DISPARITY LIMITS OF STEREOPSIS

KENNETH N. OGLE, Ph.D.
ROCHESTER, MINN.

THE OLDER visual physiologists, in particular Volkmann, von Helmholtz,[1] Hering,[2] and von Kries and Auerbach,[3] knew that stereoscopic depth perception exists even when the disparities between images in the two eyes are large enough that those images appear double. It is a common experience for observers using the Hering "drop test" to report that the falling beads appear unmistakably nearer or farther than the fixation point even when the beads are seen as double images. Yet von Helmholtz stated that if the disparity was too large then indeed would stereopsis cease. Hillebrand,[4] however, in his book published in 1929 declared that stereopsis occurs only so long as the disparate images in the two eyes are fused. Also, Matsuda [5] declared that for vision in low illuminations the limit of the perception of double images coincides with the limit of stereoscopic depth perception. Tschermak [6] took a middle position when he wrote that fusion represents the most favorable case for, but not the absolute condition of, binocular depth localization by stereopsis. Indeed, Tschermak and Hoefer [7] reported that two needles, both seen as double images, could be adjusted for the same distance from the eyes with considerable precision, even with short periods of illumination. Similarly, Aall,[8] for example, was able to adjust subjectively the separations between three needles in the median plane until the two spaces appeared equal, all the while the farther and nearer needles were seen as double images. Burian [9] also showed

---

From the Section of Biophysics and Biophysical Research, Mayo Foundation, University of Minnesota, and the Mayo Clinic.

1. Southall, J. P. C.: Helmholtz's Treatise on Physiological Optics, Ed. 3, Menasha, Wis., Optical Society of America, 1925, Vol. 3, pp. 430-431.

2. Hering, E.: Spatial Sense and Movements of the Eye, translated by C. A. Radde, Ed. 1, Baltimore, American Academy of Optometry, 1942.

3. von Kries, J. V., and Auerbach, F.: Die Zeitdauer einfachster psychischer Vorgänge, Arch. Anat. u. Physiol., pp. 297-378, 1877.

4. Hillebrand, F.: Lehre von den Gesichtsempfindungen, auf Grund hinterlassener Aufzeichnungen, Wien, Springer-Verlag, 1929.

5. Matsuda, A.: Untersuchungen zur optischen Raumwahrnehmung, Ztschr. Psychol. u. Physiol. d. Sinnesorg. **61**:225-246, 1930.

6. Tschermak, A.: Optischer Raumsinn, in Bethe, A.; von Bergmann, G.; Embden, G., and Ellinger, A.: Handbuch der normalen und pathologischen Physiologie, Berlin, Springer-Verlag, 1930, Vol. 12, pp. 834-1000.

7. Tschermak, A., and Hoefer, P.: Über binoculare Tiefenwahrnehmung auf Grund von Doppelbildern, Arch. ges. Physiol. **98**:299-321, 1903.

8. Aall, A.: Über den Massstab beim Tiefensehen in Doppelbildern, Ztschr. Psychol. u. Physiol. d. Sinnesorg. **49**:108-127; 161-205, 1908.

9. Burian, H.: Studien über zweiäugiges Tiefensehen bei örtlicher Abblendung, von Graefes Arch. Ophth. **136**:172-214, 1936.

that vertical threads could be adjusted with precision so as to appear on a frontoparallel plane 15 mm. in front of (or behind) the fixation point (observation distance, 30 cm.). In this case all threads appeared double. Calculation shows that this doubling corresponds to a disparity of the order of 36 minutes of arc.

