Assignment on Stability of Algorithms

Consider the following algorithm for the computation of $\cos(x)$, which is derived from the *Taylor's* expansion of the function cos.

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \tag{1}$$

1. Write a MATLAB function that computes $\cos\left(\frac{\pi}{4}(x-1)\right)$ in *single* and *double precision* using (1) for $x = 48$. The correct value is 0.707106781186548.

2. Why are your results inaccurate? Why is the double precision result more accurate. Provide *numerical evidence* for your arguments.

3. Based on this evidence find a way to compute $\cos\left(\frac{\pi}{4}(x-1)\right)$ for $x = 48$ in single precision, accurately.

When you implement (1) you must consider the following:

1. Since (1) is an infinite series, your algorithm theoretically should work forever. Stop when the sum of say the 50 first terms has been computed, or if two consecutive *partial sums* agree to 5 *significant* digits.

2. Do not use brute force in order to compute the individual terms of (1), **find an efficient way to implement** (1).