539)***	(−1.6848)*	(-0.0771)
Cointegration	0.0038	0.0054	−0.0174	−0.0214	−0.0312	−0.0383	−0.0002	0.0003
	(6.4530)***	(0.3983)	(−1.0756)	(−0.9314)	(−2.8805)***	(−3.6670)***	(−0.6737)	(0.6717)
Copula	0.0005	0.0188	−0.0097	−0.0073	−0.0129	−0.0044	−0.0002	0.0002
	(1.6229)	(2.2878)**	(−0.9288)	(−0.5435)	(−1.8637)*	(−0.7576)	(−1.3997)	(0.8898)
Panel F: Before transaction costs								
Distance	0.0098	0.0050	−0.0277	−0.0010	−0.0314	−0.0458	−0.0006	−0.0001
	(12.8650)***	(0.2938)	(−1.4603)	(−0.0341)	(−2.0577)**	(−3.5961)***	(−2.1341)**	(−0.1778)
Cointegration	0.0090	0.0012	−0.0135	−0.0129	−0.0308	−0.0458	−0.0003	0.0002
	(12.1689)***	(0.0830)	(−0.7529)	(−0.4914)	(−2.5201)**	(−4.1141)***	(−1.1944)	(0.4646)
Copula	0.0044	0.0148	−0.0062	0.0003	−0.0095	−0.0064	−0.0003	0.0001
	(10.8094)***	(1.7631)*	(−0.5395)	(0.0227)	(−1.3372)	(−1.0278)	(−1.8470)*	(0.4151)

Notes: This table shows results of regressing monthly return series (after and before transaction costs) against Fama-French 3 factors plus momentum and liquidity factors (Panels A and B), Fama-French 5 factors (Panels C and D), and Fama-French 5 factor plus momentum and liquidity factors (Fama and French 2015) (Panels E and F). The column labeled 'Alpha' reports the estimated regression intercept, while columns 'MKT', 'SMB', 'HML', 'MO', 'LIQ', 'RMW', and 'CMA' report the estimated coefficients for the following factors respectively: Market Excess Return, Small minus Big, High minus Low, Momentum, Liquidity, Robust minus Weak, and Conservative minus Aggressive (CMA) portfolios. For the liquidity factor, the Innovations in Aggregate liquidity measure (Pástor and Stambaugh 2003) from WRDS is applied. The *t-statatistics* for each regression coefficient, are given in parentheses and calculated using Newey-West standard errors with 6 lags.
*significant at 10% level.
**significant at 5% level.
***significant at 1% level.

return, although economically insignificant. Similar to prior results, alphas remain large and statistically significant at 1% for all strategies, with the exception of after-costs copula method, which shows that the risk factors are unable to account for the profits generated by the strategies.

Finally, by combining the two prior regressions, we attempt to examine the effects of all mentioned factors, i.e. Fama French five factors plus momentum and liquidity factors, on monthly returns. We find that, even after taking into account all the factors, the alphas for all strategies remain large and significant, highlighting the fact that neither of these factors can explain away the returns of the three strategies. Panels E and F of table 6 shows that momentum and liquidity are both negatively correlated to the DM and cointegration methods' returns as expected, though neither of the factors shows a significant correlation with the copula method's returns.

Table 7. Copula method's sensitivity analysis.

Opening threshold	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Mean	0.0001	0.0004	0.0006	0.0005	0.0006	0.0007	0.0007
Min	−0.0291	−0.0286	−0.0254	−0.0455	−0.0391	−0.0479	−0.0513
Max	0.0210	0.0258	0.0252	0.0231	0.0243	0.0355	0.0318
Standard deviation	0.0061	0.0060	0.0061	0.0067	0.0073	0.0080	0.0086

Note: This table reports statistics for sensitivity of the copula method's monthly return on employed capital after transaction cost to different opening thresholds.

Table 8. Modified threshold distance method.

Strategy	Mean	Median	Std. Dev.	Sharpe ratio	Skewness	Kurtosis	VaR (95%)	CVaR (95%)
Modified threshold distance	0.0038	0.0035	0.0121	0.3173	−0.3806	11.5077	−0.0132	−0.0222
Distance	0.0038	0.0032	0.0110	0.3498	−0.3491	13.2586	−0.0106	−0.0190

Notes: This table reports key distribution statistics for the monthly excess return time series of the MTDM and DM. MTDM is a form of DM where the opening threshold is increased from two standard deviations to two standard deviations plus transaction costs.