So far as I can learn, however, no systematic data have been reported for those limiting values of disparities within which stereopsis exists. This information is important for one's understanding of the binocular visual processes. Rough measurements by early writers, such as Schoeler [10] and Schoen,[11] cannot be quantitatively interpreted. Prandtl [12] employed the test patterns of the "limiting case" of Panum,[13] in which one eye sees a single vertical line and the other eye sees two parallel vertical lines of the same length in the stereoscope. He reported that when the separation of the parallel lines exceeded 1 arc degree no stereoscopic depth effect could be observed. Eye movements were, of course, permitted. French [14] also used patterns of Panum, these consisting of two pairs of vertical parallel lines with different separations, one pair seen by each eye. With free stereoscopy, he reported that stereoscopic depth ceased when the ratio of the angular separation of the lines seen by one eye to that seen by the other was in the ratio of 1 ± 0.6. Thus, if the angular separation of the pair seen by the left eye was 1 arc degree, stereopsis ceased when the separation of the lines seen by the other eye was made 1.6 arc degrees, or when the disparity was about 0.4 arc degree, or 24 minutes of arc.

During the last war Koch [15] devised a stereoscopic vision test based on finding the limit of disparity at which stereoscopic depth failed. With the use of anaglyphs, his test figures were of the familiar pyramid viewed toward the apex end. The angular dimensions of the base of the pyramid were not given, but on the basis of his tests and those of Weissig,[16] who used the same apparatus, limiting disparities of more than 4 degrees were frequently found. Eye movements were, of course, permitted in the test. Such a pyramid is not a simple configuration from which one can learn the basic capacity of the eyes for stereoscopic depth.

Though no systematic study of the limiting disparities within which stereopsis exists has been made, even in those studies mentioned, and in many others dealing with the localization of double images, the experimental technique usually could not eliminate some empirical factors in spatial localization. As the test objects are moved behind or in front of the point of fixation, these empirical motives for depth localization would be a change in their angular size, a change in their brightness (and color), a blurring of their images, which also could act as a stimulus for a

10. Schoeler, H.: Grenzen der Correspondenz beider Sehfelder bei Betrachtung (*a*) linearer resp. flächenhafter, (*b*) körperlicher Objecte; Messung der Disparität an Schielenden und Entdeckung neues, bisher latenten Schielformen durch das Princip der stereoscopischen Paralaxe, von Graefes Arch. Ophth. **19**:1-55, 1873.

11. Schoen: Zur Lehre vom binocularen Sehen, von Graefes Arch. Ophth. **24**:27-130, 1878.

12. Prandtl, A.: Die spezifische Tiefenauffassung des Einzelauges und das Tiefensehen mit zwei Augen, Fortschr. d. Psychol. **4**:257-326, 1917.

13. Panum, P. L.: Physiologische Untersuchungen über das Sehen mit zwei Augen, Kiel, Schwers, 1858.

14. French, J. W.: Stereoscopy Re-Stated, Tr. Optic. Soc. (London) **24**:226-256, 1923.

15. Koch, E.: Ein neues Raumseh-Prüfgerät, Luftfahrtmedizin **5**:317-321, 1941.

16. Weissig, E.: Vergleichende Untersuchungen über das Raumsehvermögen mit dem Raumsehprüfgerät nach Koch und dem Zeiss-Stereoskop mit den Prüftafeln nach Pulfrich, Luftfahrtmedizin **6**:166-173, 1942.

change in accommodation, and the influence of surroundings. The hand stereoscope also involves artificial conditions, and even the texture of the paper on which figures would be drawn could provide inhibiting clues. The extent to which large disparities could be introduced was usually restricted also because of mechanical limitations. The empirical factors must be eliminated so far as possible if the basic capacity of the binocular visual processes for stereoscopic depth is to be found.

In this paper experimental equipment is described which eliminates effectively these empirical factors. The limiting disparities within which stereopsis exists and

