LECTURE 4

Group homomorphism: Let $(G, *)$ & $(H, \cdot)$ be two abelian groups, then:

$$h : G \longrightarrow H$$

is a group homomorphism if $\forall a, b \in G$

$$h(a * b) = h(a) \cdot h(b)$$

Isomorphism is a bijective homomorphism

Field:

Let $(F, +, \cdot)$ & $(F, \oplus, \odot)$ be two fields Then.

$h : F_1 \longmapsto F_2$ is an homomorphism

$h(x+y) = h(x) \oplus h(y)$

$h(x \cdot y) = h(x) \odot h(y)$

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It is said to be isomorphism if it is also bijective.

Same as Vector space. Let $(V, F, +, \cdot)$ & $(V_2, F, \oplus, \odot)$ be two vector spaces.

Then, $h: V_1 \longmapsto V_2$ is said to be vector spaces homomorphism if

$$h(v_1 + v_2) = h(v_1) \oplus h(v_2)$$

$$h(\alpha \cdot v_1) = \alpha \odot h(v_1)$$

Example: Let C^k be vector space of all polynom of degree at most k.

Also, let $D: C^n \longrightarrow C^{n-1}$ be given by

$$D(a_0 + a_1 x + \dots + a_n x^n)$$

$$= a_1 + a_2 x + \dots + a_n x^{n-1}$$

Then D is a vector space homomorphism

Homomorphism between Simplicial Complexes

Let K & Δ be two simplicial Complexes

Let $V(K)$ & $V(\Delta)$ be vertex sets

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Then $h: V(K) \longmapsto V(\Delta)$ is a homomorphism of $\mathcal{S}$ if

$\{v_0, \ldots, v_d\} \in K$ i.e. a ~~frame~~ any implies

$\{h(v_0), \ldots, h(v_d)\} \in \Delta$

If bijective, then isomorphic

Subgroup: Given a group $(G, *)$ a subset H of G is said to be a subgroup if $(H, *)$ is also a group.

H is a subgroup iff ① H is non-empty, ② It contains the inverse for any $a \in H$, and it contains the identity & also closed in H

Claim: Let V & W be two vector spaces. Suppose $T: V \to W$ is a v.s isomorphism

Then $T^{-1}: W \longmapsto V$ is also a vector space isomorphism.

Proof: T^{-1} is a bijection.

So, if $w_1, w_2 \in W$ & $\alpha_1, \alpha_2 \in F$, then

$T^{-1}(\alpha_1 \odot w_1 \oplus \alpha_2 \odot w_2) = \alpha_1 \cdot T^{-1}(w_1) + \alpha_2 \cdot T^{-1}(w_2)$

$\Rightarrow$ To show.

For isomorphism: $h: V(K) \longmapsto V(\Delta)$

① $\{v_0, \ldots, v_d\} \in K \Leftrightarrow \{h(v_0), \ldots, h(v_d)\} \in \Delta$

② h is a bijection

Direct sum of groups: Let $(G,+)$ & $(H,\cdot)$ be two groups, their direct sum is the group $(G \oplus H, *)$, where

$$G \oplus H := \{(g,h) : g \in G, h \in H\}$$ &

$$(g_1,h_1) * (g_2,h_2) = (g_1+g_2, h_1 \cdot h_2)$$

In the same spirit, Let $V_1, V_2, \ldots, V_k$ be F-vector spaces, Then,

$$V = V_1 \oplus V_2 \oplus \cdots \oplus V_k$$ is another F-vector space. Here, $V = \{(v_1,\ldots,v_k) : v_i \in V_i\}$

Further, $(v_1,\ldots,v_k) + (w_1,\ldots,w_k)$

$$:= (v_1+w_1, v_2+w_2, \ldots, v_k+w_k)$$

& $\alpha(v_1,\ldots,v_k) := (\alpha v_1,\ldots,\alpha v_k)$

So, $\mathbb{R}^2 = \mathbb{R}^1 \oplus \mathbb{R}^1$