5.5. Sensitivity analysis

In prior analysis, for DM and the cointegration method we apply a constant threshold of 2 standard deviations, and for the copula method a value of 0.5. We analyse the sensitivity of the PTS outcomes when the thresholds are changed. Figure 5 shows how various opening thresholds affect the behaviour of the PTS. For all strategies, as the opening threshold increases, the average number of days for a pair to converge also increases. This is due to the fact that when a larger divergence is required to open a trade, it takes more time for this divergence to reverse. Moreover, again as expected, average trades per pair per six month period decreases with the increase in the opening threshold for all strategies.

The robustness of the copula method's performance is studied using multiple opening thresholds[†] (0.2, 0.3, 0.4, 0.5, 0.6, 0.7, and 0.8). Table 7 demonstrates that the copula method's performance is robust to the opening threshold with the mean monthly return on employed capital after transaction costs (varying from 1 to 7 bps). The minimum, maximum and standard deviation of the returns do not vary substantially across multiple opening thresholds. Figure 6 shows that cumulative excess return of the copula method is robust to different opening thresholds.

As stated in section 4.1, the DM's opening criterion is the divergence of the pair's spread from the two historical standard deviation threshold. The trades will then be closed upon the convergence of the spread to zero. For the pairs with low spread volatility, this method of opening and closing trades can result in negative returns after the transaction costs are deducted. As an alternative, we modified the opening threshold of the DM from two historical standard deviations to two standard deviations plus the transaction costs of the trades. The performance of this Modified Threshold Distance Method (MTDM) and its comparison with the DM is presented in table 8.

Although the mean monthly return of the DM and MTDM are equal, the DM has a higher Sharpe ratio, as the MTDM has

a higher standard deviation. The VaR and CVaR of the MTDM are also slightly inferior to DM. Thus, increasing the opening threshold of the strategy leads to increases in the volatility of monthly returns due to a decrease in the number of trades.

5.6. Crisis vs. non-crisis

Next, we compare the performance of PTS in 'Crisis' and 'Normal' periods. The 'Crisis' period is defined as a subsample that comprises the lowest quintile of years where the US equity market exhibited its worst performance.[‡] The analysis of trading strategies during crisis periods is increasingly important due to risk-averse investors seeking assets that are safe havens or hedges during market downturns (Low, Yao *et al.* 2016). Normal refers to the subsample that excludes the 'Crisis' period. Figure 7 reports the risk-adjusted performance of the strategies in these periods.

With the exception of the DM's Sortino ratio, all strategies show better performance in the 'Crisis' period compared to the 'Normal' period. While the cointegration method shows a slightly inferior performance compared to the DM in the 'Normal' period, it clearly outperforms all strategies in the 'Crisis' period. This superiority is most obvious in the Sortino ratio which rises from 0.63 in the 'Normal' period to 0.84 in the 'Crisis' period. In comparison, the DM experiences a drop in its Sortino ratio from 0.69 to 0.64 for the same periods. These results concur with the work of Do and Faff (2010) that motivate the use of such market neutral strategies in portfolios as protection against turbulent market conditions. In times that other passive investment strategies tend to perform poorly, an investment strategy that includes a PTS such as the cointegration method, can provide the benefits of diversification.

5.7. Performance in different time periods

Figure 8 reports each strategy's mean monthly excess return over 5-year periods for the duration of the study, both before

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[†]Opening threshold refers to the cumulative mispriced indicies M_1 and M_2, formulated in section 4.3.2. These indicies are calculated by accumulating the pairs' daily conditional probability, h_1 and h_2.