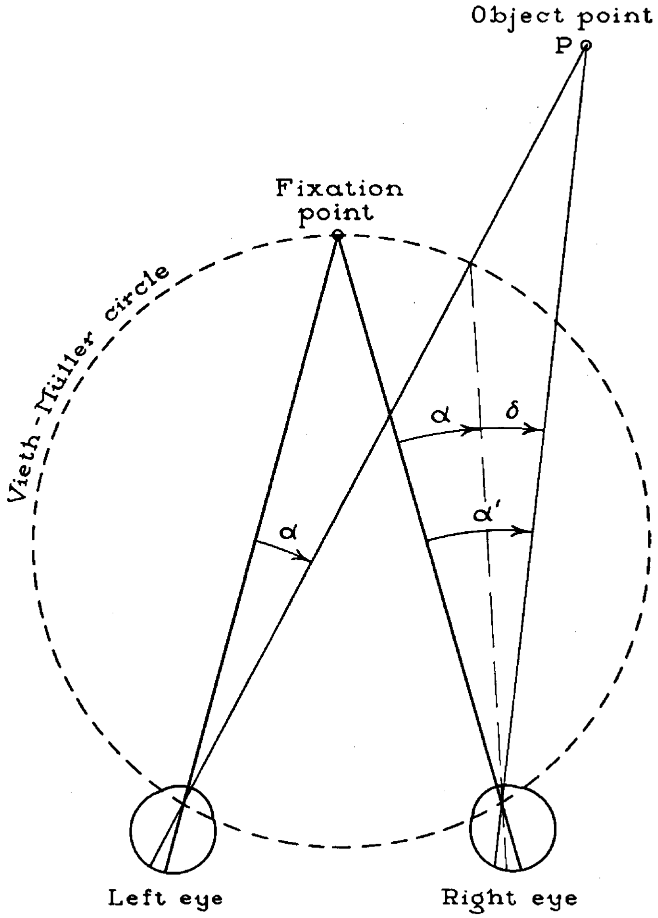

Fig. 1.—Relationships of the spatial positions of points in space and the corresponding visual angles to the geometric disparity.

within which a qualitative appearance of depth from double images exists are measured for central vision and for peripheral angles out to 8 degrees.

Figure 1 illustrates the relationships between the positions of two object points in space and the corresponding visual angles, and the geometric measure of the angular disparity between the images in the two eyes. The eyes fixate steadily a point, *F*. To one side, for example, to the right, an object point, *P*, lies beyond the fixation point. The visual angles for the spatial separation, *F-P*, to each of the two eyes are denoted by α and α'. The geometric disparity between the images in the two eyes is defined as the difference of these angles; $\delta = \alpha' - \alpha$. The physiologic disparity probably differs slightly from this, not exceeding 1.5% for a peripheral angle of 8 degrees. The error in using the geometric disparity instead of the physiologic disparity is negligible in most of the measurements obtained in this experiment. In order for the geometric disparity to be zero, the object point, *P*, must lie on the circle shown, for only then will $\alpha' = \alpha$.

