Assignment on stability of Algorithms:

"Bahare - Zare - COMP 6906 — Assignment 3

Consider the following algorithm for the Computation of Cos(x), which is derived from the Tylor's expansion of the function cos.

$$\cos(x) = 1 - \frac{x^2}{2!} - \frac{x^4}{4!} - \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \quad (1)$$

the Code is attached.

① MATLAB Function: $\cos(\frac{\pi}{4}(x-1))$
- in single precision = 0,7995
- in double precision = 0,799541

② why are your results inaccurate? why is the double precision result more accurate. provide numerical evidence for your arguments. the double precision floating point requires a 64 bit word, which may be represented as numbered from 0 to 63, left to right. in the double precision the rounding of numbers are more accurate and by adding substracting and multiplying the numbers can be more accurate, but still there is an error between the numbers and numbers that stored in a computer. and as we see in the text book, numbers in a computer are stored with some errors

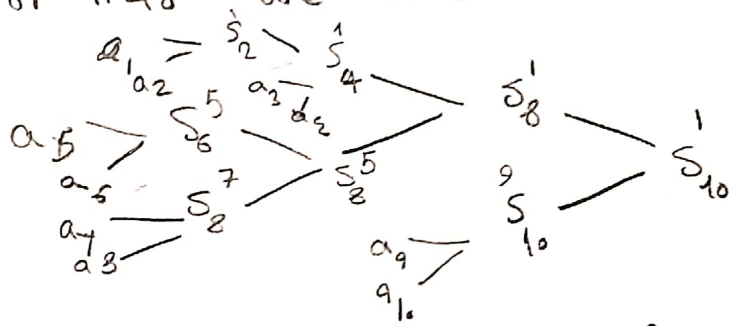
$$fl\left(\sum_{i=1}^{n} \alpha_i\right) = fl(\dots fl(fl(\alpha_1 + \alpha_2) + \alpha_3) + \dots + \alpha_n)$$

(3) Based on this evidence find a way to Compute $\cos(\frac{\pi}{2}(n+1))$ for $x = f_n$ in single precision, accurately.

an efficient way to Compute S is by using a binary tree as follows.

suppose $\alpha_i < \alpha_{i+1}$ for all i, $S_j^i = \sum_{k=i}^{j} \alpha_k$ For example

For $n=10$ we have



So the Max number of addition that a particular term is participitating is $\lceil \log_2^n \rceil$

in this way the absolute error satisfies

$$|f l(s) - s| \leq \lceil \log_2^n \rceil (|\alpha_1 \eta_1| + |\alpha_2 \eta_2| + \cdots + |\alpha_n \eta_n|)$$

So adding the terms in increasing order still reduces the error. the usefulness of algorithm is abvious when n is very large.