[‡]This is a total of 11 out of 53 years consisting of 1969, 1972, 1973, 1977, 1981, 1983, 1987, 2000, 2001, 2007 and 2008.
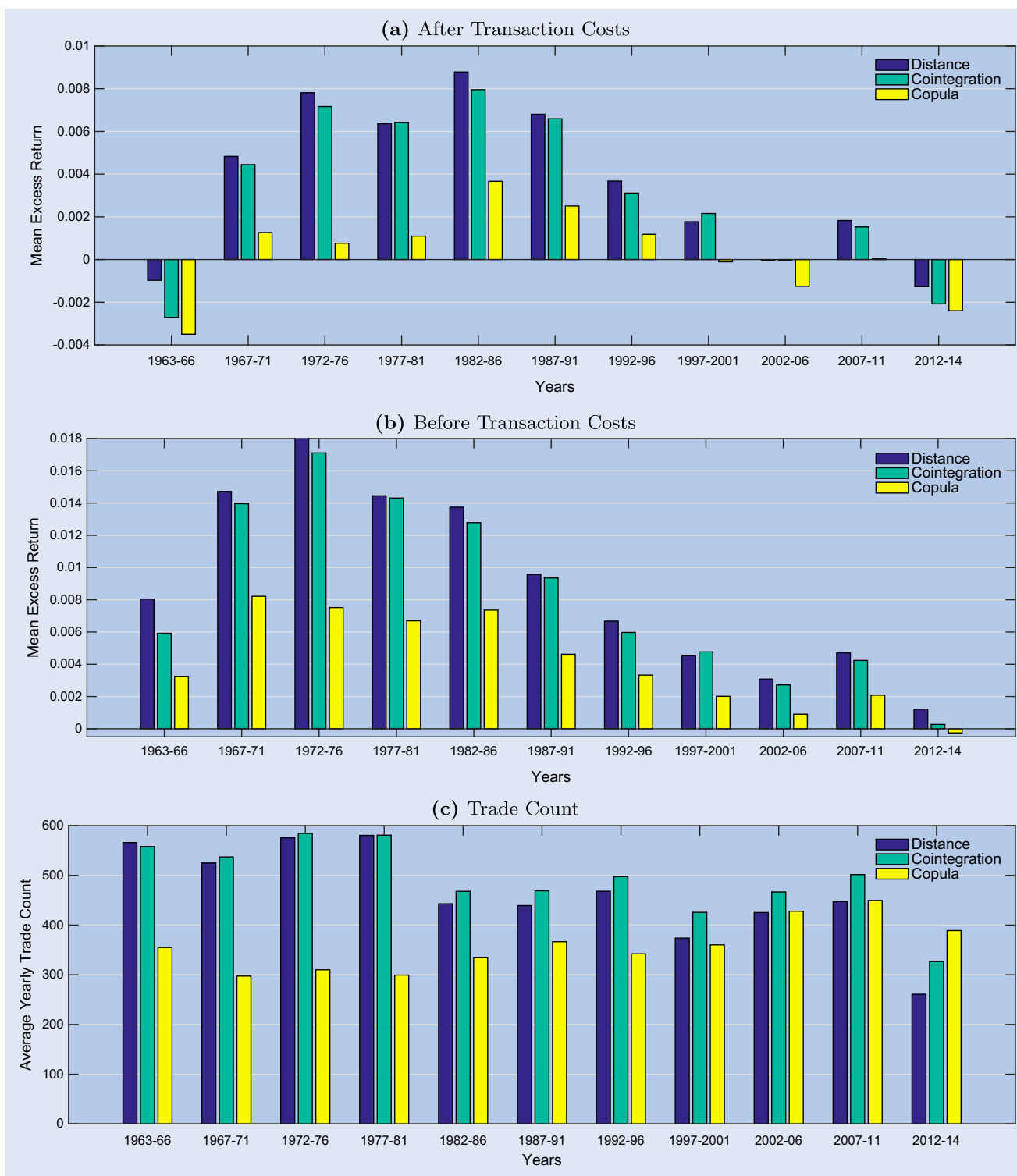
Figure 8. Sub-period performance of pairs trading strategies.
Notes: This figure shows the performance metrics of each strategy in five-year periods. (a), (b) shows the mean monthly excess returns after (before) transaction costs. (c) shows the trade count.

and after transaction costs. The strategies best performance period is 1982–1986 if transaction costs are taken into account (Figure 8(a)), but 1972–1976 if they are not (Figure 8(b)). Interestingly, we can see a decrease in the trade count in the 1982–1986 period (Figure 8(c)). Since the magnitude of trading costs is considerable, this decrease causes the after-cost performance to rise. For the rest of the periods, as the average trade count remains relatively constant, the decline in before-cost performance directly affects the after-cost performance.

Notably, the DM and cointegration method have experienced a 40 and a 35% drop in their average number of trades from the 2007–2011 to the 2012–2014 period respectively, whereas the copula method's average trade count has dropped by 15% in the same period.

This shows that in recent years, pairs whose prices closely follow each other diverge less frequently than they used to before. However, more complex trading strategies are able to produce trading opportunities more frequently. We propose that

PTS can be improved by reducing the amount that pairs need to deviate from the equilibrium before a strategy triggers a pairs trade. This reduction on the 'restrictiveness' of the PTS will result in a higher number of trades and turnover, but as trading costs have declined dramatically over the years, there should be sufficient profit in each trade to cover transaction costs. Admittedly, this reduction in the trade trigger amount will reduce the average profit per trade, however the increased frequency of trades will compensate for the smaller average return per trade, thus enhancing the overall profit of the strategy. The reduction in the trade trigger amount will also reverse the trend of the decreasing number of trades, as observed in recent years.