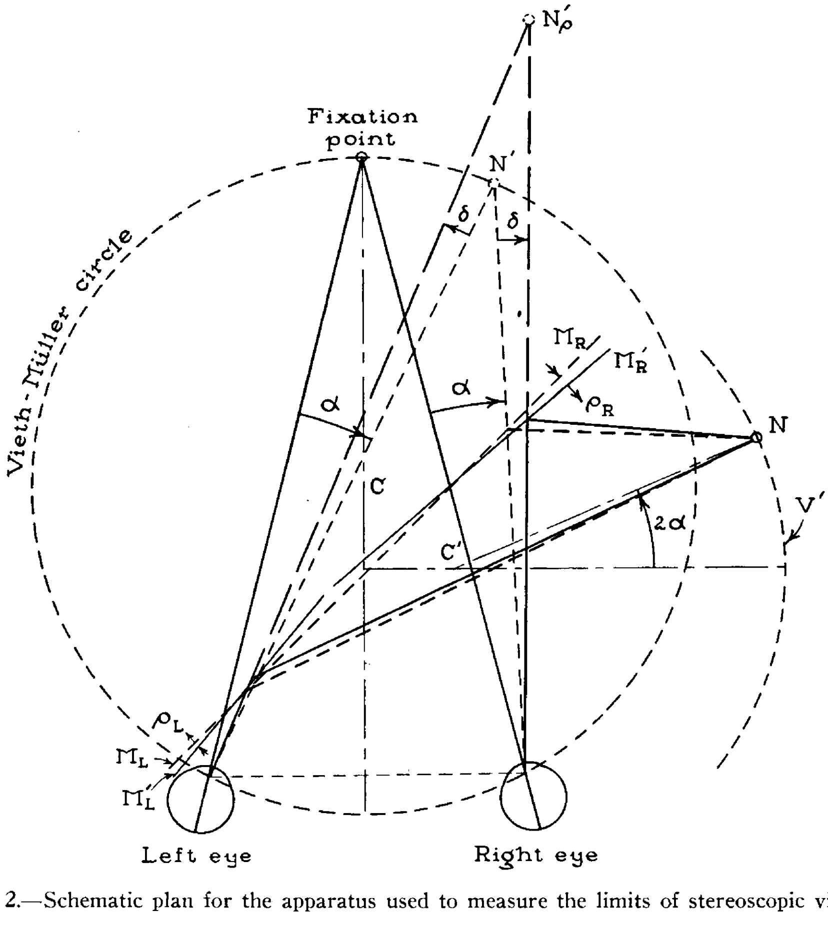

Fig. 2.—Schematic plan for the apparatus used to measure the limits of stereoscopic vision.

APPARATUS

Figure 2 illustrates schematically the apparatus used for obtaining the data to be reported here. The eyes, which converge to a suitable fixation point located 50 cm. from the eyes, see also the test object at *N* by reflection from specially treated half-aluminized (50% transmission; 50% reflection) mirrors at M_L and M_R. These mirrors are treated so that reflections from the second surface are eliminated. The mirrors are linked by suitable levers and a hand screw so that they can be turned and adjusted about vertical axes in directions opposite each other, the degree of turning being accurately indicated on a suitable dial. When the mirrors are turned from the initial positions M_L and M_R, through angles ρ_L and ρ_R to positions M_L' and M_R', a disparity (2δ) is introduced between the images of *N*, this disparity always being symmetrical with respect to the initial position of *N*′ on the Vieth-Müller circle. *N* is then seen as though these images originated at a point $N\rho'$. The position of the fixation point is virtually unchanged by the small rotations involved. Measurements at any desired lateral visual angle can be obtained by changing the angular position of N on the arm about C' (the image of the center of the Vieth-Müller circle). This apparatus provides, then, the possibility for a wide range of disparity, and within this range the retinal images of the test object do not change in brightness, in size, or in blur as the disparity is changed.

A very slender polished needle mounted vertically at N is used for the test object. This needle is appropriately illuminated by a small projector, with the result that the test object consists of a brilliant narrow line about 2 arc degrees in height. The fixation point is a small bright dot (0.25 mm. in diameter), mounted on a sheet of clear plate glass and independently illuminated by a small hidden projector. A fixation object similar in form to that of the test needle has to be avoided because of the possible intrusion of the Panum "limiting case phenomenon." Suitable horizontal slit apertures and backgrounds restrict the entire field of vision to the fixation point and the test needle. Head movements are avoided by a suitable chin cup and head rest. Empirical factors in spatial localization are thus practically eliminated. The tests were conducted in a semidarkened room, the walls of which were painted black.

The observation and study of stereoscopic depth could be made in three ways: 1. The mirrors could be continuously rotated by a motor and a special motor control [17] at any desired

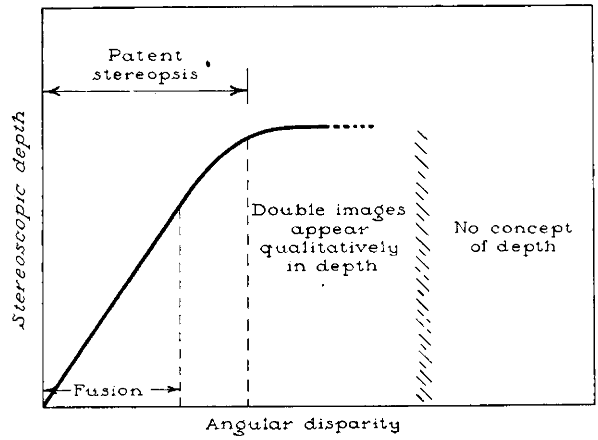

Fig. 3.—Graphic scheme for representing the phases of the stereoscopic experience as the disparity between the images in the two eyes is increased.

