4) We need to show i (inclusion map) is a homotopy equivalence i.e $\exists$ an inverse $g : X \to A$ for which $ig \simeq \mathbb{1}_X$ and $gi \simeq \mathbb{1}_A$ (~~Note~~) Since $f_1(X) \subset A$, consider $\cancel{f_1(X)}$ $f_1 : X \to A$.

Now take $g(x) = f_1(x)$ $\forall x \in X$. Take $F : X \times [0,1]$. $F(x,0) = f_0(x) = \mathbb{1}_X$,

$F(x,1) = g(x)$, but $i \circ g = g$. So,

$F(x,1) = \cancel{g} f_1(x) = g(x) = i \circ g$. Hence

$i \circ g \simeq \mathbb{1}_X$

$G : A \times [0,1] \to A$

$G(a,t) = f_t(a)$

$G(a,0) = f_0(a) = \mathbb{1}_X$

NOTES

Record the list of things you might want to do.

But a is restricted to A, Hence $G(a,0) = f_0(a) = \mathbb{1}_A$

Similarly $G(a,1) = f_1(a) = g(a) = g \circ i$

Hence $\mathbb{1}_A \simeq g \circ i$

$\Rightarrow$ i is a homotopy equivalence.