6. Conclusion

Pairs trading is a popular market neutral trading strategy that purports to profit regardless of market conditions. The strategy was first pioneered in the 1980s by traders Gerry Bamberger and the quantitative trading group headed by Nunzio Tartaglia at Morgan Stanley (Bookstaber 2007). Prior to the spectacular series of events that led to Long-Term Capital Management (LTCM)s demise, LTCM itself engaged in pairs trading strategies (Lowenstein 2000). Although Gatev *et al.* (2006) find that a simple pairs trading strategy (i.e. the DM) generates profits from 1962 to 2002, Do and Faff (2010, 2012) report their declining profitability, after accounting for transaction costs. Other studies (Vidyamurthy 2004, Wu 2013) introduce more sophisticated methodologies to improve the strategies using cointegration and copula frameworks, respectively. However, these studies are limited in terms of stock pairs investigated, sample period, and robustness of analysis. Thus, in this paper it leads us to ask, in a long-term study in the US market, do pairs trading strategies using sophisticated divergence and convergence models lead to an increase in profits? Are these profits sustainable, even after transaction costs? What risk-factors are these pairs-trading strategies exposed to? What is the performance of these pairs trading strategies during highly turbulent market conditions?

Our study examines and compares the performance of three pairs trading strategies using daily US stock data from July 1962 to December 2014: the distance, cointegration and copula methods. We use a time-varying series of trading costs. We find that the DM shows a slightly higher monthly return than the cointegration method, but the cointegration method exhibits a slightly higher Sharpe ratio. However, on a risk-adjusted basis, the two strategies perform equally well over the full sample period. The copula method does not perform as well as the other two methods in either economic or risk-adjusted performance.

We find that the market's excess return fails to account for the performance of the three strategies. This further demonstrates the market neutrality of such strategies that motivates their use by practitioners as an alternative investment strategy for reducing exposure to market risk and support by academics in asset allocation that pairs trading strategy is an effective diversifier in an investment portfolio. We provide evidence that the DM and cointegration methods' economic and risk-adjusted performance are higher in crisis periods. Therefore, such strategies can be included in investment portfolios in turbulent market conditions for both reducing downside risk and capturing alpha.

Although we find the performance of the copula method to be weaker than the DM and cointegration methods', certain attributes of this strategy deserves further attention. First, in recent years the DM and cointegration strategies suffer from a decline in trading opportunities, whereas the copula method remains stable in presenting such opportunities. Thus, the factors affecting the decline in the frequency of trades in other methods have not affected the copula method, making it a reliable substitute for the less sophisticated methods. It also shows that while market participants might have traded away the arbitrage opportunities captured by less sophisticated methods, leading to fewer such trading opportunities, the copula method shows a steady trend in the number of trading opportunities. Second, the copula method shows returns comparable to those of other methods in its converged trades, even though its relatively high proportion of unconverged trades countervails a considerable portion of such profits. Therefore, any attempt to increase the ratio of converged trades or limit their losses would result in enhanced performance outcomes. This can be done by implementing a stop-loss mechanism that limits the losses or by optimizing the formation and trading periods. Third, the copula method's unconverged trades exhibit higher risk-adjusted performance than those of any other strategy which further motivates the use of such strategies. Finally, we find that the Student-*t* is selected as the copula that provides a parsimonious fit for the dependence structure across stock pairs in pairs trading on the US market (61% of all pairs investigated). This is due to the flexibility of the Student-*t* copula in modelling correlation (positive and negative) and fat tails (leptokurtosis). Thus, we highlight the advantage of using copula in flexibly modelling the dependence structure and marginal models across stock pairs.

We design a computationally feasible copula PTS that incorporates aspects of the DM and copula technique. Our approach uses the SSD to identify the stock pairs, and from a range of marginal and copula models, selects a suitably parsimonious fit for each of the stock pairs. Such an approach is designed to ease the implementation for use by practitioners where speed and computational efficiency is an important consideration when implementing a trading strategy (Clark 2012, Angel 2014, Brogaard *et al.* 2014). Further studies in the application of copulas in pairs trading should involve investigating the out-of-sample performance of a trading strategy that is solely relies on copulas for the selection of its pairs. This investigation will lead to further understanding on whether the application of copulas sufficiently enhances PTS to justify the cost of the additional resources required by its computational complexity.

In conclusion, pairs trading strategies still continue to be a profitable trading strategy that remains robust in highly volatile market conditions. Although the profitability of strategies based on distance and cointegration is declining in recent years, an application of copulas to pairs trading strategy has been stable. In this respect, the copula method is promising and is deserving of further attention by practitioners and the academic community.

Disclosure statement

No potential conflict of interest was reported by the authors.

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