speed to introduce a continuous change of disparity in any one direction. The motor control could be disengaged so that the movements of the mirrors could be adjusted manually. 2. In the circuit for illuminating the needle, N, a tap key or a telechron® timing switch could be inserted. Thus, momentary illumination of the test object could be had for observing images adjusted manually or by motor control. 3. A suitable sliding-aperture arrangement could also be set up between N and the mirrors, so that the images of N could be exposed to one or the other eye only. This permitted the exposure of only one of the half-images if desired. Accordingly, with steady fixation the disparity between the images could be altered, during which time the subject could observe whether or not he experienced the stereoscopic depth. The predominant difficulty is, of course, to maintain constant fixation on the fixation point, and some training is essential. The muscle imbalance of the observer should be at a minimum.

RESULTS

Subjective Results.—The apparent behavior of the stereoscopic perception as the disparity is increased continuously in one direction can be divided into three ranges and can be conveniently described with reference to Figure 3.

17. A Servo-Tek motor control was used to insure immediate and positive braking.

1. From a position near the apparent frontoparallel plane, the test line appears to recede from (or approach) the observer, and the observer experiences a strong feeling of stereoscopic depth and of plasticity. He has the subjective impression that the depth difference increases proportionally as the disparity increases. The test line then appears rather suddenly to separate into two images as the limits of Panum's fusional areas are reached.

2. The stereoscopic sense of depth continues to increase, however—the plasticity continues—even though the images now appear slightly separated. The apparent increase in depth (between the fixation point and the test line) then rather suddenly stops; the strong sense of plasticity fades.

3. Then, as the separation of double images increases, the observer still has a definite subjective impression that both images are farther (or nearer) for quite a range of disparity. Then, rather abruptly, even this impression of farther (or nearer) than the fixation point ceases, and the two half-images seem to be indefinitely localized.

Quantitative Data.—The measurement of these three limits, which was the goal of this study, was obtained mostly by the method of adjustment, though some data were obtained by the method of constant stimuli. The data (which must be considered preliminary) were taken under three conditions:

1. The stereoscopic behavior of the test object was observed for a continuous change in disparity as introduced by the motor control, and the movement stopped abruptly the instant the limit of any one of the ranges described in the previous section was judged to have been reached.

2. The test line was momentarily exposed, after an adjustment of the disparity either by motor control or by manual operation.

3. One of the half-images was momentarily exposed after being occluded by suitable sliding apertures, after an adjustment of the disparity either by motor control or by manual operation.

The boundaries of the several ranges described are, of course, not sharply defined, and under a given set of conditions a probability function of experiencing or not experiencing depth perception exists. Furthermore, the limits vary somewhat with conditions of the illumination, as well as the method used. However, these variations appeared to be small, at least for the limits of fusion and for the limits at which all impression of depth of the double images ceased. The mean variation of adjustments increased with peripheral angle. For the patent stereopsis and simultaneous exposure of both half-images, this was about 1 minute of arc, at the macula, to 5 minutes of arc, at a peripheral angle of 6 degrees. For the qualitative depth perception of double images, and momentary exposure of one half-image, this mean variation was 4 to 25 minutes of arc for the same peripheral angles.

Failure to keep exact fixation during a determination of a given limit probably does not affect the measurements as much as a direct function of the eye movements themselves, because such movements involve a slight change in convergence and a corresponding increase in the apparent range of stereoscopic depth. The instantaneous exposure of either both half-images or one half-image tends to eliminate this problem. Even then, it must be borne in mind that the detail of the fixation spot for fusion is small as compared with that of the test line and a disparity of the images of the test line could give rise to a compulsion innervation for the eyes to make a fusional movement to eliminate that disparity.

When only one of the half-images is exposed momentarily a suppression of the unoccluded one sometimes occurs, and this interferes with the observer's judgment of depth. This suppression seemed to occur oftener when the disparity was near the limits of a patent stereoscopic sense of depth.

