LECTURE 2

08 THU JAN

008-357 • WEEK 02

1	8	15	22	29	M	D
2	9	16	23	30	T	E
3	10	17	24	31	W	C
4	11	18	25		T	2
5	12	19	26		F	0
6	13	20	27		S	1
7	14	21	28		S	4

Overview of Algebraic Topology

An Abstract simplicial Complexes Δ is a collection of non-empty subsets such that it is closed under subset operation.

i.e if $A \subset \Delta$, & B is an non-empty subset of A, then $B \in \Delta$

Example $\Delta = \{ \{a,b,c\}, \{a,b\}, \{a\}, \{b\}, \{c\}, \{b,c\}, \{a,b\}, \{c,a\} \}$

Hypergraph: A hypergraph is a pair (V,E) where V denotes the vertex set & E is a collection of non-empty subsets of V called hyperedges

~~Hence~~, hypergraph, edge set is more than size 2. So, $E = \{\{1,2,3\}\}$, but for graph it is modeled as size 2 i.e $\{1,2\}, \{1,3\}$ etc.

Hence a ~~hypergraph~~ simplicial complex is a hypergraph. As a hypergraph, need not imposed closed under subset.

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Examples:

① Let $V = \{1, 2, \dots, n\}$

& $\Delta := 2^{V} \setminus \emptyset$

$= \{\{1\}, \{2\}, \dots, \{1,2\}, \dots \dots \{1,2,\dots,n\}\}$

Δ is closed under subset operation

Dimension of a face $\sigma \in \Delta$

(elements in Δ as faces)

$\dim(\sigma) = |\sigma| - 1$

0-dimensional faces are set of vertices

1-dim faces are 2 element edges

2-dim .. are triangles

⋮

Higher dim are called simplices

Dimension of simplicial complex is the maximum among all dimension of faces in it.

So, for $\Delta = \{1, 2, 3, 12, 13, 23, 123\}$

$\dim(\Delta) = |123| - 1 = 3 - 1 = 2$

A simplicial complex of at most 1 is a graph

For $k \geq -1$, the k-th skeleton $\Delta^{(k)}$ of Δ is

the subcomplex obtained by removing all

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faces of dimension greater than k

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So, 1-skeleton of a simplicial complex is a graph

Focus on simplicial complexes with finite vertices

$\boxed{\text{Geometric Realization of Simplicial Complex}}$

Formally, a geometric realization is a way to associate a topological space to a simplicial complex

* Some Basic definitions of Topology.

Now, One formal way to define a geometric realization of a given complex Δ is as follows

Suppose V is the vertex set of a Δ.

Let $f : V \longrightarrow \mathbb{R}^n$

$$v \longrightarrow \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

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For every d-dimensional face (i.e d-face $\sigma : \{v_0, \dots, v_d\}$ of Δ, f induces a map

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$f_\sigma : X^d \longmapsto \mathbb{R}^n$
given by

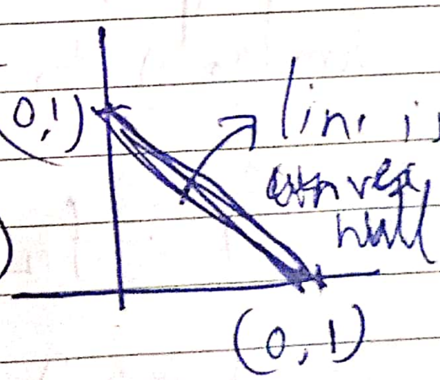
$f_\sigma(\lambda_0, \dots, \lambda_d)$

$:= \lambda_0 f(v_0) + \dots + \lambda_d f(v_d)$

where X^d is the standard d-simplex or the convex hull of points $(1,0,\dots,0)$, $(0,1,0\dots0)$ ..... $(0,0,\dots,1)$

X^0 is the convex hull of (1) in $\mathbb{R}$

X^1 is $(1,0)$ $(0,1)$ in $\mathbb{R}^2$



X^2 is $(1,0,0)$ $(0,1,0)$, $(0,0,1)$