The data for one observer for the limits of the two ranges of stereoscopic depth perception are illustrated in Figure 4, showing the change with lateral (peripheral) visual angle. The lowest series of points indicates the limits of disparity for fusion or where double images first appear. The next two sets are for the limits of a patent stereoscopic sense, the lower one obtained by exposing momentarily one of the half-images and the upper one by exposing momentarily both half-images. The upper

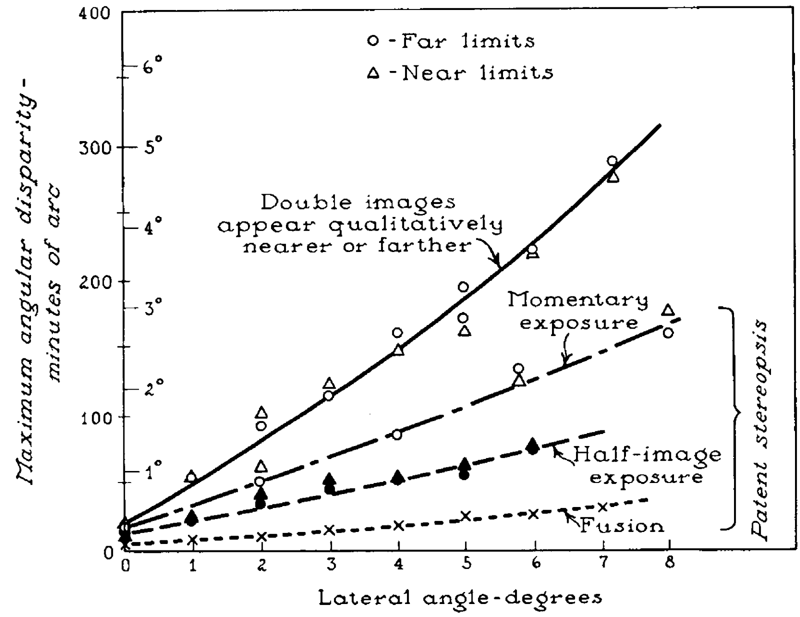

Fig. 4.—Graphic representation of the data obtained for the limits of disparity corresponding to fusion, patent stereoscopic perception of depth, and the qualitative appearance of depth of double images, as well as the change with peripheral angle (author's data).

set of data gives the limits at which an impression of depth for the two double images ceases. The last data then indicate the limits of disparity within which an interaction occurs in the visual processes for stimuli that arise in the two eyes.

These limits also vary a little with experimental conditions, especially time of exposure, adaptation, and fatigue, and with the individual subject. With continued observation of the double images, the plastic or stereoscopic perception of depth may fade, a fact frequently reported in the literature. Care must also be taken to avoid the influence of after-images. The observer frequently looks away from the fixation points when the test images are not exposed.

Inspection of Figure 4 shows the following facts:

1. At the maculae, the range of disparities within which there is a definite (patent) stereoscopic perception of depth amounts to about ± 15 minutes of arc, and

a qualitative appearance of depth of the two half-images occurs within an additional disparity of 5 minutes of arc. The double images in this case occur on each side of the point of fixation, and the cortical representations of those images would therefore be presumed to be on the opposite lobes of the occipital cortex.

2. The momentary exposure of both the half-images yields a larger range of disparities within which a patent stereopsis is experienced than does the brief

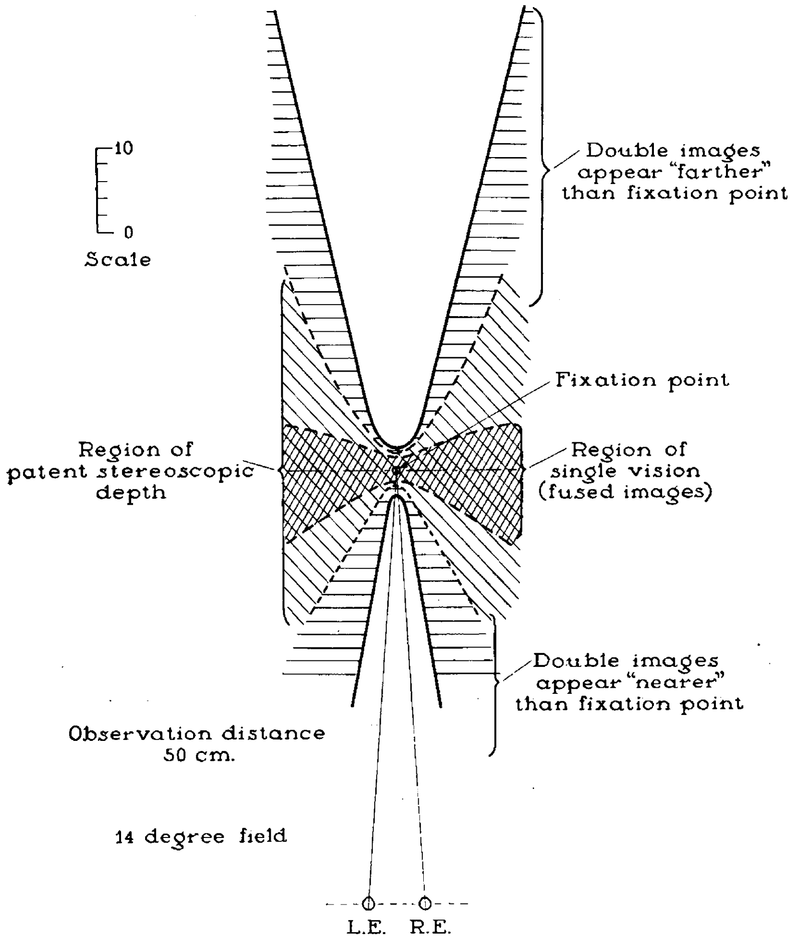

Fig. 5.—Spatial regions corresponding to the several limits of stereoscopic depth perception calculated from the data illustrated in Figure 4.

exposure of one of the half-images. It is highly possible that this difference rests in the fact that with momentary exposure of both images there is a subjective identification of these as belonging to the same external object. On exposure of the one half-image, this identity would not be so evident.

3. The range of limiting disparities increases rapidly with lateral angle in the periphery, but the range within which there is the weak qualitative impression of depth of the two half-images increases the more rapidly.

4. The limits of disparity within which fusion exists (Panum's areas) are of the same order as those previously found.[18]

In that part of the experiment in which one of the half-images was occluded and then momentarily exposed, it was interesting to note that the unoccluded half-image would be indefinitely localized, or could be localized at will. It could be made to appear at the same distance as the fixation point, or even to drift nearer than that point; but when the second half-image was exposed (and with an uncrossed disparity), the first half-image would appear to jump back of the fixation point, as would its fellow, the exposed half-image.

The equivalent spatial regions corresponding to the limits of disparity shown in Figure 4 can be obtained by calculation. These spatial limits are illustrated in Figure 5. One can easily identify the spatial region of binocular single vision, the spatial region of a patent stereoscopic vision, and, finally, the spatial region within which there exists a qualitative sense that both the double images are farther or nearer than the fixation point. The limits of a patent stereopsis represent a mean of the two sets of data for these limits of disparity, shown in Figure 4. The two blank wedge-shaped regions beyond and nearer than the fixation point, symmetrical with respect to the median plane of the observer, are regions where no stereoscopic depth perception is experienced.

If these disparity limits are constant for all observation distances, the corresponding spatial regions will increase in size greatly as the observation distance increases—in fact, nearly in proportion to the square of that distance. On the basis of these data, if the eyes were fixated on a very distant object, then at a peripheral angle of 6 degrees the range of patent stereopsis would be roughly from infinity to a distance of about 2 meters from the observer.

COMMENT

The fact that a defined range of disparities exists within which the perception of stereoscopic depth occurs is evidence of neuroanatomic limitations and suggests further that the processes involved may be basically physiologic. If the phenomenon of stereopsis were entirely psychologic, no such limitations in the disparities would be expected. These neuroanatomic ranges could be accounted for by the extent to which the neural paths that arise at the retinas of the two eyes are multiplied, and specifically by the extent to which these multiplied fibers overlap at the occipital cortex or at some other region of the brain. The rapid increase in the limiting disparities with peripheral angle could not be accounted for on the basis of the decrease in visual acuity in the periphery or the extent to which an increasing number of receptor elements have single neurons, but could be explained by the pattern of the overlapping of the terminal fibers at the cortex. The site of such neuroanatomic overlapping probably cannot be, at least wholly, in the occipital areas of the cortex, because the disparity limits for stereopsis exist to the extent of 20 minutes of arc or more at the maculae of the two eyes, and hence each of the half-images would then have to be represented on opposite halves of that cortex. The data here would imply also a different order of overlapping for the two ranges of stereoscopic

18. Ogle, K. N.: Researches in Binocular Vision, Philadelphia, W. B. Saunders Company, 1950.

perception. In ordinary surroundings, of course, empirical factors to spatial localization may easily be (and probably are) dominant so far as a depth perception from double images is concerned. Furthermore, to reduce the confusion of so many double images in ordinary surroundings, some suppression must occur. Whether a stimulus for stereoscopic vision can exist when one of the half-images is apparently suppressed remains as yet an open question.

The existence of these limits and of the empirical factors usually present helps to account for the diversity of results of experiments conducted by so many research workers [19] that have dealt with the problem of where double images are localized.

Of interest, also, is the fact that the range of a patent stereopsis lies outside the region of binocular single vision—where fusion of the disparate images occurs. One can raise the question whether stereoscopic vision is a separate physiologic function, independent of the fusional processes. The alternative is the suggestion, often made, that the stereoscopic sense of depth arises from a sensory component of the innervations sent to the extrinsic muscles of the two eyes. These innervations arise in an effort to cause the eyes to make a fusional movement under the fusion compulsion reflex to eliminate double (disparate) images. Such actual movements cannot be made because of the attention to, and the fusion of the images of, the point of fixation. While subjects are frequently found for whom fusion and prism vergences exist but for whom no stereopsis can be demonstrated, there is yet to be reported one who is without fusion or prism vergences but in whom true [20] stereopsis can be found. Presently used tests to detect stereopsis would be generally inadequate, however, and more sensitive means, such as the "leaf room" test, would preferably have to be employed.

The results of this particular experiment cannot be construed to imply that there is an accurate stereoscopic spatial localization within the limits delineated. Rather, they indicate that there is a type of stereoscopic perception of depth within these limits. The stereoscopic localization undoubtedly is increasingly accurate near the apparent frontoparallel plane through the fixation point. Any general theory of binocular spatial localization must take into account the existence of these limiting disparities and the spatial regions corresponding to them, outside of which stereoscopic depth perception is not experienced.

SUMMARY

A survey of the literature shows that no systematic study has ever been made of the limiting disparities between the images in the two eyes, and the corresponding regions of space within which stereoscopic depth perception exists. An apparatus is described in this paper which permits one to determine these limiting disparities without its also introducing empirical factors for spatial localization. Experimental data obtained from two observers show that there are three ranges of disparity: In the first, stereopsis occurs with single (fused) images; in the second, stereopsis exists with double images, and, finally, there is a range of disparities within

19. Trendelenburg, W., and Drescher, K.: Über die Grenzen der beidäugigen Tiefenwahrnehmung und Doppelbildwahrnehmung, Ztschr. Biol. **84**:427-435, 1926.

20. This category excludes those squinters who, by closing one eye and then the other, note the change in parallax and thus arrive at a concept of depth differences.

which a qualitative sense of depth of the two double images occurs. For greater disparities, the images are indefinitely localized. The limiting disparity for a patent stereoscopic experience of depth at the foveae is about 20 minutes of arc, and this increases rapidly to nearly 90 minutes of arc for a peripheral angle of 6 arc degrees. The qualitative sense of a difference in distance of the double images from the point of fixation ceases for disparities of about 25 minutes of arc at the foveae and of nearly 4 arc degrees at a peripheral angle of 6 arc degrees. The existence of these limiting disparities has implications for one's understanding of the neuroanatomical organization of the two visual systems, the fusional processes, and the origin of stereopsis, and for any general theory of binocular spatial localization.